Prądy Zmienne.

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Wydz. Elektryczny. Sem. V i VI.
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Badanie odkształceń napięć i prądów. Badanie okresów nieustalonych.

„sinusoida" ta ma wyższe harmoniczne o różnych częstotliwościach i amplitudach. Jest to t.zw. krzywa periodyczna.

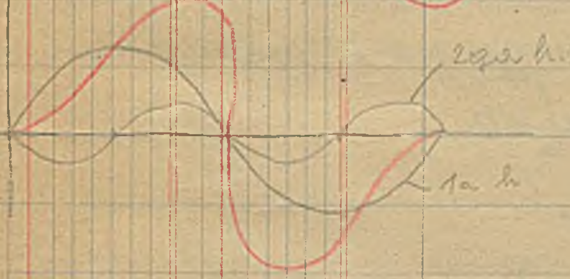
Wyższe harmoniczne.

Mają częstotliwość jako wielokrotność częstotliwości pierwszej sinusoidy.

1).

Każda harmoniczna jest czystą sinusoidą. Druga harmoniczna ma częstotliwość dwa razy większą. Tworzymy wypadkową.

2).



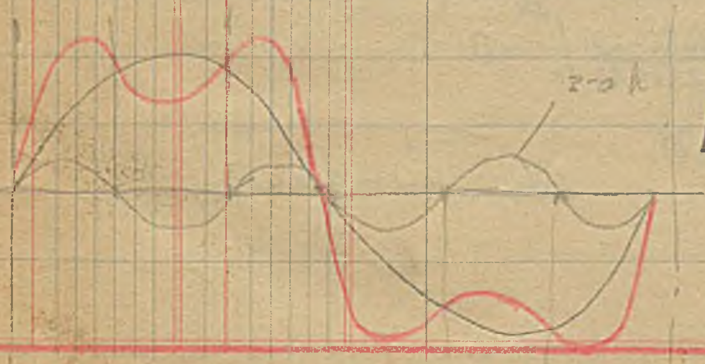
Krzywe periodyczne zwiemy odkształc-
onymi. Jak przedstawić matematycznie krzywe odkształcone.

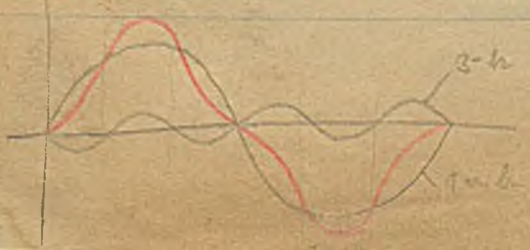
$$a_y = f(\omega t) = f\left(\frac{2\pi}{T} t\right) = f\left[\frac{2\pi}{T}(t + k\,T)\right]$$

$$= f\left(\frac{2\pi}{T} t + 2\pi k\right) = f(\omega t + 2\pi k)$$

$\boxed{f(t) = f(x + 2\pi k)}$ warunek periodyczności

3).





Jeżeli warunek periodyczności jest spełniony to taką krzywą możemy rozłożyć na szereg wielu sinusoid (wyższych harmonicznych) czyli szereg Fouriera.

$$f(x)=A_0+A_1\cos x+A_2\cos 2x+A_3\cos 3x+\ldots\ldots+A_k\cos k_x+\ldots+A_{k+1}\cos(k+1)x+\ldots\ldots+$$

$$+A_n\cos nx+\ldots\ldots+B_1\sin x+B_2\sin 2x+B_3\sin 3x+\ldots\ldots+B_k\sin kx+B_{k+1}\sin(k+1)x+\ldots+B_n\sin nx+$$

$$f(x)=A_0+\sum_{k=1}^{k=n}A_k\cos kx+\sum_{k=1}^{k=n}B_k\sin kx$$

$$f(x)=A_0+\sum_{k=1}^{k=n}D_x\sin(kx+\alpha_k)\quad;\quad D_k=\sqrt{A_k^2+B_k^2}\qquad \operatorname{tg}\alpha_k=\frac{A_k}{B_k}$$

mnożymy obustronnie przez dx i całkujemy od 0 do 2π.

$$\int_0^{2\pi}f(x)dx=\int_0^{2\pi}A_0dx+\ldots\ldots+\int_0^{2\pi}\underbrace{A_k\cos kx\,dx}_{0}+\ldots\ldots+\int_0^{2\pi}B_k\underbrace{\sin kx\,dx}_{0}+\ldots\ldots$$

$$\boxed{A_0=\frac{1}{2\pi}\int_0^{2\pi}f(x)dx}$$ można określić A_0

mnożymy przez cos kx dx i całkujemy od 0 do 2π.

$$\int_0^{2\pi}f(x)\cos kx\,dx=\int_0^{2\pi}A_0\underbrace{\cos kx\,dx}_{\substack{=0\\ \text{[illegible] całkowity}}}+\ldots\ldots+\int_0^{2\pi}A_i\underbrace{\cos ix\cos kx\,dx}_{=0}+\ldots+\int_0^{2\pi}A_k\cos^2kx\,dx+\ldots$$

$$\ldots+\int_0^{2\pi}A_n\underbrace{\cos nx\cos kx\,dx}_{=0}+\ldots\ldots+\int_0^{2\pi}B_i\underbrace{\sin ix\cos kx}_{=0}dx+\ldots\ldots+\int_0^{2\pi}B_k\underbrace{\sin kx\cos kx}_{=0}dx+\ldots$$

$$\ldots+\int_0^{2\pi}B_n\underbrace{\sin nx\cos kx}_{=0}dx+\ldots\ldots$$

Typy całek.

1). $\int_0^{2\pi}\cos ix\cos kx\,dx=\frac{\sin(i-k)x}{2(i-k)}\Big|_0^{2\pi}+\frac{\sin(i+k)x}{2(i+k)}\Big|_0^{2\pi}=0$

2). $\int_0^{2\pi}\cos^2kx\,dx=\int_0^{2\pi}\frac{1}{2}dx+\int_0^{2\pi}\frac{1}{2}\underbrace{\cos 2kx}_{=0}dx=\pi$

3). $\int_0^{2\pi}\sin ix\cos kx\,dx=-\frac{\cos(i-k)x}{2(i-k)}\Big|_0^{2\pi}-\frac{\cos(i-k)x}{2(i+k)}\Big|_0^{2\pi}=0$

4). $\int_0^{2\pi}\sin kx\cos kx\,dx=\frac{1}{2}\int_0^{2\pi}\sin 2kx\,dx=0$

$\sin\alpha\cos\alpha=\frac{1}{2}\sin 2\alpha$

$$\int_0^{2\pi}f(x)\cos kx\,dx=A_k\pi$$

$$\boxed{A_k=\frac{1}{\pi}\int_0^{2\pi}f(x)\cos kx\,dx}$$

A_k określone

mnożymy przez sin kx dx i całkujemy od 0 - 2π.

$$\int_0^{2\pi} f(x)\sin kx\,dx=\int_0^{2\pi} A_0\underbrace{\sin kx\,dx}_{=0}+\dots+\int_0^{2\pi} A_i\underbrace{\cos ix\sin kx}_{=0}dx+\dots+\int_0^{2\pi} A_k\underbrace{\cos kx\sin kx}_{=0}dx+\dots$$

$$\dots+\int_0^{2\pi} A_n\underbrace{\cos nx\sin kx\,dx}_{=0}+\dots+\int_0^{2\pi} B_i\underbrace{\sin ix\sin kx}_{=0}dx+\dots+\int_0^{2\pi} B_k\sin^2 kx\,dx+\dots+\int_0^{2\pi} B_n\underbrace{\sin nx\sin kx}_{0}dx$$

5). $\int_0^{2\pi}\sin ix\sin kx\,dx=\frac{\sin(i-k)x}{2(i-k)}\Big|_0^{2\pi}-\frac{\sin(i+k)x}{2(i+k)}\Big|_0^{2\pi}=0.$

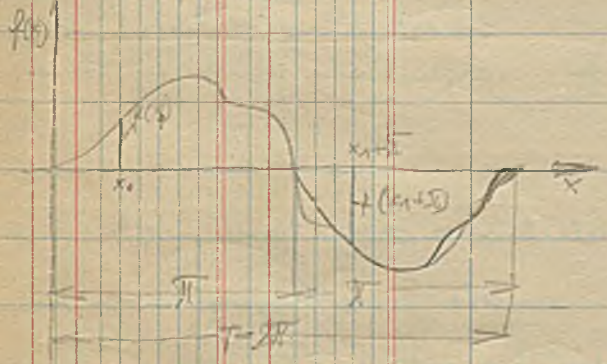
6). $\int_0^{2\pi}\sin^2 kx\,dx=\int_0^{2\pi}\frac{1}{2}dx-\int_0^{2\pi}\underbrace{\frac{1}{2}\cos 2kx\,dx}_{=0}=\pi$

$$\boxed{B_k=\frac{1}{\pi}\int_0^{2\pi} f(x)\sin kx\,dx}$$ B_k określone

Jeżeli: $f(x)=f(x+2\pi k)$ to: $f(x)=A_0+\sum_{k=1}^{k=n}A_k\cos kx+\sum_{k=1}^{k=n}B_k\sin kx$

$$\boxed{A_0=\frac{1}{2\pi}\int_0^{2\pi} f(x)dx}\qquad\boxed{A_k=\frac{1}{\pi}\int_0^{2\pi} f(x)\cos kx\,dx}\qquad\boxed{B_k=\frac{1}{\pi}\int_0^{2\pi} f(x)\sin kx\,dx}$$

Czy można istnieć krzywa odkształcona symetryczna wzgl. osi x-ów.



Krzywa symetryczna wzgl. osi x.

Jest periodyczna, czyli spełnia równanie $f(x)=f[x+2\pi k]$

$f(x)=-f(x+\pi)$

$f(x)=-f[x+(2k+1)\pi]$

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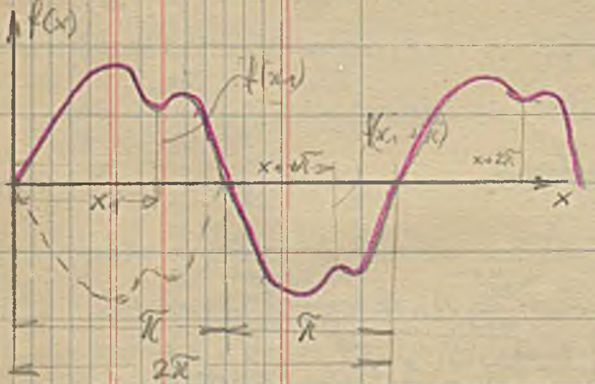
$$f(x)=f(x+2\pi k)$$

$$f(x)=A_0+\sum_{k=1}^{k=n}A_k\cos kx+\sum_{k=1}^{k=n}B_k\sin kx$$

$$A_0=\frac{1}{2\pi}\int^{2\pi}f(x)\,dx\ ;\quad A_m=\frac{1}{\pi}\int_0^{2\pi}f(x)\cos kx\,dx\ ;\quad B_n=\frac{1}{\pi}\int_0^{2\pi}f(x)\sin kx\,dx$$

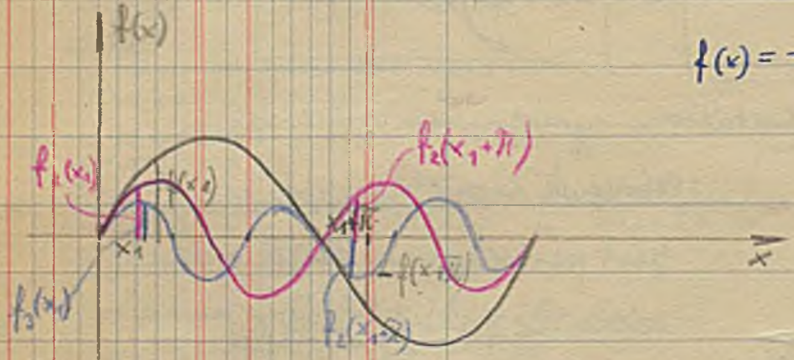
Krzywa symetryczna wzgl. osi x.



$f(x)=-f(x+\pi)$

$f(x)=f(x+2\pi k)$ warunek identyczno

$f(x)=-f[x+(2k-1)\pi]$ - - - symetryczności wzgl.

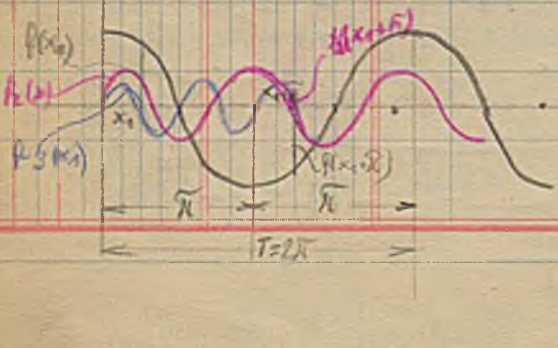


$f(x)=-f[x+(2k-1)\pi]$ warunek pierwszej harmonicznej nie spełnia się dla drugiej harmonicz

trzecia krzywa harmoniczna odpowiada równaniu 1 k. h

Harmoniczne cosinusowe.

Jeżeli krzywa jest sym. wzgl. osi x to nie może mieć krzywych wyższych rzędów parzystych

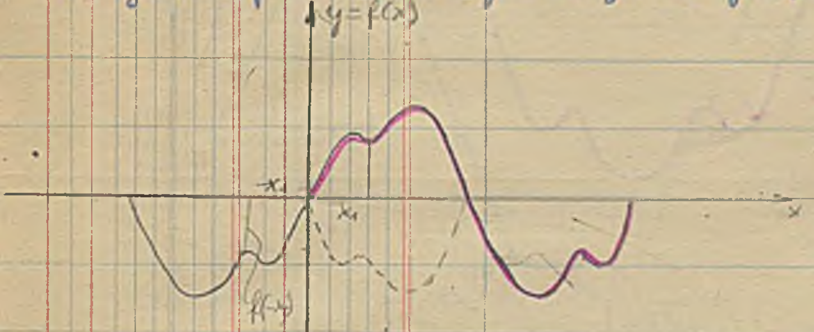


$$A_0=0\ ;\quad A_{2k-1}=\frac{1}{\pi}\int_0^{2\pi} f(x)\cos(2k-1)x\,dx$$

$$A_{2k-1}=\frac{2}{\pi}\int_0^{\pi} f(x)\cos(2k-1)x\,dx\quad ;\ B_{2k-1}=\frac{2}{\pi}\int_0^{\pi} f(x)\sin(2k-1)x\,dx$$

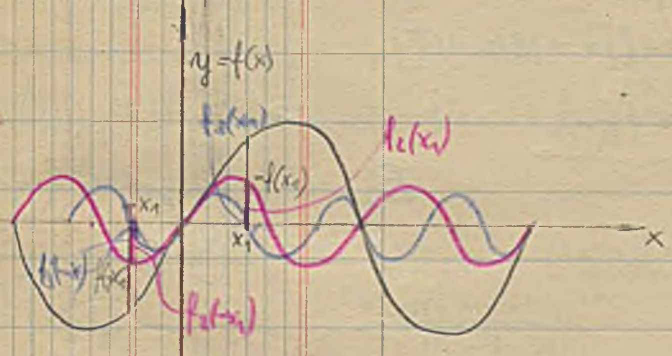
$$f(x)=\sum_{k=1}^{k=n} A_{2k-1}\cos(2k-1)x+\sum_{k=1}^{k=n} B_{2k-1}\sin(2k-1)x$$

Krzywa sym. odkąt. sym. wzgl. osi y i pocz. układu.



$f(x)=-f(-x)$

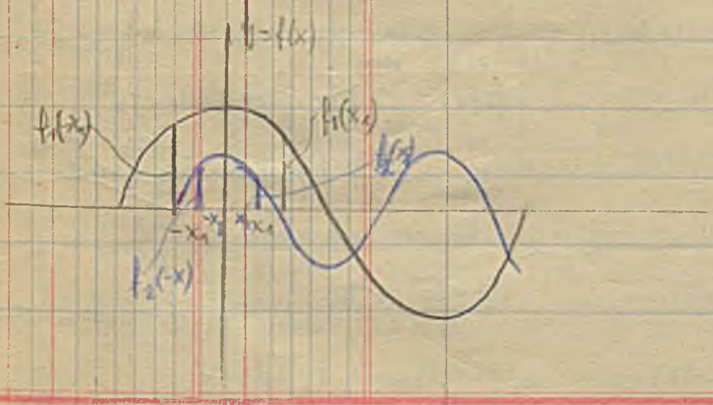
$f(x)=f(x+2\pi k)$



$\cos\alpha=\cos(-\alpha)$

$\sin x=-\sin(-x)$

nie może być żadnych harmonicznych cosinusowych.



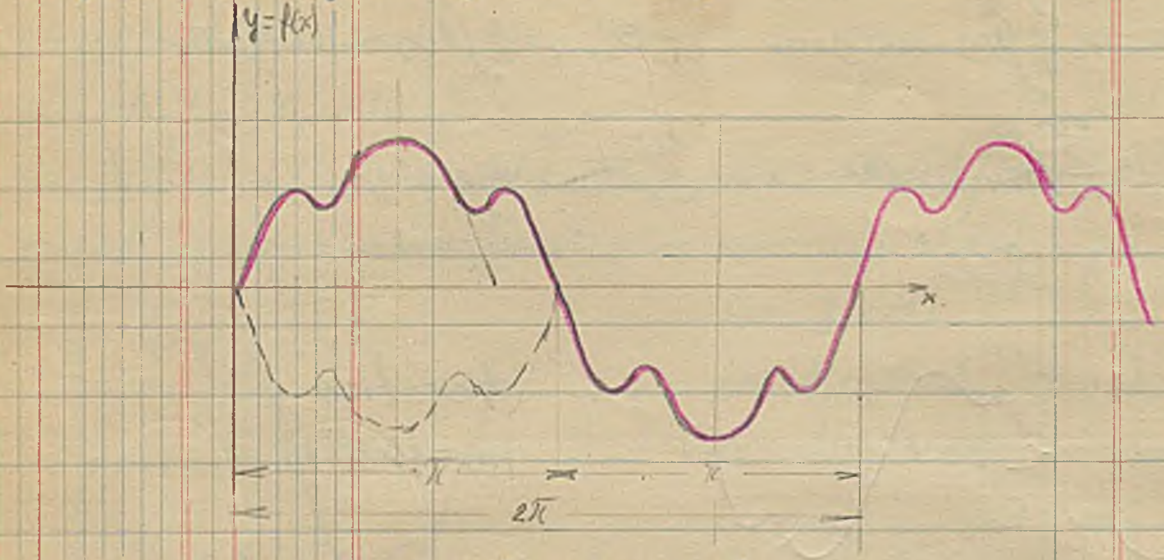
dla sym. wzgl. osi y $A_0=0$

$A_k=0$ nie ma cosinusów

$$B_k=\frac{2}{\pi}\int_0^{\pi} f(x)\sin kx\,dx$$

$$f(x)=\sum_{k=1}^{k=n} B_k \sin k_x$$

Krzywa harm. sym. wzgl. osi x i osi y.



warunki spełnione bez.: $f(x) = f(x + 2\pi K)$

$f(x) = -f[x + (2K-1)\pi]$

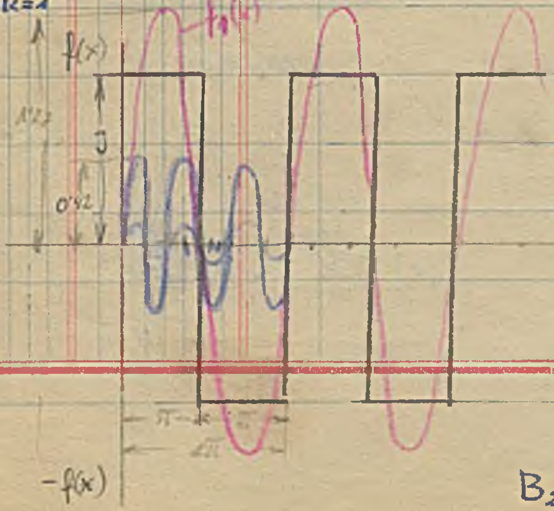
$f(x) = -f(-x)$

$A_0 = 0 \quad A_k = 0$

$B_{2k-1} = \frac{4}{\pi} \int_0^{\frac{\pi}{2}} f(x) \sin(2k-1)x\,dx$ $\qquad$ $B_{2k} = 0$ wszystkie parzyste $= 0$

$f(x) = \sum_{k=1}^{k=n} B_k \sin kx$

$f(x) = \sum_{k=1}^{k=n} B_{2k-1} \sin(2k-1)x$



$f(x) = \sum_{k=1}^{k=n} B_{2k-1} \sin(2k-1)x$

$B_{2k-1} = \frac{4}{\pi} \int_0^{\frac{\pi}{2}} f(x) \sin(2k-1)x\,dx$

$B_{2k-1} = \frac{4}{\pi} \int_0^{\frac{\pi}{2}} J \sin(2k-1)x\,dx$

$B_{2k-1} = \frac{4}{\pi} J \int_0^{\frac{\pi}{2}} \sin(2k-1)x\,dx$

$B_{2k-1} = \frac{4}{\pi} J \frac{1}{2k-1} \left[\underbrace{-\cos(2k-1)x}_{1} \right]_0^{\frac{\pi}{2}}$

$B_{2k-1} = \frac{4}{\pi} J \frac{1}{2k-1}$

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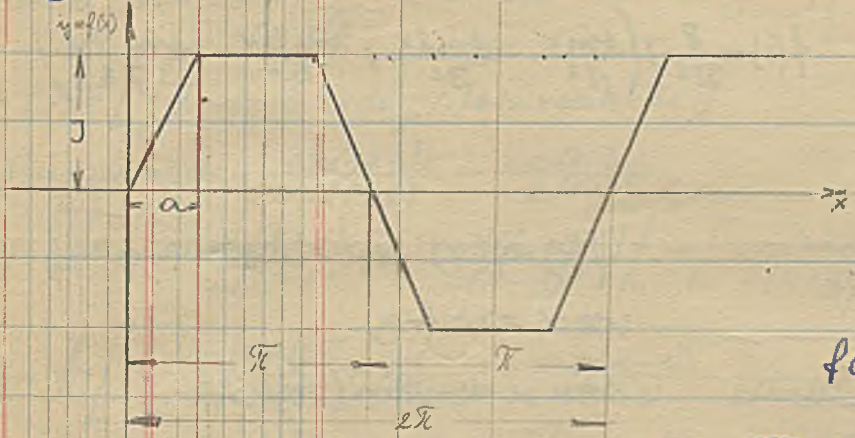
$$B_{2k-1} = \frac{4}{\pi} J \frac{1}{2k-1}$$

$$f(x) = \frac{4}{\pi} J \left(\frac{\sin x}{1} + \frac{\sin 3x}{3} + \frac{\sin 5x}{5} + \frac{\sin 7x}{7} + \ldots\ldots \right)$$

$$B_1 = 1'27\, J$$

$$B_3 = 0'42\, J$$



$$\frac{a}{J} = \frac{x}{f(x)}$$

$$f(x) = \frac{J}{a} x$$

$$B_{2k-1} = \frac{4}{\pi} \int_0^a \frac{J}{a} x \sin(2k-1)x\,dx + $$
$$+ \frac{4}{\pi} \int_a^{\frac{\pi}{2}} J \sin(2k-1)x\,dx$$

$$B_{2k-1} = \frac{4}{\pi} \frac{J}{a} \int_0^a x \sin(2k-1)x\,dx + \frac{4}{\pi} J \int_a^{\frac{\pi}{2}} \sin(2k-1)x\,dx$$

$$\int_a^{\frac{\pi}{2}} \sin(2k-1)x\,dx = -\frac{1}{2k-1} \cos(2k-1)x \Big|_a^{\frac{\pi}{2}} =$$
$$= + \frac{\cos(2k-1)a}{2k-1}$$

$$(2k-1)x = m$$

$$J = \int_0^a x \sin(2k-1)x\,dx = \frac{1}{(2k-1)^2} \int_0^a (2k-1)x \sin(2k-1)x\, d\left[(2k-1)x\right]$$

$u = m$; $du = dm$; $v = -\cos m$; $dv = \sin m\, dm$

$$J = \frac{1}{(2k-1)^2} \int \frac{m}{1} \underbrace{\sin m\, dm}_{dv} = \frac{1}{(2k-1)} \left[-m \cos m \cdot \int -\cos dm \right]$$

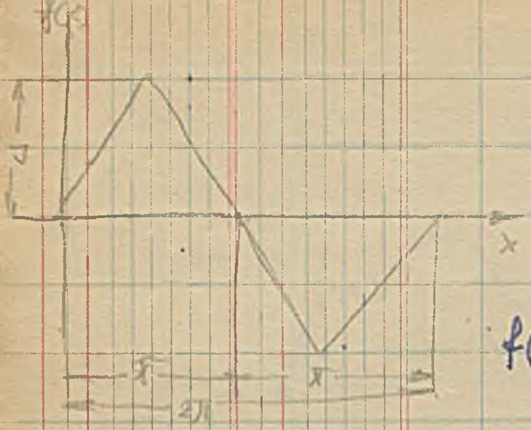
$$J = \frac{1}{(2k-1)^2} (\sin m - m \cos m) = \frac{1}{(2k-1)^2} \left[\sin(2k-1)x - (2k-1)x \cos(2k-1)x \right]$$

$$J = \frac{\sin(2k-1)x}{(2k-1)^2} \Big|_0^a - \frac{x \cos(2k-1)x}{(2k-1)} \Big|_0^a = \frac{\sin(2k-1)a}{(2k-1)^2} - \frac{a \cos(2k-1)a}{2k-1}$$

$$B_{2k-1} = \frac{4J}{\pi a} \left[\frac{\sin(2k-1)a}{(2k-1)^2} - \frac{a \cos(2k-1)a}{2k-1} + \frac{a \cos(2k-1)a}{2k-1} \right]$$

$$\boxed{B_{2k-1} = \frac{4}{\pi} \frac{J}{a} \frac{\sin(2k-1)a}{(2k-1)^2}}$$

$$\boxed{f(x) = \frac{4}{\pi} \frac{J}{a} \left(\frac{\sin a}{1^2} \sin x + \frac{\sin 3a}{3^2} \sin 3x + \frac{\sin 5a}{5^2} \sin 5x \ldots\ldots \right.}$$



$$a=\frac{\pi}{2}$$

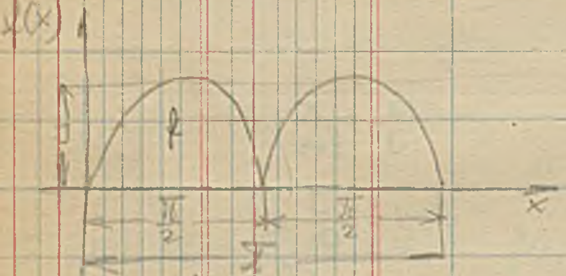
$$B_{2k-1}=\frac{8}{\pi^2}J\frac{\sin(2k-1)\frac{\pi}{2}}{(2k-1)^2}=\frac{8}{\pi^2}J\frac{\sin(k\pi-\frac{\pi}{2})}{(2k-1)^2}=$$

$$=-\frac{8}{\pi^2}J\frac{\cos k\pi}{(2k-1)^2}$$

$\cos k\pi=(-1)^k$

$$B_{2k-1}=-\frac{8}{\pi^8}J\frac{(-1)^k}{(2k-1)^2}$$

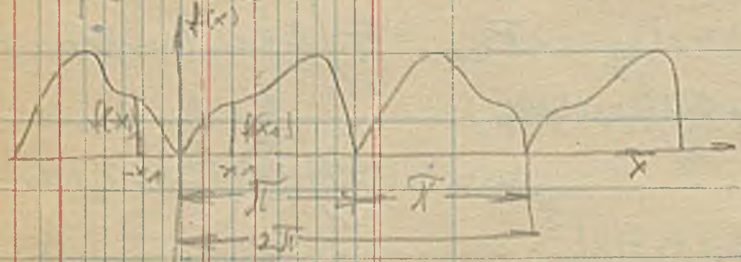
$$f(t)=\frac{8}{\pi^2}J\left(\frac{\sin x}{1^2}-\frac{\sin 8x}{3^2}+\frac{\sin 5x}{5^2}-\frac{\sin 7x}{7^2}\right)$$



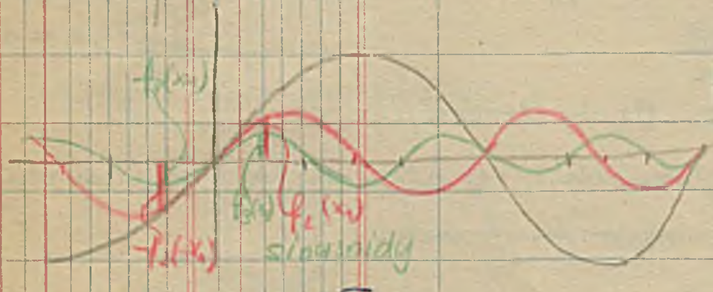
$f(x)=f(-x)$ symetryczna wzgl. y-ów

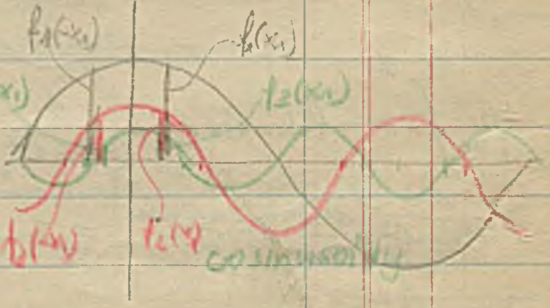
$\cos\alpha=\cos(-\alpha)$

$\sin\alpha=-\sin(-\alpha)$



$f(x)=f(-x)$

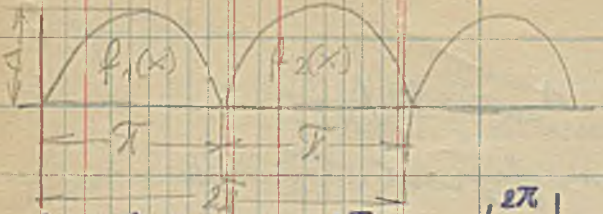




$$A_0=\frac{1}{2\pi}\int_0^{2\pi}f(x)\,dx$$

$$A_k=\frac{1}{\pi}\int_0^{\pi}f(x)\cos kx\,dx$$

$$f(x)=A_0+\sum_{k=1}^{k=n}A_k\cos kx$$



$f_1(x) = J \sin \omega t; \quad f_2(x) = -J \sin \omega t$

$$A_0 = \frac{1}{2\pi}\int_0^{\pi} J \sin x\,dx - \frac{1}{2\pi}\int_{\pi}^{2\pi} J \sin x\,dx$$

$$A_0 = \frac{1}{2\pi} J\Big(\underbrace{-\cos x \Big|_0^{\pi}}_{=2} + \underbrace{\cos x}_{=2}\Big|_{\pi}^{2\pi}\Big) = \frac{2}{\pi} J$$

$$A_k = \frac{1}{\pi}\left[\int_0^{\pi} J \sin x \cos kx\,dx - \int_{\pi}^{2\pi} J \sin x \cos kx\,dx\right]$$

$$\int \sin x \cos kx\,dx = -\frac{\cos(1+k)x}{2(1+k)} - \frac{\cos(1-k)x}{2(1-k)}$$

$$A_k = \frac{J}{\pi}\left\{\left[-\frac{\cos(k+1)x}{2(k+1)} + \frac{\cos(k-1)x}{2(k-1)}\right]\Bigg|_0^{\pi} + \left[\frac{\cos(k+1)x}{2(k+1)} - \frac{\cos(k-1)}{2(k-1)}\right]\Bigg|_{\pi}^{2\pi}\right\} =$$

$$= \frac{J}{\pi}\Big[-\frac{\cos(k+1)\pi}{2(k+1)} + \frac{1}{2(k+1)} + \frac{\cos(k-1)\pi}{2(k-1)} - \frac{1}{2(k-1)} + \frac{\cos(k+1)2\pi}{2(k+1)} - \frac{\cos(k+1)\pi}{2(k+1)} -$$

$$- \frac{\cos(k-1)2\pi}{2(k-1)} + \frac{\cos(k-1)\pi}{2(k-1)}\Big] = \frac{J}{\pi}\Big[-\frac{\cos(k+1)\pi}{(k+1)} + \frac{1}{2(k+1)} + \frac{\cos(k-1)\pi}{(k-1)} -$$

$$- \frac{1}{2(k-1)} + \frac{\cos(k+1)2\pi}{2(k+1)} - \frac{\cos(k-1)2\pi}{2(k-1)}\Big] =$$

$$= \frac{J}{\pi}\left[\frac{1}{2(k+1)} - \frac{1}{2(k-1)} - \frac{\cos(k+1)\pi}{k+1} + \frac{\cos(k-1)\pi}{k-1} + \frac{1}{2(k+1)} - \frac{1}{2(k-1)}\right] =$$

$$= \frac{J}{\pi}\left[\frac{1}{k+1} - \frac{1}{k-1} - \frac{\cos(k+1)\pi}{k+1} + \frac{\cos(k-1)\pi}{k-1}\right]$$

o ile $k=1$ to $A_1 = 0$

$k=2 \quad A_2 = \frac{J}{\pi}\left(\frac{1}{3} - \frac{1}{1} + \frac{1}{3} - \frac{1}{1}\right)$

$k=3 \quad A_3 = 0$

$$A_{(2k-1)} = 0; \quad A_{2k} = \frac{J}{\pi}\left(\frac{2}{k+1} - \frac{2}{k-1}\right) = \frac{2}{\pi} J\left(\frac{1}{k+1} - \frac{1}{k-1}\right) =$$

$$= \frac{2}{\pi} J \frac{-2}{k^2-1} = -\frac{4}{\pi} J \frac{1}{k^2-1}$$

$$f(x) = \frac{4}{\pi} J \left(1 - \frac{2}{3}\cos 2x - \frac{2\cos 4x}{3\cdot 5} - \frac{2\cos 6x}{5.7} - \frac{2\cos 8x}{7.9} \ldots\ldots\right)$$

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$$A_0 = \frac{2}{\pi} J \quad A_{(2k-1)} = 0 \qquad B_k = 0 \qquad A_{2k} = -\frac{4}{\pi} J \frac{1}{k^2-1}$$

$$f(x) = \frac{2}{\pi} J \left[1 - \frac{2}{3}\cos 2x - \frac{2}{3.5}\cos 4x + \frac{2}{5.7}\cos 6x - \dots \right]$$

$\hat{J}'$ i $\hat{J}''$ są odkształcone w jednakowym stopniu

$\hat{J}'$ jest przesunięty wzgl. $\hat{J}''$ o kąt α.

czyli jego pierwsza harm. jest przesunięta

o α wzgl. -"- -"- $\hat{J}''$.

$i_1' = J_1' \sin \omega t$

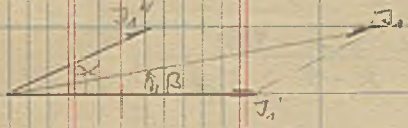
$i_1'' = J_1'' \sin(\omega t + \alpha)$

$i_1' + i_1'' = i_1$

$J_1' \sin \omega t + J_1'' \sin(\omega t + \alpha) = J_1 \sin(\omega t + \beta)$ $\qquad \beta =$ przesunięcie wypadkow

$J_1' \sin \omega t + J_1'' \sin \omega t \cdot \cos\alpha + J_1'' \cos \omega t \sin\alpha =$

$= J_1 \sin \omega t \cos\beta + J_1 \cos \omega t \sin\beta$



dla $t = 0$

$$\left.\begin{aligned} J_1'' \sin\alpha &= J_1 \sin\beta \\ t &= \tfrac{1}{4}\pi \\ J_1' + J_1'' \cos\alpha &= J_1 \cos\beta \end{aligned}\right\} \quad \operatorname{tg}\beta = \frac{J_1'' \sin\alpha}{J_1' + J_1'' \cos\alpha}$$

$J_1''^2 \sin^2\alpha = J_1 \sin^2\beta$

$J_1'^2 + 2 J_1' J_1'' \cos\alpha + J_1''^2 \cos^2\alpha = J_1 \cos^2\beta$

$J_1'^2 + 2 J_1' J_1'' \cos\alpha + J_1''^2 = J_1^2 \quad \searrow$

niech $J_1' = J_1''$

$\operatorname{tg}\beta = \frac{\sin\alpha}{1+\cos\alpha} = \operatorname{tg}\frac{\alpha}{2}$

$\beta = \frac{1}{2}\alpha$

$J_1^2 = 2 J_1'^2 + 2 J_1'^2 \cos\alpha$

$J_1^2 = 2 J_1'^2 (1 + \cos\alpha)$

$J_1^2 = 4 J_1'^2 \cos^2 \frac{\alpha}{2}$

$J_1 = 2 J_1' \cos \frac{1}{2}\alpha$ $\qquad$ dla pierwszej harmonicznej.

i_K = wypadkowa

$B_K = \frac{1}{2}\alpha K$

$J_K = 2J'_K \cos\frac{1}{2}\alpha_K$ $\Big\}$ $i'_K = J'_K \sin K\omega t$

$i''_K = J''_K \sin K(\omega t + \alpha) = J''_K \sin(K\omega t + \underbrace{K\alpha}_{\alpha_K})$. $K\alpha = \alpha_K$

$i_K = 2J'_K \cos\frac{K\alpha}{2} \sin(K\omega t + K\alpha)$

$2J'_K \cos\frac{K\alpha}{2} = 0$ gdy $\cos\frac{K\alpha}{2} = 0$ czyli gdy $\cos(2n-1)\frac{\pi}{2}$

gdy $K\alpha = (2n-1)\pi$

1). $\boxed{K = \frac{(2n-1)\pi}{\alpha}}$ ta harmoniczna będzie równa zero. (znosi się).

2). $\cos\frac{K\alpha}{2} = 1 = \cos(2n)\frac{\pi}{2}$

$\frac{K\alpha}{2} = \frac{2n\pi}{2}$

$\boxed{K = \frac{2n\pi}{\alpha}}$ wartość maksymalna (harmoniczne nakładają się).

Niektóre harmoniczne będą tłumione, a inne akcentowane.

SEM odkształcona.

$u_K = U_{Km} \sin K\omega t$

$i_K = J_{Km} \sin(K\omega t - \varphi_K)$

$J_{Km} = \frac{U_{Km}}{\sqrt{r^2 + (K\omega L - \frac{1}{K\omega C})^2}} = \frac{U_{mK}}{Z_K}$

$\operatorname{tg}\varphi_K - \frac{K\omega L - \frac{1}{K\omega C}}{R}$

może się trafić harmoniczna, która ma rezonans.

$K\omega L = \frac{1}{K\omega C}$; $K\omega\sqrt{LC} = 1$ $K = \frac{1}{\omega\sqrt{LC}}$

n.p. $L = 300\,mH$

$C = 0'135\,\mu F$

$LC = 0'3 \cdot 0'135 \cdot 10^{-6} = = 0'405 \cdot 10^{-6} = 0'0405 \cdot 10^{-5}$

$= 40'5\ 10^{-8}$

$\sqrt{LC} = 10^{-4}\sqrt{40'5} = 6'36 \cdot 10^{-4}$

$K = \frac{10^4}{314 \cdot 6'36}$

$K = 5$

piąta harmoniczna będzie w rezonansie

$$J_K = \frac{U_K}{\sqrt{R^2 + K^2\omega^2 L^2}}$$

jeżeli **mam** w obwodzie tylko indukcyjność

gdy $R \cong 0$

$$J_K \cong \frac{U_K}{K\omega L}$$

im większe L, tym **mniejsze** amplitudy

indukcyjność
pojemność łagodzi harmoniczne (odkształcenia)

$$J_K = \frac{U_K}{\sqrt{R^2 + \frac{1}{K^2\omega^2 C^2}}}$$

dla $R \cong 0$

$$J_K \cong \frac{U_K}{\frac{1}{K\omega C}} = U_K K\omega C$$

pojemność powiększa amplitudy prądu, akcentuje wyższe harmoniczne.

Wartość skuteczna

$$U = \sqrt{\frac{1}{2\pi}\int_0^{2\pi} u^2\, dx}$$

$$u^2 = \left[\sum_{k=1}^{k=n} A_k \cos kx + \sum_{k=1}^{k=n} B_k \sin kx\right]^2$$

Typy 1)

$A_k^2 \cos^2 kx$

2). $A_i B_k \cos ix \sin kx$

3). $A_k B_i \cos kx \sin ix$

4). $A_k B_k \cos kx \sin kx$

5). $A_i B_i \cos ix \sin ix$

6). $B_k^2 \sin^2 kx$

7). $A_i A_k \cos ix \cos kx$

8). $B_i B_k \sin ix \sin kx$

$$U = \sqrt{\frac{1}{2\pi}\int_0^{2\pi}\left[\sum_{k=1}^{k=n} A_k \cos kx + \sum_{k=1}^{k=n} B_k \sin kx\right]^2 dx}$$

$$A_i B_k \int_0^{2\pi} \cos ix \sin kx\, dx = 0$$

$$A_k B_i \int_0^{2\pi} \cos kx \sin ix = 0$$

$$A_i A_k \int_0^{2\pi} \cos ix \cos kx\, dx = 0$$

$$B_i B_k \int_0^{2\pi} \sin ix \sin kx\, dx = 0$$

$$A_k^2\int_0^{2\pi}\cos^2 kx\,dx = A_k^2\left[\int_0^{2\pi}\frac{1}{2}dx\,\underbrace{\int_0^{2\pi}\frac{1}{2}\cos 2kx\,dx}_{=0}\right]$$

$$= A_k^2\pi$$

$$B_k^2\int_0^{2\pi}\sin^2 kx\,dx = B_k^2\pi$$

$$\int_0^{2\pi}\left[\sum_{k=1}^{k=n}A_k\cos kx + \sum_{k=1}^{k=n}B_k\sin kx\right]^2dx = \sum_{k=1}^{k=n}(A_k^2+B_k^2)\pi$$

A_k i B_k są amplitudą poszcz. harmonicznych

$$U=\sqrt{\frac{1}{2}\sum(A_k^2+B_k^2)} =$$

$$u=\sum_{k=1}^{k=n}A_k\cos kx + \sum_{k=1}^{k=n}B_k\sin kx$$

$$=\sum_{k=1}^{k=n}U_{k_{max}}\sin(kx+\psi_k) \qquad U_{k_{max}}=\sqrt{A_k^2+B_k^2} \qquad \mathrm{tg}\,\psi_k=\frac{B_k}{A_k}$$

$$U=\sqrt{\frac{1}{2}\sum_{k=1}^{k=n}U_{k_{max}}^2} = \sqrt{U_1^2+U_2^2+U_3^2+U_4^2+\dots}$$

$$\boxed{U=\sqrt{\sum U_k^2}}$$

wartość skut. krzywych odkszt. równa się sumie kwadratów wart. skut. krzywych harmonicznych.

$$J=\sqrt{\sum J_k^2} \qquad J=\sqrt{J_1^2+J_2^2+\dots}$$

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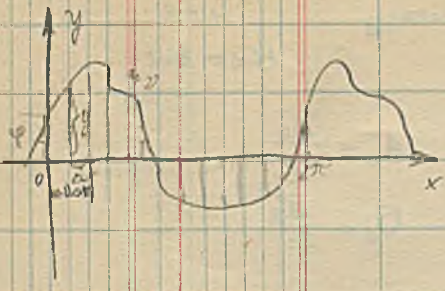


$y = f(x)$

$$A_0 = \frac{1}{2\pi}\int_0^{2\pi} f(x)\,dx \approx \frac{1}{2\pi}\sum_{\nu=0}^{n-1} y_\nu \cdot \Delta x = \frac{\Delta x}{2\pi}\sum y_\nu =$$

$$= \frac{\frac{2\pi}{n}}{2\pi}\sum y_\nu = \frac{1}{n}\sum_{\nu=0}^{n-1} y_\nu$$ średnia wartość

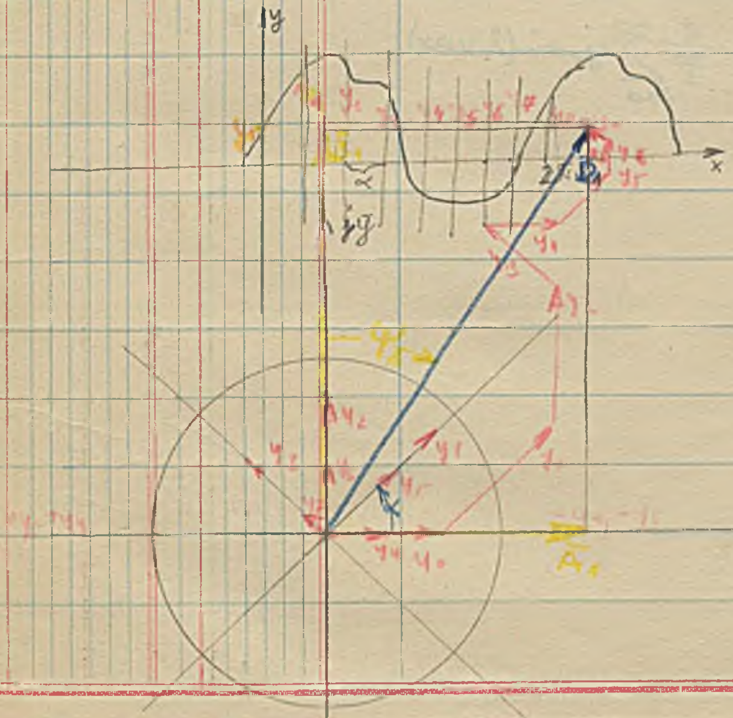
$$A_k = \frac{1}{\pi}\int_0^{2\pi} f(x)\cos kx\,dx \approx \frac{1}{\pi}\sum_{\nu=0}^{n-1}\left[y_\nu \cos(k\nu\Delta x)\right]\Delta x$$

$$= \frac{\Delta x}{\pi}\sum_{\nu=0}^{n-1} y_\nu \cos(k\nu\Delta x)$$

$$A_k = \frac{1}{\frac{n}{2}}\sum_{\nu=0}^{n-1} y_\nu \cos(k\nu\Delta x)$$

$$B_k = \frac{1}{\frac{n}{2}}\sum_{\nu=0}^{n-1} y_\nu \sin(k\nu\Delta x)$$

Metoda Rothe'go.
metoda geometryczna.



n=2

dzielimy na n [illegible] równych części.

$$\frac{2\pi}{8} = 45°$$

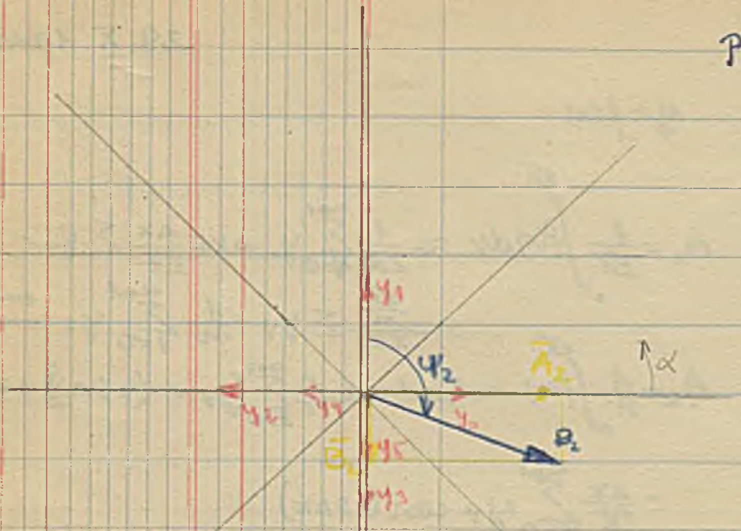
$$A_1 = \frac{\overline{A_1}}{\frac{n}{2}} \;;\; \frac{\overline{B_1}}{\frac{n}{2}} = B_1$$

$$D_1 = \frac{\overline{D_1}}{\frac{n}{2}}$$

$$tg\,\varphi_1 = \frac{A_1}{B_1}$$

przesuwany $k=2$

$K\alpha = 2\alpha$



$$A_2 = \frac{\bar{A}_2}{\frac{n}{2}}$$

$$B_2 = \frac{\bar{B}_2}{\frac{n}{2}}$$

$$D_2 = \frac{\bar{D}_2}{\frac{n}{2}}$$

$$tg\,\Psi_2 = \frac{\bar{A}_2}{B_2}$$

$$\bar{D}_k = y_0 + y_1 e^{jk\alpha} + y_2 e^{j2k\alpha} + \ldots + y_{n-1} e^{j(n-1)k\alpha} =$$

$$= y_0 + y_1 \cos k\alpha + \ldots + y_{n-1} \cos k(n-1)\alpha + j\left[y_1 \sin k\alpha + \ldots + y_{n-1} \sin k(n-1)\alpha \right]$$

$$\bar{D}_k = \sum_{\nu=0}^{n-1} y_\nu \cos(k\cdot\nu\cdot\alpha) + \sum_{\nu=0}^{n-1} y_\nu \sin(k\nu\cdot\alpha)$$

$$A_k = \frac{1}{\frac{n}{2}} \sum_{\nu=0}^{n-1} y_\nu \cos(k\nu\Delta x); \qquad B_k = \frac{1}{\frac{n}{2}} \sum_{\nu=0}^{n-1} y_\nu \sin(k\nu\Delta x)$$

$\Delta x = \alpha$

$$\bar{D}_k = \frac{n}{2}(A_k + j B_k)$$

$$A_k + j B_k = \frac{\bar{D}_k}{\frac{n}{2}}$$

Układy trójfazowe.

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Moc $P_1 = U_1 J_1 \cos\varphi_1$ moc pierwszej harmonicznej.

$$P = U_1 J_1 \cos\varphi_1 + U_3 J_1 \cos\varphi_3 + U_5 J_1 \cos\varphi_5$$

$P = UJ\cos\varphi$ moc krzywej ściśle sinusoidalnej. U i J wartości krzywych sinusoidalnych.

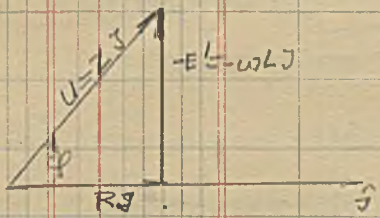
U' = wartość skuteczna $U' = \sqrt{U_1^2 + U_3^2 + U_5^2 + \dots}$

$$J' = \sqrt{J_1^2 + J_3^2 + J_5^2 + \dots}$$

$P = U'J'\cos\varphi'$ moc zmierzona przyrządem.

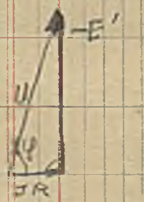
$$P = \sqrt{\sum_{k=1}^{k=n} U_{(2k-1)}^2 \cdot \sum_{k=1}^{k=n} J_{(2k-1)}^2}\,\cos\varphi'$$

$$\cos\varphi' = \frac{P}{U'J'} = \frac{U_1J_1\cos\varphi_1 + U_3J_3\cos\varphi_3 + U_5J_5\cos\varphi_5}{\sqrt{\sum_{k=1}^{k=n} U_{(2k-1)}^2 \sum_{k=1}^{k=n} J_{(2k-1)}^2}}.$$



$$\cos\varphi' < 1$$

R≅0



Im mniejszy opór, tym dokładniej $\hat{U} \cong -E' = \omega L J$

$$\hat{U} = -\hat{E}' = j\omega L \hat{J}$$

$$U = \omega L J$$

$L = \frac{U}{\omega J}$ jeśli U i J są wartościami czysto sinusoidalnymi.

a zatem $u = U_{1m}\sin\omega t + U_{3m}\sin 3\omega t + U_{5m}\sin 5\omega t$ krzywa odkształcona.

$$i = \frac{U_{1m}}{\omega L}\sin\left(\omega t - \frac{\pi}{2}\right) + \frac{U_{3m}}{3\omega L}\sin\left(3\omega t - \frac{3\pi}{2}\right) + \frac{U_{5m}}{5\omega L}\sin\left(5\omega t - \frac{5\pi}{2}\right) + \dots$$

$J' =$ skuteczne.

$$\left.\begin{aligned} J' &= \frac{1}{\omega L}\sqrt{\frac{U_1^2}{1^2} + \frac{U_3^2}{3^2} + \frac{U_5^2}{5^2} + \dots} \\ U' &= \sqrt{U_1^2 + U_3^2 + U_5} . \end{aligned}\right\}$$ wartości skuteczne krzywej odkształconej.

$U = \omega L J$ $\quad J' = \frac{1}{\omega L'}U' = \frac{1}{\omega L'}\sqrt{U_1^2 + U_3^2 + U_5^2 + \dots}$

$$J = \frac{1}{\omega L}U$$

$L' \neq L$

$L' > L$ $\quad L' = L\sqrt{\frac{U_1^2 + U_3^2 + U_5^2 + \dots}{\frac{U_1^2}{1^2} + \frac{U_3^2}{3^2} + \frac{U_5^2}{5^2}}}$

Pomiar pojemności!

$J = \omega C U$ $\quad C = \frac{J}{\omega U}$ $\quad$ pojemność inna przy każdej frekwencji.

$u = U_{m1}\sin\omega t + U_{m3}\sin 3\omega t + U_{m5}\sin 5\omega t + \ldots$

$i = \omega C U_{m1}\sin(\omega t + \frac{\pi}{2}) + 3\omega C U_{m3}\sin(3\omega t + \frac{1}{3}\frac{5\pi}{2}) + 5\omega C U_{m5}\sin(5\omega t + \frac{5\pi}{2})$

J' wartość skuteczna $\quad \varphi_1 = \frac{\pi}{2}$; $\varphi_3 = \frac{1}{3}\cdot\frac{\pi}{2}$; $\varphi_5 = \frac{1}{5}\frac{\pi}{3}$

$J' = \omega C\sqrt{U_1^2 + (3U_3)^2 + (5U_5)^2 + \ldots}$ $\quad$ | $\quad i_k = J_{k m}\sin(k\omega t - \varphi_k)$

$U' = \sqrt{U_1^2 + U_3^2 + U_5^2}$ $\quad tg\,\varphi_k = \frac{k\omega L - \frac{1}{k\omega C}}{R} \neq tg\,k\varphi$

$C' = \frac{J'}{\omega U'}$ $\quad C' = \frac{\omega C\sqrt{U_1^2 + 9U_3^2 + 25U_5^2}}{\omega\sqrt{U_1^2 + U_3^2 + U_5^2}}$

$C' > C$ $\quad C' = C\frac{\sqrt{U_1^2 + 9U_3^2 + 25U_5^2}}{\sqrt{U_1^2 + U_3^2 + U_5^2}}$

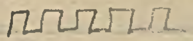
Spółczynnik kształtu ; Spółczynnik szczytu.

1). $K = \frac{J_{sk}}{J_{śr}}$ $\quad S = \frac{J_{max}}{J_{sk}}$

$S = \sqrt{2} = 1{\cdot}41$

dla 

$K = \frac{J_{max}\cdot\pi}{\sqrt{2}\cdot 2J_{max}} = \frac{\pi}{2\sqrt{2}}$

n.p.: 

$K = \frac{\pi}{2\sqrt{2}} = 1{\cdot}11$

2). $f(x) = \frac{4}{\pi}J\left(\frac{\sin x}{1} + \frac{\sin 3x}{3} + \frac{\sin 5x}{5}\right)$

$J_{max} = J = J_{sk}$

$K = 1 \quad S = 1$

3). 

$f(x) = \frac{8}{\pi}J\left(\frac{\sin x}{1^2} - \frac{\sin 3x}{3^2} + \frac{\sin 5x}{5^2}\right)$

$J = \sqrt{\frac{1}{\frac{1}{4}T}\int_0^{\frac{1}{4}T} i^2 dt}$ $\quad \frac{i}{J_{max}} = \frac{t}{\frac{1}{4}T}$ $\quad i = J_{max}\frac{4}{T}t$. $\quad J = \frac{J_{max}}{\sqrt{3}}$

$J = \sqrt{\frac{4}{T}\int_0^{\frac{1}{4}T} J_{m}^2\frac{16}{T^2}t^2 dt} = \sqrt{\frac{64}{T^3}J_m^2\left|\frac{1}{3}t^3\right|_0^{\frac{1}{4}T}} = \sqrt{\frac{64}{T^3}J_m^2\cdot\frac{1}{3}\cdot\frac{1}{64}T^3} = \frac{J_{m}}{\sqrt{3}}$

$J = \frac{J_{max}}{\sqrt{3}}$ $\quad J_{śr}' = \frac{4}{\pi}\int^{\frac{1}{4}T} i\,dt = \frac{4}{\pi}\int^{\frac{1}{4}T} J_{max}\frac{4}{T}\,t\,dt$

$J_{śr} = \frac{16}{T^2} J_{max} \left|\frac{1}{2}t^2\right|_0^{\frac{1}{4}T} = \frac{16}{T^2} J_{max} \frac{1}{2}\frac{1}{16} T^2$

$J_{śr}' = \frac{J_{max}}{2}$

$$K = \frac{J}{J_{max}} = \frac{J_{max}\cdot 2}{\sqrt{3}\,J_{max}} = \frac{2}{\sqrt{3}} = 1{,}16$$

$$S = \frac{J_{max}}{J_{skut}} = \frac{J_{max}\sqrt{3}}{J_{max}} = \sqrt{3} = 1{,}73$$

ukł. trójfazowy

$e' = E_{1max}\sin\omega t + E_{3max}\sin 3\omega t + E_{5max}\sin 5\omega t + E_{7m}\sin 7\omega t + \dots$

$e'' = E_{1m}''\sin(\omega t - 120) + E_{3m}''\sin(3\omega t - 360) + E_{5m}''\sin(5\omega t - 600°) + E_{7m}''\sin(7\omega t - 7\cdot 120) +$

$e''' = E_{1m}'''\sin(\omega t + 120) + E_{3m}'''\sin(3\omega t + 360) + E_{5m}'''\sin(5\omega t + 600) + E_{7m}'''(7\omega t + 7\cdot 120°)$

$e' = E_{1m}'\sin\omega t + E_{3m}'\sin 3\omega t + E_{5m}'\sin 5\omega t + E_{7m}'\sin 7\omega t + E_{9m}\sin 9\omega t$

$e'' = E_1''\sin(\omega t - 120) + E_{3m}''\sin 3\omega t + E_{5m}''\sin(5\omega t + 120) + E_{7m}''\sin(7\omega t - 120) + E_{9m}''\sin 9\omega t + \dots$

$e''' = E_1'''\sin(\omega t + 120) + E_{3m}'''\sin 3\omega t + E_{5m}'''\sin(5\omega t - 120) + E_{7m}'''(7\omega t + 120) + E_{9m}'''\sin 9\omega t +$

Trzecia harmoniczna są w fazie, piąta o 120 wstecz

Wskaźnik harmonicznej jest podzielny przez 3, te harmoniczne są w fazie.

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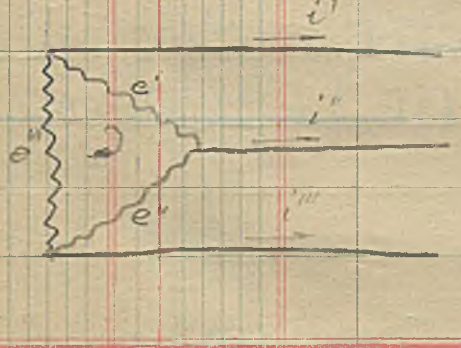
Połączenie w trójkąt



Prąd pierwszej harmonicznej

$i_1 = \frac{e_1' + e_1'' + e_1'''}{3R} = 0$

dla trzeciej

$i_3 = \frac{e_3' + e_3'' + e_3'''}{3R} = \frac{3E_3\sin 3\omega t}{3R}$

$J_3' = \frac{3E_3'}{3Z_3}$ $\quad i_3 = \frac{E_3}{\sqrt{R^2 + X_3^2}}\sin(3\omega t - \varphi_3)$

$$i = \sum_{K=1}^{K=n} \frac{E_{(6K-3)}}{\sqrt{R^2 + X_{(6K-3)}^2}} \sin\left[(6K-3)\omega t - \varphi_{(6K-3)}\right]$$

$X_3 = 3\omega L - \frac{1}{3\omega C}$

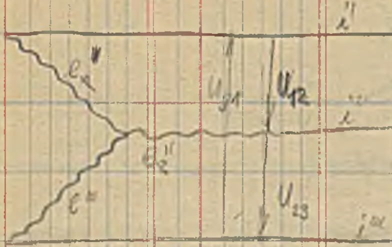
$tg\,\varphi_3 = \frac{X_3}{R}$

$$tg\,\varphi_{(6K-3)} = \frac{(6K-3)\omega L - \frac{1}{(6K-3)\omega C}}{R}$$

7.L

i', i'', i''' są pozbawione wyższych harmonicznych podzielnych

SEM połączone w gwiazdę



$u_{12} = \sqrt{3} E_{1m} \sin(\omega t + 30°)$

$u_{23} = \sqrt{3} E_{1m} \sin(\omega t - 90)$

$u^{12} = e' - e''$

$U'^{12} = E'_{m} \sin \omega t - E''_{1} \sin(\omega t - 120)$

$U_1^{12} = E'_{1m} \sin \omega t - E''_{1m} \sin(\omega t - 120$

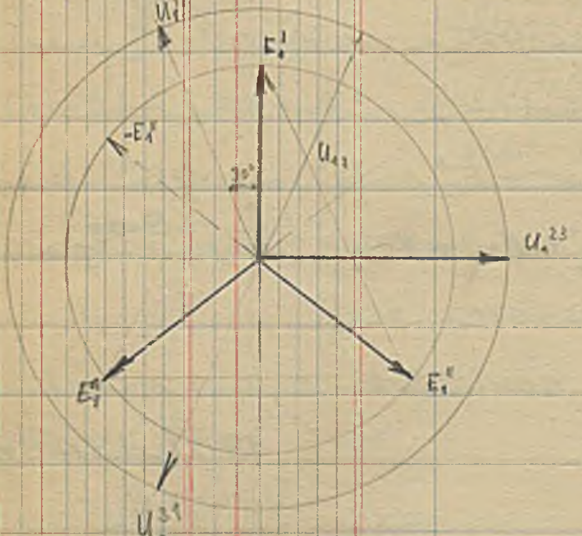
$U_1^{12} = E'_{1m}[\sin \omega t - \sin(\omega t - 120)]$

$\sin m - \sin n = 2 \cos \frac{1}{2}(m+n) \sin \frac{1}{2}(m$

$m = \omega t$
$n = \omega t - 120$

$U_1^{12} = E_{1m}\left[2 \cos \frac{1}{2}(m+n) \sin \frac{1}{2}(m-n)\right] = E_{1m}\left[2 \cos \frac{1}{2}(\omega t + \omega t - 120) \sin 60°\right]$

$U_1^{12} = E_{1m} \sqrt{3} \cos(\omega t - 60°) = E_{1m} \sqrt{3} \cos(\omega t - 90 + 30) = E_{1m} \sqrt{3} \sin(\omega t + 30°)$



$u_1^{23} = e'' - e'''$

$U_1^{23} = E_1'' \sin(\omega t - 120) - E_1''' \sin(\omega t + 120)$

$U_1^{23} = E_{1max}[\sin(\omega t - 120) - \sin(\omega t + 120)$

$U_1^{23} = E_{1max} 2 \cos \omega t \cdot \sin \frac{1}{2}(-240)$

$U_1^{23} = 2 E_{max} \cos \omega t \sin(-120)$

$= -2 E_{1max} \cos \omega t \sin 120 =$

$= -2 E_{1max} \cos \omega t \sin 60° =$

$= -\sqrt{3} E_{1max} \cos \omega t$

$= -\sqrt{3} E_{1max} \sin(90 - \omega t)$

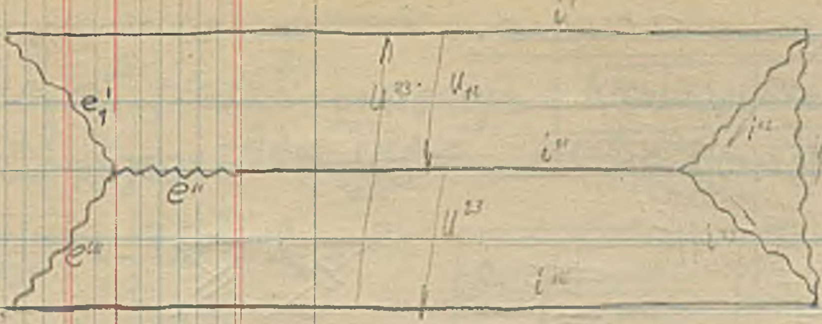
$= \sqrt{3} E_{1max} \sin(\omega t - 90)$

3-a harmoniczna.

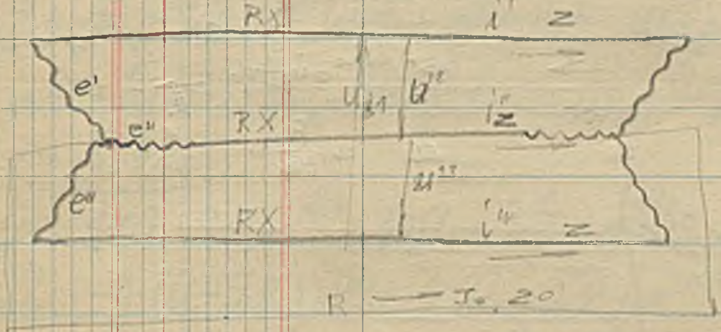
$U_3^{12} = e_3' - e_3'' = E_3' \sin 3\omega t - E_3'' \sin 3\omega t = 0$

5-a harmoniczna

TL



jeżeli brakuje harmonicznej podzielnej przez 3 to wszyst. jest połączony w gw.



$$\hat{E}' = \hat{J}'\hat{Z} - \hat{J}^{\circ}\hat{Z}^{\circ}$$
$$\hat{E}'' = \hat{J}''\hat{Z} - \hat{J}^{\circ}\hat{Z}^{\circ}$$
$$\hat{E}''' = \hat{J}'''\hat{Z} - \hat{J}^{\circ}\hat{Z}^{\circ}$$

$$\hat{J}' + \hat{J}'' + \hat{J}''' = -\hat{J}^{\circ}$$

$$\hat{E}' + \hat{E}'' + \hat{E}''' = \hat{Z}\underbrace{(\hat{J}' + \hat{J}'' + \hat{J}''')}_{-J^{0}} - 3\hat{J}^{\circ}\hat{Z}^{\circ}$$
$$\hat{E}' + \hat{E}'' + \hat{E}''' = -\hat{J}^{\circ}(3\hat{Z}^{\circ} + \hat{Z})$$
$$\boxed{\hat{J}^{0} = \frac{\hat{E}' + \hat{E}'' + \hat{E}'''}{3\hat{Z}^{0} + \hat{Z}}}$$

$$\hat{J}_1^{\circ} = 0$$
$$\hat{J}_3^{\circ} = \frac{3\hat{E}_3}{3\hat{Z}_3^{\circ} + \hat{Z}_3}$$
$$\hat{J}_5^{\circ} = 0 \; ; \; \hat{J}_7^{\circ} = 0 ; \; \hat{J}_9^{\circ} = 0$$

$$\hat{J}^{\circ} = \sum_{K=1}^{K=n} \frac{3\hat{E}_{(6K-1)}}{3\hat{Z}_{(6K-3)} + \hat{Z}'_{(6K-3)}}$$
$$= \sum_{K=1}^{K=n} \frac{3\hat{E}_{(6K-3)}}{3R + \hat{Z}_{(6K-5)}}$$

$$i^{\circ} = \sum_{K=1}^{K=n} \frac{3E_{(6K-3)}}{\sqrt{(3R^{0}+R)^2 + X^2_{(6K-3)}}} \sin\left[(6K-3)\omega t - \varphi_{(6K-3)}\right]$$

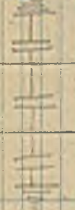
$$X = (6K-3)\omega L - \frac{1}{(6K-3)\omega C}$$

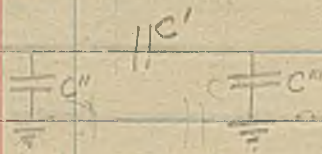
Jeżeli 3 siły EM odkształcone symetrycznie względ. osi x i y połączone w Δ, to wewn. Δ płyną prądy harm. (6K-3). Napięcia międzyprzewodowe mają wszystkie harmoniczne, prądy przewodowe harm. (6K-3)

Linie długie.

Posiadają opór, indukcyjność i pojemność i przewodność rozłożoną R, L, C, G

Oporność między z odcinkami równa tak samo z mierzyć indukcyjność



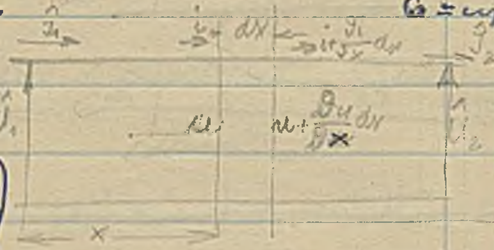


układ zastępczy.

Napięcie i prąd na linii są funkcjami czasu i miejsca

$u = f_1(t,x)$ $\quad i = f_2(t,x)$

G = upływność



Z drugiego prawa Kirchhoffa.

$$u + e' = iR\,dx + u + \frac{\partial u}{\partial x}dx \qquad e' = -L\frac{\partial i}{\partial t}dx$$

$$-L\,dx\frac{\partial i}{\partial t} = iR\,dx + \frac{\partial u}{\partial x}dx$$

$$-\frac{\partial u}{\partial x} = iR + L\frac{\partial i}{\partial t}$$

Z pierwszego prawa Kirchhoffa.

$$i = i + \frac{\partial i}{\partial x}dx + C\frac{\partial u}{\partial t}dx + Gu\,dx$$

$$-\frac{\partial i}{\partial x} = C\frac{\partial u}{\partial t} + Gu$$

Z obu równań otrzymujemy przez zastosowanie rachunku symbolicznego (dla czystych sinusoid)

$$-\frac{d\hat{U}}{dx} = \hat{J}R + L\frac{d\hat{J}}{dt} = \hat{J}R + j\omega L\hat{J}$$

$$-\frac{d\hat{J}}{dx} = \hat{U}G + C\frac{d\hat{U}}{dt} = \hat{U}G + j\omega C\hat{U}$$

$$\left.\begin{aligned}-\frac{d\hat{U}}{dx} &= \hat{J}(R+j\omega L)\\ -\frac{d\hat{J}}{dx} &= \hat{U}(G+j\omega C)\end{aligned}\right\}\text{różniczkowych ukł. równ.}$$

$$-\frac{d^2\hat{U}_x}{dx^2} = \frac{d\hat{J}}{dx}(R+j\omega L) = -\hat{U}_x(R+j\omega L)(G+j\omega C)$$

$$-\frac{d^2\hat{J}}{dx^2} = \frac{d\hat{U}}{dx}(G+j\omega C) = -\hat{J}(R+j\omega L)(G+j\omega C)$$

rozwiązywanie tych równań.

$$\hat{U}_x = \hat{A}_1 e^{\hat{\varkappa}x} + A_2 e^{-\hat{\varkappa}x}$$ gdzie $$\boxed{\sqrt{(R+j\omega L)(G+j\omega L)} = \hat{\varkappa}}$$

$$\hat{J} = -\frac{d\hat{U}}{dx}\frac{1}{R+j\omega L} = \frac{-\hat{\varkappa}A_1 e^{\hat{\varkappa}x} + \hat{\varkappa}A_2 e^{-\varkappa x}}{R+j\omega L} = \frac{\varkappa(-\hat{A}_1 e^{\varkappa x} + \hat{A}_2 e^{-\varkappa x})}{R+j\omega L} = \sqrt{\frac{G+j\omega C}{R+j\omega L}}(\hat{A}_2 e^{-\hat{\varkappa}x} - A_1 e^{-\hat{\varkappa}x})$$

$$\hat{J}\sqrt{\frac{R+j\omega L}{G+j\omega C}} = -\hat{A}_1 e^{\hat{\varkappa}x} + \hat{A}_2 e^{-\hat{\varkappa}x}$$

$$\boxed{\hat{Z} = \sqrt{\frac{R+j\omega L}{G+j\omega C}}}$$ Oporność falowa linii niezależna od długości linii.

$$\left.\begin{aligned}\hat{U}_x &= \hat{A}_1 e^{\hat{\varkappa}x} + \hat{A}_2 e^{-\hat{\varkappa}x}\\ \hat{J}_x\hat{Z} &= -\hat{A}_1 e^{\varkappa x} + \hat{A}_2 e^{-\varkappa x}\end{aligned}\right\}$$ Wyznaczanie stałych całkowania dla $x=0$ $\hat{U}_x = \hat{U}_1$ $\hat{J}_x = \hat{J}_1$

$$\hat{U}_1 = \hat{A}_1 + \hat{A}_2 \qquad \hat{A}_2 = \frac{\hat{U}_1 + \hat{J}_1\hat{Z}}{2} \qquad \hat{A}_1 = \frac{\hat{U}_1 - \hat{J}_1\hat{Z}}{2}$$

$$\underline{\hat{J}_1\hat{Z} = -\hat{A}_1 + \hat{A}_2}$$

$$\left\{\begin{aligned}\hat{U}_x &= \tfrac{1}{2}(\hat{U}_1 - \hat{J}_1\hat{Z})e^{\varkappa x} + \tfrac{1}{2}(\hat{U}_1 + \hat{J}_1\hat{Z}_1)e^{-\varkappa x}\\ \hat{J}_x\hat{Z} &= -\tfrac{1}{2}(\hat{U}_1 - \hat{J}_1\hat{Z})e^{\varkappa x} + \tfrac{1}{2}(\hat{U}_1 + \hat{J}_1\hat{Z})e^{-\hat{\varkappa}x}\end{aligned}\right\}$$

$$\left\{\begin{aligned}\hat{U}_x &= \hat{U}_1\frac{e^{\hat{\varkappa}x} + e^{-\hat{\varkappa}x}}{2} - \hat{J}_1\hat{Z}_1\frac{e^{\hat{\varkappa}x} - e^{-\hat{\varkappa}x}}{2}\\ \hat{J}_1\hat{Z} &= -\hat{U}_1\frac{e^{\hat{\varkappa}x} - e^{-\hat{\varkappa}x}}{2} + \hat{J}_1\hat{Z}\frac{e^{\hat{\varkappa}x} + e^{-\hat{\varkappa}x}}{2}\end{aligned}\right\}$$

$$\left.\begin{aligned}\hat{U}_x &= \hat{U}_1 \operatorname{ch}\hat{\varkappa}x - \hat{J}_1\hat{Z}\operatorname{sh}\hat{\varkappa}x\\ \hat{J}_x\hat{Z} &= -\hat{U}_1 \operatorname{sh}\hat{\varkappa}x + \hat{J}_1 Z\operatorname{ch}\hat{\varkappa}x\end{aligned}\right\}$$ dla x liczonego od początku linii.

Jeżeli liczymy x od końca linii

$$\hat{U}_x = \hat{A}_1' e^{-\varkappa x} + \hat{A}_2' e^{\varkappa x} \qquad \hat{J}_1\hat{Z} = -\hat{A}_1' e^{-\hat{\varkappa}x} + \hat{A}_2' e^{\varkappa x}$$

$$A_1' = \tfrac{1}{2}(\hat{U}_1 - \hat{J}_1\hat{Z}) \qquad A_2' = \tfrac{1}{2}(\hat{U}_1 + \hat{J}_1\hat{Z})$$

$$\left.\begin{aligned}\hat{U}_x &= \tfrac{1}{2}(\hat{U}_1 - \hat{J}_1\hat{Z})e^{-\varkappa x} + \tfrac{1}{2}(\hat{U}_1 + \hat{J}_1\hat{Z})e^{\varkappa x}\\ \hat{J}_x\hat{Z} &= -\tfrac{1}{2}(\hat{U}_1 - \hat{J}_1\hat{Z})e^{-\hat{\varkappa}x} + \tfrac{1}{2}(\hat{U}_1 + \hat{J}_1\hat{Z})e^{\hat{\varkappa}x}\end{aligned}\right\}$$ stąd

$$\left\{\begin{aligned}\hat{U}_x &= \hat{U}_1 \operatorname{ch}\hat{\varkappa}x + \hat{J}_1\hat{Z}\operatorname{sh}\hat{\varkappa}x\\ \hat{J}_x\hat{Z} &= \hat{U}_2 \operatorname{sh}\hat{\varkappa}x + \hat{J}_1\hat{Z}\operatorname{ch}\hat{\varkappa}x\end{aligned}\right\}$$ dla x liczonego od końca linii

$$\hat{Z}^2=\frac{R+j\omega L}{G+j\omega C}=a^2+2jab-b^2=a^2-b^2+j2ab$$

$$\frac{(R+j\omega L)(G-j\omega C)}{G^2+\omega^2C^2}=a^2-b^2+2jab \qquad a=? \quad b=?$$

$$\hat{Z}=\sqrt[4]{\frac{R^2+\omega^2L^2}{G^2+\omega^2C^2}}\,e^{j\alpha} \qquad \text{bo} \quad \hat{Z}=\hat{Z}(\cos\alpha+j\sin\alpha)$$

$$\hat{Z}^2=\frac{R+j\omega L}{G+j\omega L}=\hat{Z}^2(\cos 2\alpha+j\sin 2\alpha)=\frac{(R+j\omega L)(G-j\omega C)}{G^2+\omega^2G^2}=\frac{RG+\omega^2LC}{G^2+\omega^2C^2}+j\frac{\omega(GL-RC)}{G^2+\omega^2C^2}$$

$$\hat{Z}^2=Z^2\cos 2\alpha+jZ^2\sin 2\alpha$$

$$\hat{Z}^2=\sqrt{\frac{(RG+\omega^2LC)^2+\omega^2(GL-RC)^2}{(G^2+\omega^2C^2)^2}}=\sqrt{\frac{R^2G^2+2\omega^2LCRG+\omega^2L^2C^2+\omega^2G^2L^2-2\omega^2GLCR}{(G^2+\omega^2C^2)^2+R^2C^2\omega^2}}$$

$$\hat{Z}^2=\sqrt{\frac{R^2(G^2+\omega^2C^2)+\omega^2L^2(G^2+\omega^2C^2)}{(G^2+\omega^2C^2)^2}}=\sqrt{\frac{R^2+\omega^2L^2}{G^2+\omega^2C^2}}$$

$$Z=\sqrt[4]{\frac{R^2+\omega^2L^2}{G^2+\omega^2C^2}} \qquad tg\,2\alpha=\omega^2\frac{GL-RC}{RG+\omega^2LC}$$

$$\hat{\mathcal{H}}=\sqrt{(R+j\omega L)(G+j\omega C)}=a+jb$$

$$\hat{\mathcal{H}}^2=(R+j\omega L)(C+j\omega C)=a^2+2jab-b^2$$

$$\hat{\mathcal{H}}^2=(RG-\omega^2LC)+j\omega(RC+LG)=a^2-b^2+2jab$$

$$RG-\omega^2LC=a^2-b^2 \qquad \omega(LG+RC)=2ab$$

$$(RG-\omega^2LC)^2=a^4-2a^2b^2+b^4$$

$$\underline{\omega^2(GL+RC)^2=4a^2b^2} \qquad \text{dodajemy}$$

$$(RG-\omega^2LC)^2+\omega^2(GL+RC)^2=(a^2+b^2)^2$$

$$(a^2+b^2)^2=\sqrt{R^2G^2-2\omega^2RGLC}+\omega^4L^2C^2+\omega^2G^2L^2+2\omega^2RGLC+\omega^2R^2C^2$$

$$(a^2+b^2)^2=R^2(G^2+\omega^2C^2)+\omega^2L^2(G^2+\omega^2C^2)$$

$$\sqrt{(R^2+\omega^2L^2)(G^2+\omega^2C^2)}=a^2+b^2 \quad \text{dla} \quad (RG-\omega^2LC)=a^2-b^2$$

$$2a^2=\sqrt{(R^2+\omega^2L^2)(G^2+\omega^2C^2)}+(RG-\omega^2LC)$$

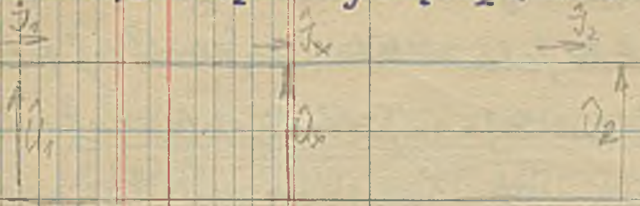
$$2b^2=\sqrt{(R^2+\omega^2L^2)(G^2+\omega^2C^2)}-(RG-\omega^2LC)$$

$$a=\sqrt{\frac{1}{2}\left[\sqrt{(R^2+\omega^2L^2)(G^2+\omega^2C^2)}+(RG-\omega^2LC)\right.} \;;\; b=\sqrt{\frac{1}{2}\left[\sqrt{(R^2+\omega^2L^2)(G^2+\omega^2C^2)}-(RG-\omega^2LC)\right.}$$

$$\mathcal{H}=a+jb \qquad e^{\mathcal{H}x}=e^{ax}\cdot e^{jbx}$$

a = współczynnik tłumień ; b = współczynnik długości fali

$$\left.\begin{aligned}\hat U_x &= \underbrace{\overset{\alpha}{\hat A_1} e^{\hat\gamma x}}_{u_x'} + \underbrace{A_2 e^{-\hat\gamma x}}_{u_x''} \\ \hat J_x \hat Z &= -A_1 e^{\hat\gamma x} + A_2 e^{-\hat\gamma x}\end{aligned}\right\} \begin{aligned}A_1 &= \tfrac{1}{2}(\hat U_1 - \hat J_1 \hat Z) \\ A_2 &= \tfrac{1}{2}(\hat U_1 + \hat J_1 \hat Z)\end{aligned}$$



$u = f(t,x)$ } musimy uważać u i i jako
$i = f(t,x)$ } funkcja czasu i miejsca.

Interpretacja równań linii

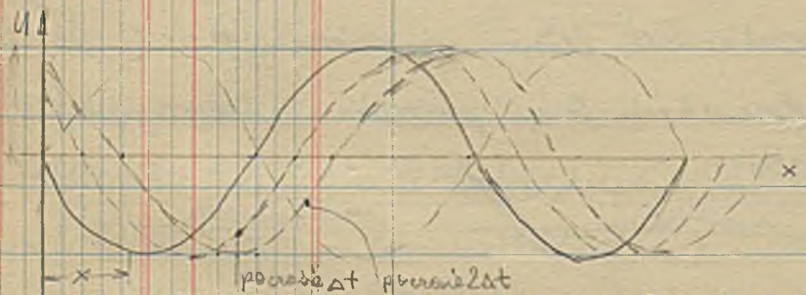
$\hat U_x'' = \hat A_2 e^{-\hat\gamma x}$ niech $\hat A_2 = A_2\sqrt{2}\, e^{j\omega t}$ $\hat u_x = A_2\sqrt{2}\, e^{j\omega t} e^{-ax} \cdot e^{-jbx}$

$\hat U_x'' = \underbrace{\hat A_2 \sqrt{2} e^{-ax}}_{\text{amplituda}} \underbrace{e^{j(\omega t - bx)}}_{\text{sinusoida}} \longrightarrow$ fala główna

$$\boxed{U_x'' = A_2\sqrt{2} e^{-ax} \sin(\omega t - bx)}$$

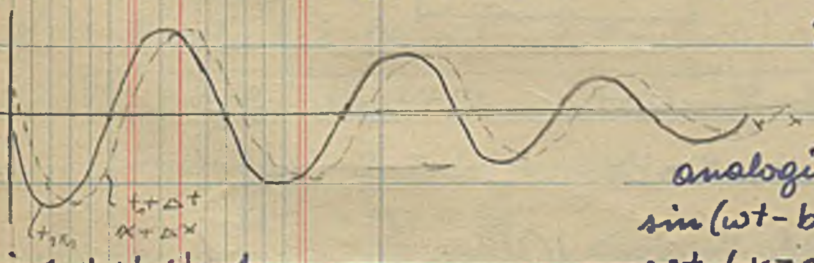
Amplituda sinusoidy maleje w miarę wzrostu x.

Niech dla prostoty a = 0, wówczas



Są to wartości u w zależności od miejsca rozłożenia.

Amplitudy tych sinusoid przesuwają się w czasie. Otrzymujemy falę biegnącą. W ogólnym wypadku fala biegnąca stłumiona.



Prędkość przesuwania się fali.

$\sin(\omega t - bx) = 1$

$\sin[\omega(t+\Delta t) - b(x+\Delta x)] = 1$

$\omega t - bx = \omega t + \omega dt - bx - b dx$

$b\,dx = \omega\,dt$

$\frac{dx}{dt} = \frac{\omega}{b} = \frac{2\pi f}{b} = v$ $\boxed{v = \frac{2\pi f}{b}}$

analogicznie długość fali:

$\sin(\omega t - bx) = \sin[\omega(t+T) - b(x+\lambda)]$

$\omega t - bx = \omega t + \omega T - bx - b\lambda$

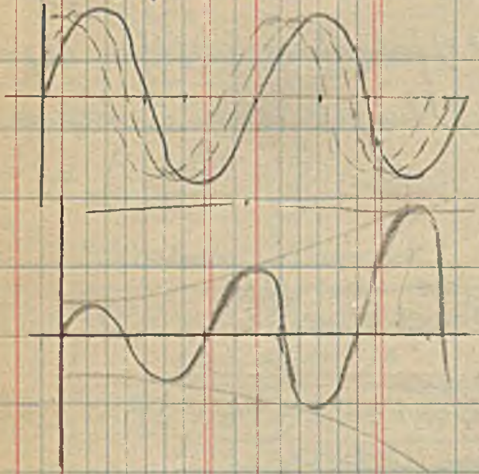
$b\lambda = \omega T$

$\boxed{\lambda = \frac{2\pi}{b}}$ ale $v = \frac{2\pi f}{b}$

$$\boxed{v = f\lambda}$$

Dla pierwszej części równania $\hat{A}_1 e^{\hat{\gamma}x}$

$U' = A_1\sqrt{2}\, e^{ax} e^{j(\omega t + bx)}$



Amplitudy biegną w kier przeciwnym jest to fala odbita. Prędkość tej fali:

$\sin(\omega t + bx) = \sin[\omega(t + dt) + b(x + dx)]$

$\omega t + bx = \omega t + \omega dt + bx + bdx$

$-bdx = \omega dt$

$\frac{dx}{dt} = v = -\frac{\omega}{b} = -\frac{2\pi f}{b}$ Prędkość przeciwna

Fala odbita biegnie od końca do początku przy zmniejszającej się amplitudzie

$A_1 = \frac{1}{2}(\hat{U}_1 - \hat{J}_1\hat{Z})$ amplituda fali głównej jest większa od amplitudy fali odbitej.

$A_2 = \frac{1}{2}(\hat{U}_1 + \hat{J}_1\hat{Z})$ fala odbita.

dla dużego L, a małego C $v \approx c$ (linia napowietrzna)

Tłumienie określamy stosunkiem 2 amplitud następujących

$\hat{A}_1 e^{-ax}$ $\hat{A}_2 e^{-n_1}$ $\hat{A}_2 e^{-n_2}$

$\frac{\hat{A}_2' e^{-n_1}}{A_2'' e^{-n_2}} = e^{n_2 - n_1} = e^{n}$

$P_1 = U'J'\cos\varphi'$

$\boxed{n = \ln\frac{U_1}{U_2}}$ w neperach; $\boxed{db = 10 \lg\frac{\hat{P}_1}{P_2}}$ $P_2 = U''J''\cos\varphi''$

$P' = \frac{U'^2}{Z'}\cos\varphi''$ $P'' = \frac{U''^2}{Z''}\cos\varphi''$

zmniejszając Z' i $\cos\varphi' \equiv Z''\cos\varphi''$

$db = 10\lg\frac{U'^2}{U''^2} = 20\lg\frac{U'}{U''} \Longrightarrow 10^{\frac{db}{20}} = \frac{U'}{U''} = e^{n}$

$db = n \lg e$

$\boxed{db = 20 \lg e\, n = 8{,}686 n}$

Przypadki szczególne linii długiej.

$$\hat{U}_x = \hat{U}_2 ch\,\hat{\gamma}x + sh\,\hat{\gamma}x\, J_2 Z$$

$$\hat{J}_x Z = \hat{J}_2 \hat{Z} ch\,\hat{\gamma}x + U_2 sh\,\hat{\gamma}x$$

Linie bez strat $R \approx 0$ $G \approx 0$ wówczas

$$a = \sqrt{\frac{1}{2}\left[\sqrt{(R^2+\omega^2L^2)(G^2+\omega^2C^2)} + (RG - \omega^2LC)\right]} = 0$$

$$b = \sqrt{\frac{1}{2}\left[\sqrt{(R^2+\omega^2L^2)(G^2+\omega^2C^2)} - (RG - \omega^2LC)\right]} = \omega\sqrt{LC}$$

$$v = \frac{\omega}{b} = \frac{1}{\sqrt{LC}}$$ (niezależnie od ω!)

Dla linii bez strat szybkość rozchodzenia się fali nie zależy od frekwencji !!

$$\hat{\gamma} = jb \qquad \hat{Z} = \sqrt{\frac{L}{C}} = Z$$

$$\left.\begin{aligned} \hat{U}_x &= \hat{U}_2 ch\,jbx + \hat{J}_2 Z\, sh\,jbx \\ \hat{J}_x Z &= \hat{J}_2 Z ch\,jbx + \hat{U}_2 sh\,jbx \end{aligned}\right\} \text{dla } a=0$$

$$ch\,jbx = \cos bx \; ; \; sh\,jbx = j \sin bx$$

$$\left.\hat{U}_x = \hat{U}_2 \cos bx + j \hat{J}_2 Z \sin bx \; ; \; \hat{J}_x Z = \hat{J}_2 Z \cos bx + j \hat{U}_2 \sin bx \right\} \begin{aligned} &\text{niech } \hat{U}_2 = \hat{U} \\ &\hat{J}_2 = J_2(\cos\varphi_2 - j\sin\varphi_2) \end{aligned}$$

$$\hat{U}_x = U_2 \cos bx + j J_2 Z(\cos\varphi_2 - j\sin\varphi_2)\sin bx$$

$$\hat{U}_x = \underbrace{U_2 \cos bx + J_2 Z \sin\varphi_2 \sin bx}_{\text{rzeczywista}} + j \underbrace{J_2 Z \cos\varphi_2 \sin bx}_{\text{urojona}}$$

$$\check{U}_x = \sqrt{(U_2 \cos bx + J_2 Z \sin\varphi_2 \sin bx)^2 + (J_2 Z \cos\varphi_2 \sin bx)^2}$$

$$tg\,\varphi_1 = \frac{J_2 Z \cos\varphi_2 \sin bx}{U_2 \cos bx + J_2 Z \sin\varphi_2 \sin bx}$$ $\varphi > \sphericalangle(U_2, U_x)$ Kąt przesunięcia między napięciem na końcu a napięciem w miejscu x

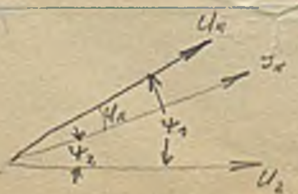
$$\hat{J}_x Z = \hat{J}_2 Z(\cos\varphi_2 - j\sin\varphi_3)\cos bx + j U_2 \sin bx$$

$$\hat{J}_x Z = J_2 Z \cos\varphi \cos bx + j(U_2 \sin bx - \hat{J}_2 Z \sin\varphi_2 \cos bx)$$

$$J_x Z = \sqrt{(J_2 Z \cos\varphi_2 \cos bx)^2 + (U_2 \sin bx - J_2 Z \sin\varphi_2 \cos bx)^2}$$

$$tg\,\varphi_2 = \frac{U_2 \sin bx - J_2 Z \sin\varphi_2 \cos bx}{J_2 Z \cos\varphi_2 \cos bx}$$

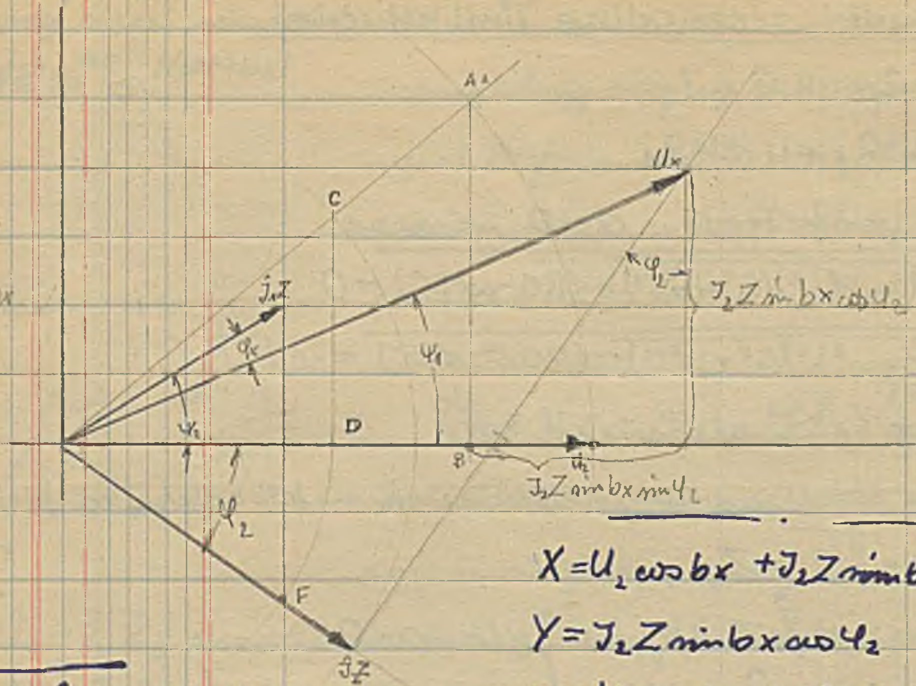
Kąt przesunięcia między U_2 a J_x $\quad \varphi_x = \varphi_1 - \varphi_2$



$OB = U_2 \cos bx$

$CD = J_2 Z \sin bx$

$BE = j Z \hat{J}_2 \sin bx$



$$X = U_1 \cos bx + J_2 Z \sin bx \sin \varphi_2$$

$$Y = J_2 Z \sin bx \cos \varphi_2$$

$$x = U_2 \cos bx + y \, tg \, \varphi_2$$

$$X = U_2 \sqrt{1 - \frac{y^2}{J_2^2 Z^2 \cos^2 \varphi_2}} + y \, tg \, \varphi_2$$

$$\sin bx = \frac{y}{J_2 Z \cos \varphi_2} ; \quad \cos bx = \sqrt{1 - \sin^2 bx} = \sqrt{1 - \frac{y^2}{J_2^2 Z^2 \cos^2 \varphi_2}}$$

$$(x - y \, tg \, \varphi_2)^2 = U_2^2 - \frac{y^2 U_2^2}{J_2^2 Z^2 \cos^2 \varphi_2}$$

$$x^2 - 2xy + y^2 tg^2 \varphi_2 = U_2^2 - \frac{y^2 U_2^2}{J_2^2 Z^2 \cos^2 \varphi_2}$$

$$x^2 - 2 \, tg \, \varphi_2 \, x y + y^2 \left(tg^2 \varphi_2 + \frac{U_2^2}{J_2^2 Z^2 \cos^2 \varphi_2} \right) - U_2^2 = 0$$

$$A x^2 + B x y + C y^2 + D = 0$$

$$A = 1 ; \quad B = -2 \, tg \, \varphi_2 ; \quad C = tg^2 \varphi_2 + \frac{U_2^2}{\frac{J_2^2 Z^2}{\cos^2 \varphi_2}}$$

$$D = -U_2^2$$

$$4 \, tg^2 \varphi_2 - 4 \, tg^2 \varphi_2 - \frac{U_2^2}{J_2^2 Z_1^2 \cos^2 \varphi_2} < 0$$

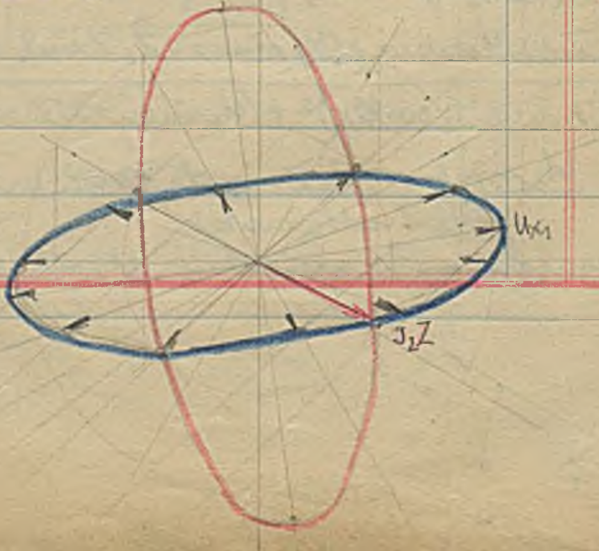
Elipsa $B^2 - 4AC < 0$

Miejscem geometrycznym końców U_x jest elipsa.

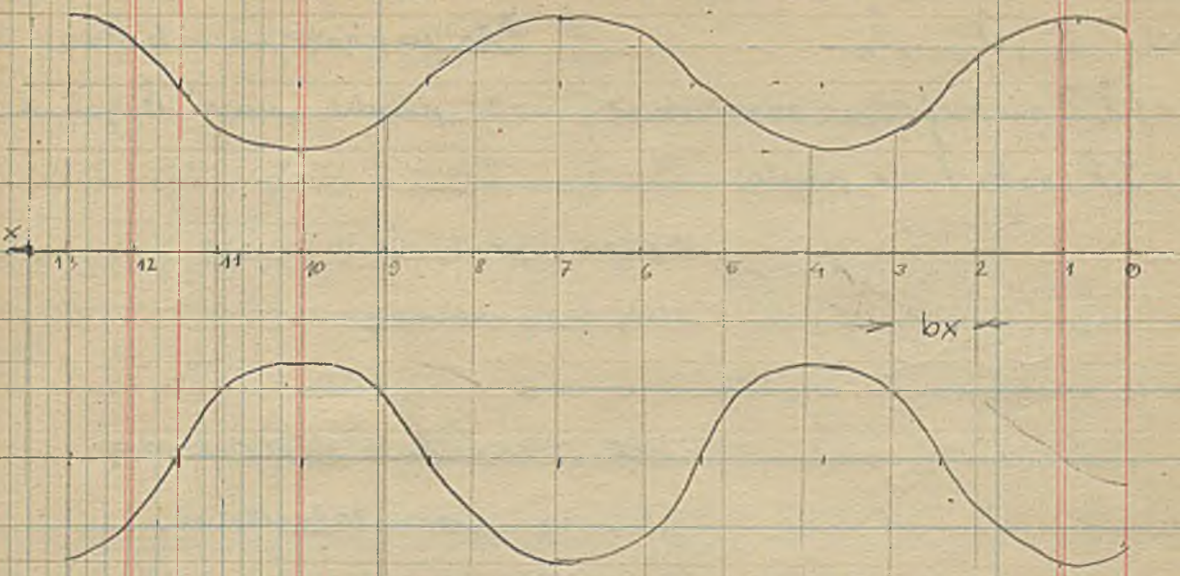
Jeżeli $\varphi_2 = 0$ (obciążenie czysto omowe)

to $x^2 + \frac{y^2 U_2^2}{J_2^2 Z^2} - U_2^2 = 0$

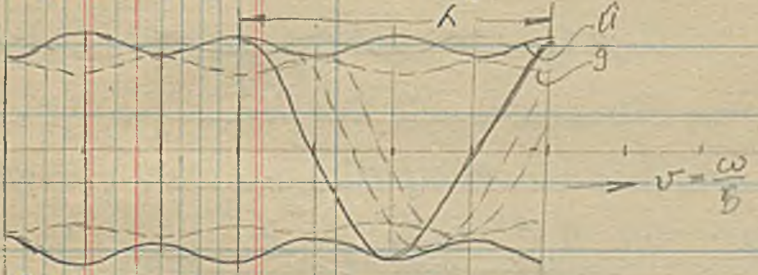
$$\frac{x^2}{U_2^2} + \frac{y^2}{J_2^2 Z^2} = 1$$



Przebiegi powtarzają się co $bx = 2\pi$; $x = \frac{2\pi}{b} = \Lambda$



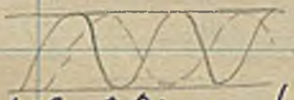
Oś elipsy napięć ⊥ do elipsy prądów.



Obie fale razem tworzą jedną falę zmieniającą się w obwiedniach powyższych. Na drodze prąd jest przesunięty wzgl. napięcia o 90°.

1). $\hat{U}_2 = \hat{J}\cdot\hat{Z}$ elipsa zamienia się w koło

wówczas mamy



brak fali odbitej $A_1 = \frac{1}{2}(\hat{U}_2 - \underbrace{\hat{J}_2\hat{Z}}_{U_2}) = 0$

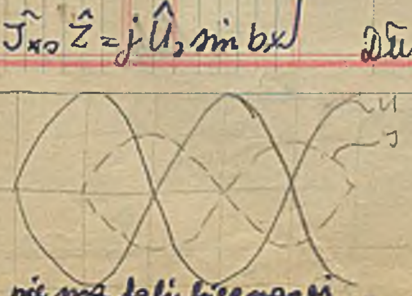
2) Bieg jałowy $\hat{J}_2 = 0$, $\hat{U}_2, \hat{U}_{10}, \hat{J}_{10}$

$\hat{J}_{x0} = \hat{U}_2 \cos bx$

$\tilde{J}_{x0}\hat{Z} = j\hat{U}_2 \sin bx$

} elipsa degeneruje się do prostej (o nowej osi podłużnej)

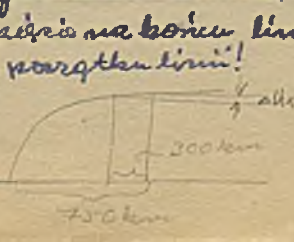
Długość prostej zmienia się cosinusoidalnie.



nie ma fali biegnącej
mamy falę stojącą. Napięcie na węzłach = 0

Zjawisko Ferranti'ego

Napięcie na końcu linii wyższe, od napięcia na początku linii!



3). Zwarcie na końcu

$\hat{U}_2 = 0$, $\hat{J}_1$, $\hat{U}_{12}$, $\hat{J}_{12}$ Dla napięcia na oś krótszą

$\hat{U}_{x2} = j\hat{J}_2\hat{Z}\sin bx$ | Elipsa zamienia się | -"- prądu na oś długą

$\hat{J}x_2 = \hat{J}_2\hat{Z}\cos bx$ | na prostą.

Napięcia i prądy nie zmieniają swej fazy czasowej

4). $\cos\varphi_2$ na końcu $= 0$ $\varphi_2 = 90^0$

Obciążenie czysto indukcyjne: Elipsa zamienia się na prostą.

Dla napięcia na oś dużą } Mamy znowu fale stojące.

-"- prądu -"- -"- krótszą } [illegible]

Fale stojące, gdy moc otrzymana na końcu równa się zeru.

$P_2 = 0$, $P_2 = U_2 J_2 \cos\varphi_2$ (Jeżeli na końcu nie pobiera się żadnej energii).

Z chwilą pobierania mocy fale zaczynają biegnąć.

Fale biegnące powstają, gdy faza czasowa fali zmienia się po drodze.

Najkorzystniejsze wartości R, L, G, C by otrzymać

$a =$ minimum; $\mathcal{H} = a + jb = \sqrt{\frac{1}{2}\left[\sqrt{(R^2+\omega^2L^2)(G^2+\omega^2C^2)} + (RG - \omega^2LC)\right.}$

$$a = \sqrt{\frac{1}{2}\left[\sqrt{\omega^2L^2\left(\frac{R^2}{\omega^2L^2}+1\right)\omega^2C^2\left(\frac{G^2}{\omega^2C^2}+1\right)} + (RG - \omega^2LC)\right]}$$

$$a = \sqrt{\frac{1}{2}\left[\omega^2LC\left(1+\frac{R^2}{\omega^2C^2}\right)^{\frac{1}{2}}\left(1+\frac{G^2}{\omega^2C^2}\right)^{\frac{1}{2}} + (RG - \omega^2LC)\right.}$$

$$(a+b)^n = a^n + \frac{n}{1}a^{n-1}b + \frac{n(n-1)}{2}a^{n-2}b^2 + \frac{n(n-1)(n-2)}{3}a^{n-3}b^3 + \dots\dots$$

$\left(1+\frac{R^2}{\omega^2L^2}\right)^{\frac{1}{2}} = 1 + \frac{1}{2}\frac{R^2}{\omega^2L^2} + \dots$ pomijamy

$$\left(1+\frac{G^2}{\omega^2C^2}\right)^{\frac{1}{2}} = 1 + \frac{1}{2}\frac{G^2}{\omega^2C^2}$$

$$a=\sqrt{\frac{1}{2}\left[\omega^2LC\left(1+\frac{1}{2}\frac{G^2}{\omega^2C^2}+\frac{1}{2}\frac{R^2}{\omega^2L^2}+\underbrace{\frac{R^2G^2}{4\omega^4C^2C^2}}_{pomijamy}+(RG-\omega^2LC)\right]}$$

$$a\approx\sqrt{\frac{1}{2}\left[\omega^2LC\left(1+\frac{1}{2}\frac{G^2}{\omega^2C^2}+\frac{1}{2}\frac{R^2}{\omega^2L^2}\dots\right.\right.}$$

$$a\approx\sqrt{\frac{1}{2}\left[\omega^2LC+\frac{1}{2}\frac{G^2L}{C}+\frac{1}{2}\frac{R^2C}{L}+RG-\omega^2LC\right.}$$

$$a\approx\sqrt{\frac{1}{4}\frac{G^2L}{C}+\frac{1}{2}RG+\frac{1}{4}\frac{R^2C}{L}}=\sqrt{\left(\frac{1}{2}G\sqrt{\frac{L}{C}}+\frac{1}{2}R\sqrt{\frac{C}{L}}\right)^2}$$

$$\boxed{a\approx\frac{1}{2}\left(G\sqrt{\frac{L}{C}}+R\sqrt{\frac{C}{L}}\right)}\qquad\sqrt{\frac{L}{C}}=x\qquad\frac{da}{dx}=\frac{1}{2}G-\frac{R}{2x^2}$$

$$G=\frac{R}{x^2}=R\frac{C}{L}\qquad\frac{C}{L}=\frac{G}{R}$$

$\frac{da^2}{dx^2}=\frac{R}{x^2}>0$ dla $x>0$; warunek najmniejszego tłumienia $\frac{R}{G}=\frac{L}{C}$

wówczas $a=\frac{1}{2}\left(G\sqrt{\frac{R}{G}}+R\sqrt{\frac{G}{R}}\right)=\frac{1}{2}(\sqrt{RG}+\sqrt{RG})=\sqrt{RG}$ $\boxed{a=\sqrt{RG}}$

Linie nieodkształcające.

$$b=\sqrt{\frac{1}{2}\left[\sqrt{(R^2+\omega^2L^2)(G^2+\omega^2C^2)}-(RG-\omega^2LC)\right]}$$

$$b=\sqrt{\frac{1}{2}\left[\sqrt{R^2\left(1+\frac{\omega^2L^2}{R^2}\right)G^2\left(1+\frac{\omega^2C^2}{G^2}\right)}-(RG-\omega^2LC)\right]}$$

$$b=\sqrt{\frac{1}{2}\left[\sqrt{RG\left(1+\omega^2\frac{L^2}{R^2}\right)^2}-(RG-\omega^2LC)\right]}$$

$$b=\sqrt{\frac{1}{2}\left[\sqrt{\left(RG\left(1+\omega^2\frac{L^2}{R^2}\right)\right)^2}-(RG-\omega^2LC)\right]}$$

$$b=\sqrt{\frac{1}{2}\left[RG+\frac{\omega^2L^2G}{R}-RG+\omega^2LC\right]}$$

$$b=\sqrt{\frac{1}{2}\left[\frac{\omega^2G\,LL}{R^2}+\omega^2LC\right]}=\sqrt{\frac{1}{2}\left[\frac{\omega^2RCL}{R}+\omega^2LC\right]}$$

$\boxed{b=\omega\sqrt{LC}}$ współczynnik długości fali, taki sam jak dla linii bez strat.

$v=\frac{\omega}{b}=\frac{\omega}{\omega\sqrt{LC}}=\frac{1}{\sqrt{LC}}$ Prędkość niezależna od częstotliwości.

(To dla linii, której $\frac{C}{L}=\frac{R}{G}$). Na linii nieodkształcającej nie ma

$$\hat{Z}=\sqrt{\frac{R+j\omega L}{G+j\omega C}}=\sqrt{\frac{L\left(\frac{R}{L}+j\omega\right)}{C\left(\frac{G}{C}+j\omega\right)}}=\sqrt{\frac{L}{C}}=Z$$

$\frac{R}{L}=\frac{G}{C}$

odkształceń amplitud ani fazowych.

$\boxed{\hat{Z}=\sqrt{\frac{L}{C}}=Z}$ tak samo dla linii bez strat.

$$\hat{U}_x=\frac{1}{2}(\hat{U}_2+\hat{I}_2Z)e^{ax}e^{jbx}+\frac{1}{2}(\hat{U}_2-\hat{I}_2Z)e^{-ax}e^{-jbx}$$

$$\hat{I}_xZ=\frac{1}{2}(\hat{U}_2+\hat{I}_2Z)e^{ax}e^{jbx}-\frac{1}{2}(\hat{U}_2-\hat{I}_2Z)e^{-ax}e^{-jbx}$$

$$\hat{U}_x=\hat{U}_{x_1}+\hat{U}_{x_2}\qquad\hat{I}_x\hat{Z}=\hat{I}_{x_0}\hat{Z}+\hat{I}_{x_2}\hat{Z}$$

1). Bieg luzem $\hat{J}_2 = 0$ $\hat{U}_2, \hat{U}_{10}, \hat{J}_{10}$.

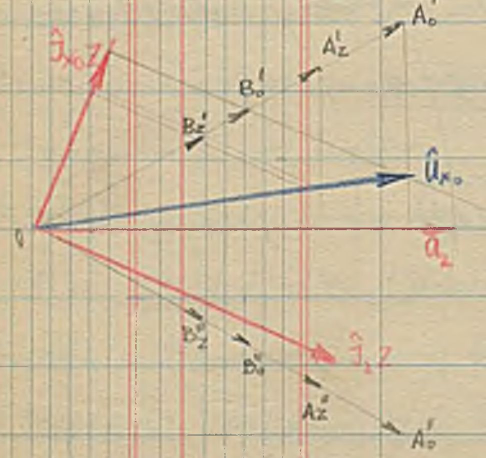
$$\hat{U}_{x0} = \frac{1}{2}\hat{U}_2 e^{ax} e^{jbx} + \frac{1}{2}\hat{U}_2 e^{-ax} e^{-jbx}$$

$$\hat{J}_{x0} Z = \frac{1}{2}\hat{U}_1 e^{ax} e^{jbx} - \frac{1}{2}\hat{U}_1 e^{-ax} e^{-jbx}$$

$$\hat{U}_{x0} = \frac{1}{2}\hat{U}_2 e^{ax}(\cos bx + j\sin bx) + \frac{1}{2}\hat{U}_2 e^{-ax}(\cos b_x - j\sin bx)$$

$$\hat{J}_{x0} Z = \frac{1}{2}\hat{U}_2 e^{ax}(\cos bx + j\sin bx) - \frac{1}{2}\hat{U}_1 e^{-ax}(\cos bx - j\sin bx)$$

napięcie jest sumą 2 fal (bieg luzem), zaś prąd (bieg luzem) jest różnicą tych samych fal.

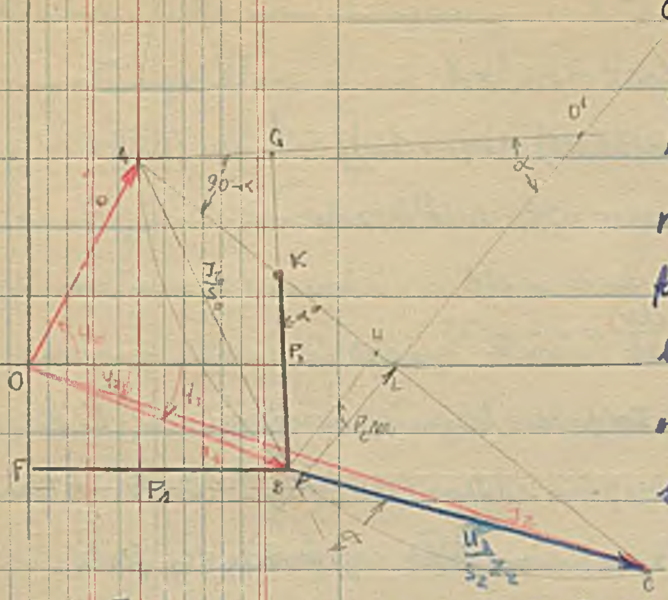


$$\overline{0A_0'} = \hat{U}_2 \cos bx + j\hat{U} \sin bx$$
$$= \hat{U}_2(\cos bx + j\sin bx)$$
$$\overline{0A_0'} = \hat{U}_2(\cos bx - j\sin bx)$$
$$\overline{0B_0'} = \frac{1}{2}\hat{U}_2(\cos bx + j\sin bx)e^{ax}$$
$$\overline{0B_0''} = \frac{1}{2}\hat{U}_2(\cos bx - j\sin bx)e^{-ax}$$

Wykres pracy.

Pod kątami φ_0 i φ_2 odkładamy $J_0 = 0A$ i $J_2 = 0C$. Z A pod ∢90° − α do AC prowadzimy A0' do przecięcia ⊥ do AC wyprowadzonej ze środka AC. W zależności od α kreślimy część koła: przy α > 0 pod AC; przy α < 0 nad AC. Łuk koła stanowi poszukiwany wykres pracy. Stąd znajdujemy wartości J_1 i U_2 dla rozmaitych J_2. Dla danej wartości J_2 obliczamy $\frac{J_2}{S_{01}}$, lub dla danej symetrii $\frac{J_2}{S_1}$.

W przyjętej skali z punktu A odcinek równy $\frac{J_1}{S_{01}}$ przecinamy wykres znajdując B.



OB daje nam bezpośrednio $\hat{J}_1$ zaś BC $\frac{\hat{U}_2}{S_2 Z_2}$, stąd obl. $\hat{U}_2$. $J_0, \varphi_0, J_2, \varphi_2$ winne być wiadome lub znalezione przez pomiary prądu i kąta przesunięcia faz w stanie jałowym i zwarcia końca obwodu albo z wiadomych oporności Z_1, Z_2, Z_3. Dla obwodu symetrycznego te dane wystarczają do obliczenia $\hat{S}$ oraz α ze wzorów:

$$S^2 = \frac{J_2}{\sqrt{J_0^2 + J_2^2 - 2 J_0 J_2 \cos(\varphi_0 - \varphi_2)}} \; ; \; tg\, 2\gamma = \frac{J_0 \sin(\varphi_0 - \varphi_2)}{J_2 - J_0 \cos(\varphi_0 - \varphi_2)} \; ; \; \alpha = \varphi_2 - \varphi_2$$

Gdy badany obwód niesymetryczny musimy jeszcze mieć dane z dwu analiz pomiarów na końcach obwodu. Gdy obwód jest symetryczny moduł $\hat{S}$ i jego γ możemy znaleźć z wykresu przy wzorem $\hat{S}^2 = \frac{\hat{J}_2}{\hat{J}_2 - \hat{J}_0}$, $2\gamma = \varphi_2 - \sphericalangle CLU_1$ stąd $\gamma = \sphericalangle CLU_1$, mierzymy kątomierzem.

$BF = J_1 \cos\varphi_1 \qquad P_1 = U_1 J_1 \cos\varphi_1 \qquad U_1 = const \qquad BF = \frac{P_1}{U_1} = cP_1$

W $\Delta ABC \quad AB = \frac{J_2}{S_0}$; $BC = \frac{U_2}{S_2 Z_2}$ $\qquad$ Pole $\Delta = \frac{1}{2} AB \cdot BC \sin\alpha = \frac{1}{2} \frac{J_2 U_2}{S_0 S_2 Z_2} \sin\alpha$

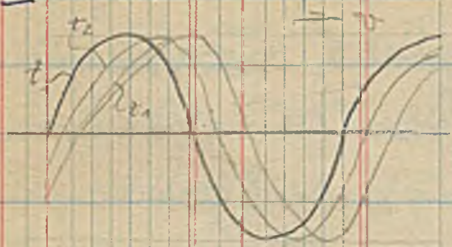
$P_2 = U_2 J_2 \cos\varphi_2$ więc $\Delta = \frac{1}{2} \frac{\sin\alpha}{S_0 S_2 Z_2 \cos\varphi_2} P_2$ } przez porównanie

Pole $\Delta_{ABC} = \frac{1}{2} AC \cdot BH$; $AC = \frac{U_1}{S_0 S_2 Z_2}$; $\Delta = \frac{1}{2} \frac{U_1}{S_0 S_2 Z_2} \cdot BH$ } obu trójkątów

$BH = \frac{P_2 \sin\alpha}{U_1 \sin\varphi_2} \qquad \frac{BH}{\sin\alpha} = \frac{P_2}{U_1 \cos\varphi_2}$

Wartość $\frac{BH}{\sin\alpha}$ znajdziemy przeprowadzając $\perp$ BG z punktu B na promień na promień O'A, ta~~pł~~ przetnie AC w punkcie K: ponieważ $\sphericalangle AKG = \alpha = \sphericalangle BKH$ przeto $BK = \frac{BH}{\sin\alpha} = \frac{P_2}{U_1 \cos\varphi_2}$. Największa moc na odbiorniku odpowiada największej wartości BH.

10.XII 1948r.



$$\sin(\omega t - bx) = 1$$

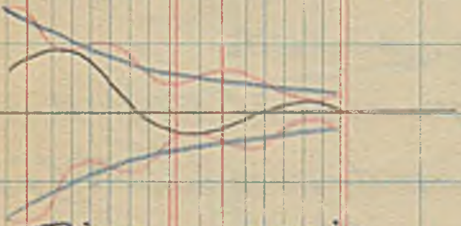
$$\sin\left[\omega(t+dt) - b(x+dx)\right] = 1$$

$$\omega t - bx = \omega t + \omega dt - b\,dx - bx$$

$$b\,dx = \omega dt$$

$$\frac{dx}{dt} = \frac{\omega}{b} = v$$

Jeśli faza zmienia się wstecz to fala biegnie naprzód. Jeśli faza zmienia się naprzód to fala biegnie wstecz (do tyłu). Warunkiem aby fala była biegnącą jest aby zmieniała się faza tej fali.



Bieg zwarcia.

$$\hat{U}_{xz} = \frac{1}{2}\hat{J}_o\hat{Z}e^{ax}(\cos bx + j\sin bx) - \frac{1}{2}\hat{J}_z\hat{Z}e^{-ax}(\cos bx - j\sin bx)$$

$\hat{U}_{xz}$ = napięcie zwarcia

$$\hat{J}_{xz}\hat{Z} = \frac{1}{2}\hat{J}_z\hat{Z}e^{ax}(\cos bx + j\sin bx) + \frac{1}{2}\hat{J}_z e^{-ax}(\cos bx - j\sin bx)$$

$\hat{U}_x = \hat{U}_{x_0} + \hat{U}_{xz}$ $\qquad$ $\hat{J}_x\hat{Z} = \hat{J}_{x_0}\hat{Z} + \hat{J}_{xz}\hat{Z}$

$v = \frac{1}{\sqrt{LC}}$ prędkość nie zależy od częstotliwości

Na liniach kablowych przeważa pojemność,

dla telef. i telegraf. L i G b. małe ≅ 0

Na liniach powietrznych przeważa indukcyjność

Na liniach telefonicznych dajemy co 80 km wzmacniacz, który nie dopuszcz
do zupełnego tłumienia fali. Tłumienie będzie najmniejsze gdy $\frac{L}{C}=\frac{R}{G}$
wtedy $a=\sqrt{RG}$. Ponieważ w liniach kablowych L i G są b. małe przeto $\frac{L}{C}<\frac{R}{G}$
dla zmniejszenia więc tłumienia należy zwiększyć L. W kablu
telef. wpina się cewki co 1,5-2 km. Cewki są wielkości pięści
i posiadają rdzeń żelazny. Rdzeń musi być z pewnego gatunku
startego na proszek żelaza i z dodatkiem pewnego materiału [illegible]

Pomiar R, L, C

Do pomiaru zastosować amperomierz, woltomierz, watomierz.

Na początku przykładamy napięcie U_1 i mierzymy jaki prąd pobiera
linia przy biegu luzem oraz moc jaką pobiera przy biegu luzem.
U_{10}, J_{10}, P_0. Następnie mierzymy U_1 zwarcia J_{1z} i P: U_{1z}, J_{1z}, P_z

$P_0 = U_{10} J_{10} \cos\varphi_0$ $\qquad$ $P_z = U_{1z} J_{1z} \cos\varphi_z$

Stąd obliczamy $\cos\varphi_0$, $\cos\varphi_z$ oraz Z_0; $Z_0=\frac{U_{10}}{J_{10}}$; $Z_z=\frac{U_{1z}}{J_{1z}}$; l = dane

Stąd znajdujemy $\hat{Z}_0$: $\quad \hat{Z}_0 = Z_0(\cos\varphi_0 - j\sin\varphi_0)$

$$\hat{Z}_z = Z_z(\cos\varphi_z - j\sin\varphi_z)$$

$$\left.\begin{aligned}\hat{U}_x &= \hat{U}_2 \operatorname{ch} \hat{\varkappa} x + \hat{J}_2 \hat{Z} \operatorname{sh} \hat{\varkappa} x\\ \hat{J}_x \hat{Z} &= \hat{J}_2 \hat{Z} \operatorname{ch} \hat{\varkappa} x + \hat{U}_2 \operatorname{sh} \hat{\varkappa} x\end{aligned}\right\}$$

$$\hat{U}_1 = \hat{U}_2 \operatorname{ch} \hat{\varkappa} l + \hat{J}_2 \hat{Z} \operatorname{sh} \hat{\varkappa} l$$

$$\hat{J}_1 \hat{Z} = \hat{J}_1 \hat{Z} \operatorname{ch} \hat{\varkappa} l + \hat{U}_2 \operatorname{sh} \hat{\varkappa} l$$

$$\left.\begin{aligned}\hat{U}_{10} &= \hat{U}_2 \operatorname{ch} \hat{\varkappa} l\\ \hat{J}_{10} \hat{Z} &= \hat{U}_2 \operatorname{sh} \hat{\varkappa} l\end{aligned}\right\} \frac{U_{10}}{J_{10} Z} = \operatorname{cotgh} \hat{\varkappa} l \qquad \boxed{\hat{Z}_0 = \hat{Z} \operatorname{cotgh} \hat{\varkappa} l}$$

dla zwarcia

$$\left.\begin{aligned}U_{1z} &= \hat{J}_1 \hat{Z} \operatorname{sh} \hat{\varkappa} l\\ \hat{J}_{1z} \hat{Z} &= \hat{J}_z \hat{Z} \operatorname{ch} \hat{\varkappa} l\end{aligned}\right\} \frac{U_{1z}}{J_{1z} \hat{Z}} = \operatorname{tgh} \hat{\varkappa} l \qquad \boxed{\hat{Z}_z = \hat{Z} \operatorname{tgh} \hat{\varkappa} l}$$

$$\operatorname{ctgh}\hat{\varkappa}l \cdot \operatorname{tgh}\hat{\varkappa}l = \frac{Z_0 Z_z}{\hat{Z}^2} = 1 \qquad \boxed{\hat{Z} = \sqrt{Z_0 Z_z}}$$

$$\operatorname{tgh}\hat{\varkappa}l = \frac{\hat{Z}_z}{\hat{Z}} = \frac{Z_z}{\sqrt{Z_0 Z_z}} = \sqrt{\frac{Z_z}{Z_0}} \qquad \boxed{\operatorname{tgh}\hat{\varkappa}l = \frac{\hat{Z}_z}{\hat{Z}_0}}$$

$$\operatorname{tgh}\hat{\varkappa}l = \frac{e^{\hat{\varkappa}l} - e^{-\hat{\varkappa}l}}{e^{\hat{\varkappa}l} + e^{-\hat{\varkappa}l}} \cdot \frac{e^{\hat{\varkappa}l}}{e^{\hat{\varkappa}l}} = \frac{e^{2\hat{\varkappa}l} - 1}{e^{2\hat{\varkappa}l} + 1}$$

$$e^{2\hat{\varkappa}l}(\operatorname{tgh}\hat{\varkappa}l - 1) = -\operatorname{tgh}\hat{\varkappa}l - 1$$

$$e^{2\hat{\varkappa}l} = \frac{\operatorname{tgh}\hat{\varkappa}l + 1}{1 - \operatorname{tgh}\hat{\varkappa}l} \qquad e^{2\hat{\varkappa}l} = \frac{1 + \sqrt{\frac{\hat{Z}_z}{\hat{Z}_0}}}{1 - \sqrt{\frac{\hat{Z}_z}{Z_0}}} = \frac{\sqrt{\hat{Z}_0} + \sqrt{\hat{Z}_z}}{\sqrt{\hat{Z}_0} - \sqrt{Z_z}}$$

$$e^{2\hat{\varkappa}l} = \frac{\sqrt{Z_0}\, e^{j\frac{\varphi_z}{2}} + \sqrt{Z_z}\, e^{j\frac{\varphi_z}{2}}}{\sqrt{Z_0}\, e^{j\frac{\varphi_0}{2}} - \sqrt{Z_z}\, e^{j\frac{\varphi_z}{2}}} \; \frac{\sqrt{Z_0}\, e^{-j\frac{\varphi_0}{2}} - \sqrt{Z_z}\, e^{-j\frac{\varphi_z}{2}}}{\sqrt{Z_0}\, e^{-j\frac{\varphi_0}{2}} - \sqrt{Z_0}\, e^{-j\frac{\varphi_z}{2}}}$$

$$e^{2\hat{\varkappa}l} = \frac{Z_0 - Z_z - \sqrt{Z_0 Z_z}\, e^{j\frac{1}{2}(\varphi_0 - \varphi_z)} + \sqrt{Z_0 Z_z}\, e^{-j\frac{1}{2}(\varphi_0 - \varphi_z)}}{Z_0 + Z_z - \sqrt{Z_0 Z_z}\, e^{j\frac{1}{2}(\varphi_0 - \varphi_z)} - \sqrt{Z_0 Z_z}\, e^{-j\frac{1}{2}(\varphi_0 - \varphi_z)}}$$

$$e^{2\hat{\varkappa}l} = \frac{Z_0 - Z_z - \sqrt{Z_0 Z_z}\;\; e^{j\frac{1}{2}(\varphi_0 - \varphi_z)} - e^{-j\frac{1}{2}(\varphi_0 - \varphi_z)}}{Z_0 + Z_z - \sqrt{Z_0 Z_z}\;\; e^{j\frac{1}{2}(\varphi_0 - \varphi_z)} + e^{-j\frac{1}{2}(\varphi_0 - \varphi_z)}}$$

$$e^{2\hat{\varkappa}l} = \frac{Z_0 - Z_z - j2\sqrt{Z_0 Z_z}\sin\frac{1}{2}(\varphi_0 - \varphi_z)}{Z_0 + Z_z - 2\sqrt{Z_0 Z_z}\cos\frac{1}{2}(\varphi_0 - \varphi_z)}$$

$$e^{2\hat{\varkappa}l} = \frac{Z_0 - Z_z}{Z_0 + Z_z - 2\sqrt{Z_0 Z_z}\cos\frac{1}{2}(\varphi_0 - \varphi_z)} - j\,\frac{2\sqrt{Z_0 Z_z}\sin\frac{1}{2}(\varphi_0 - \varphi_z)}{Z_0 + Z_z - 2\sqrt{Z_0 Z_z}\cos\frac{1}{2}(\varphi_0 - \varphi_z)}$$

$$e^{2\hat{\varkappa}l} = e^{2al}\, e^{j2bl} = e^{2al}(\cos 2bl + j\sin 2bl)$$

$$e^{4al} = \frac{(Z_0 - Z_z)^2 + 4Z_0 Z_z \sin^2\frac{1}{2}(\varphi_0 - \varphi_z)}{\left[Z_0 + Z_z - 2\sqrt{Z_0 Z_z}\cos\frac{1}{2}(\varphi_0 - \varphi_z)\right]^2} = A$$

$$4al = \ln A \qquad \boxed{a = \frac{1}{4l}\ln A}$$

$$\operatorname{tg} 2bl = \frac{-2\sqrt{Z_0 Z_z}\sin\frac{1}{2}(\varphi_0 - \varphi_z)}{Z_0 - Z_z} = B$$

$$2bl = \operatorname{arc\,tg} B \qquad \boxed{b = \frac{1}{2l}\operatorname{arc\,tg} B}$$

$\hat{\varkappa} = \sqrt{(R+j\omega L)(G+j\omega C)} = a+jb \qquad \hat{Z} = \sqrt{\frac{R+j\omega L}{G+j\omega C}}$

$$\boxed{\hat{\varkappa}\hat{Z} = R+j\omega L} \qquad \boxed{\frac{\hat{\varkappa}}{\hat{Z}} = G+j\omega C}$$

$$R+j\omega L = (a+jb)\sqrt{\hat{Z}_0\hat{Z}_z} = (a+jb)\sqrt{Z_0 Z_z}\, e^{j\frac{1}{2}\varphi_0} e^{j\frac{1}{2}\varphi_z}$$

$$= (a+jb)\sqrt{Z_0 Z_z}\, e^{j\frac{1}{2}(\varphi_0+\varphi_z)} =$$

$$= \sqrt{Z_0 Z_z}\left\{(a+jb)\left[\cos\tfrac{1}{2}(\varphi_0+\varphi_z) + j\sin\tfrac{1}{2}(\varphi_0+\varphi_z)\right]\right\}$$

$$= \sqrt{Z_0 Z_z}\left\{a\cos\tfrac{1}{2}(\varphi_0+\varphi_z) - b\sin\tfrac{1}{2}(\varphi_0+\varphi_z) + j\left[a\sin\tfrac{1}{2}(\varphi_0-\varphi_z) + b\cos\tfrac{1}{2}(\varphi_0-\varphi_z)\right]\right\}$$

$$\boxed{R = \sqrt{Z_0 Z_z}\left[a\cos\tfrac{1}{2}(\varphi_0+\varphi_z) - b\sin\tfrac{1}{2}(\varphi_0+\varphi_z)\right]}$$

$$\boxed{\omega L = \sqrt{Z_0 Z_z}\left[b\cos\tfrac{1}{2}(\varphi_0+\varphi_z) + a\sin\tfrac{1}{2}(\varphi_0+\varphi_z)\right]}$$

$$G+j\omega C = (a+jb)\frac{1}{\sqrt{Z_0 Z_z}} = \frac{a+jb}{\sqrt{Z_0 Z_z}}\, e^{-j\frac{1}{2}(\varphi_0+\varphi_z)} =$$

$$= \frac{a+jb}{\sqrt{\hat{Z}_0\hat{Z}_z}}\left[\cos\tfrac{1}{2}(\varphi_0+\varphi_z) - j\sin\tfrac{1}{2}(\varphi_0+\varphi_z) =\right.$$

$$= \frac{1}{\sqrt{Z_0 Z_z}}\left[a\cos\tfrac{1}{2}(\varphi_0+\varphi_z) + b\sin\tfrac{1}{2}(\varphi_0+\varphi_z) +\right.$$

$$+ j\frac{1}{\sqrt{Z_0 Z_z}}\left[b\cos\tfrac{1}{2}(\varphi_0+\varphi_z) - a\sin\tfrac{1}{2}(\varphi_0+\varphi_z)\right)$$

$$\boxed{G = \frac{1}{\sqrt{Z_0 Z_z}}\left[a\cos\tfrac{1}{2}(\varphi_0+\varphi_z) + b\sin\tfrac{1}{2}(\varphi_0+\varphi_z)\right)}$$

$$\boxed{\omega C = \frac{1}{\sqrt{Z_0 Z_z}}\left[b\cos\tfrac{1}{2}(\varphi_0+\varphi_z) - a\sin\tfrac{1}{2}(\varphi_0+\varphi_z)\right)}$$

— . —

$$\left.\begin{aligned} \hat{U}_{10} &= \hat{U}_z \operatorname{ch}\hat{\varkappa} l \\ \hat{J}_{1z}\hat{Z} &= \hat{J}_z \hat{Z} \operatorname{ch}\hat{\varkappa} l \end{aligned}\right\} \quad \frac{\hat{U}_{10}}{\hat{U}_z} = \frac{\hat{J}_{1z}}{\hat{J}_{zz}} = \operatorname{ch}\hat{\varkappa} l = \hat{S}$$

S = spółczynnik liniowy linii

$$\boxed{\hat{S} = \operatorname{ch}\hat{\varkappa} l}$$

$\hat{S}=S(\cos\gamma+j\sin\gamma)$

~~γ~~ γ jest kątem przesunięcia faz między (U_{10} i U_2) oraz (J_{1z} i J_2)

$$\hat{S}=\mathrm{ch}\,\hat{\varkappa}\,l=\frac{e^{al}+e^{-al}}{2}\cos bl+j\frac{e^{al}-e^{-al}}{2}\sin bl$$

$$\hat{S}=\mathrm{ch}\,al\cos bl+j\,\mathrm{sh}\,al\sin bl$$

$$S=\sqrt{\mathrm{ch}^2al\cos^2bl+\mathrm{sh}^2al\sin^2bl}$$

$$\mathrm{tg}\,\gamma=\frac{\mathrm{sh}\,al\sin bl}{\mathrm{ch}\,al\cos bl}=\mathrm{tg}\,al\,\mathrm{tg}\,bl$$

$$\boxed{\mathrm{tg}\,\gamma=\mathrm{tg}\,al\,\mathrm{tg}\,bl}$$

$\mathrm{ch}^2al=\frac{1}{2}(1+\mathrm{ch}\,2al)$ $\quad$ $\mathrm{sh}^2al=\frac{1}{2}(\mathrm{ch}\,2al-1)$

$\cos^2bl=\frac{1}{2}(1+\cos 2bl)$ $\quad$ $\sin^2bl=\frac{1}{2}(1-\cos 2bl)$

$$S=\sqrt{\frac{1}{4}\left[(1+\mathrm{ch}\,2al)(1+\cos 2bl)+(-\mathrm{ch}\,2al+1)(1-\cos bl)\right]}$$

$$S=\frac{1}{2}\sqrt{\begin{matrix}1+\mathrm{ch}\,2al+\cos 2bl+\mathrm{ch}\,al\cos 2bl+\\-1+\mathrm{ch}\,2al+\cos 2bl-\mathrm{ch}\,al\cos 2bl\end{matrix}}$$

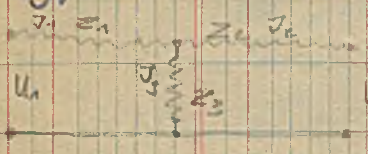
$$\boxed{S=\sqrt{\tfrac{1}{2}(\mathrm{ch}\,2al+\cos 2bl}}$$

7. I. 1949 r.

Układy zastępcze.

Czwórniki (kształtu) typu T, Π, Γ.

Typ T.



1). **Bieg luzem.** $\hat{U}_2$; $\hat{Z}_2=0$, $\hat{U}_{10}$; $\hat{J}_{10}$

$\hat{Z}_0$ oporność pozorna na początku przy biegu luzem.

$$\hat{Z}_0=\hat{Z}_1+\hat{Z}_3\ ;\ \hat{J}_{10}=\frac{\hat{U}_{10}}{\hat{Z}_0}=\frac{\hat{U}_2}{\hat{Z}_3}$$

$$\hat{U}_{10}=\frac{\hat{U}_2}{\hat{Z}_3}\hat{Z}_0=\frac{\hat{U}_2}{\hat{Z}_3}(\hat{Z}_1+\hat{Z}_3)=\hat{U}_2\Big(\underbrace{1+\frac{\hat{Z}_1}{\hat{Z}_3}}_{\hat{S}_0}\Big)$$

$\hat{S}_0$ współczynnik biegu luzem (przy czwórniku) a). $\boxed{\hat{U}_{10} = \hat{U}_2 \hat{S}_0}$ $\boxed{\hat{J}_{10} = \frac{\hat{U}_{10}}{\hat{Z}_0} = \frac{\hat{U}_2 \hat{S}_0}{\hat{Z}_0}}$ $\hat{S}_0 = S_0(\cos\gamma_0 + j\sin\gamma_0) = S_0 e^{j\gamma_0}$

2). przy biegu zwarcia: $\hat{U}_2 = 0,\ \hat{J}_2,\ \hat{U}_{1z},\ \hat{J}_{1z}.$

$\hat{Z}_z = \hat{Z}_1 + \frac{\hat{Z}_2 \hat{Z}_3}{\hat{Z}_2 + \hat{Z}_3}$ $\hat{J}_{1z} = \hat{J}_2 + \hat{J}_{3z}$

$\hat{J}_{3z} = \hat{Z}_3 - \hat{J}_2 \hat{Z}_2$; $\hat{J}_{3z} = \frac{\hat{J}_2 \hat{Z}_2}{\hat{Z}_3}$; $\hat{J}_{1z} = \hat{J}_2 + \frac{\hat{J}_2 \hat{Z}_2}{\hat{Z}_3} = \hat{J}_2 \underbrace{\left(1 + \frac{\hat{Z}_2}{\hat{Z}_3}\right)}_{S_z}$

b). $\boxed{\hat{J}_{1z} = \hat{J}_2\left(1 + \frac{\hat{Z}_2}{\hat{Z}_3}\right)}$

$\boxed{\hat{S}_z = \left(1 + \frac{\hat{Z}_2}{\hat{Z}_3}\right)}$ spółczynnik zwarcia

$\boxed{\hat{U}_{1z} = \hat{J}_2 \hat{S}_z \hat{Z}_z}$ $\hat{U}_{1z} = \hat{J}_{1z} \hat{Z}_z = \hat{J}_2 \hat{S}_z \hat{Z}_z$

3). normalne obciążenie $\hat{U}_1 - \hat{U}_2 = ?$

$\hat{U}_1 - \hat{U}_2 = \hat{J}_1 \hat{Z}_1 + \hat{J}_2 \hat{Z}_2$ $\hat{J}_1 = \hat{J}_2 + \hat{J}_3$ $\hat{J}_3 = \frac{\hat{U}_1 - \hat{J}_1 \hat{Z}_1}{\hat{Z}_3}$

$\hat{J}_1 = \hat{J}_2 + \frac{\hat{U}_1 - \hat{J}_1 \hat{Z}_1}{\hat{Z}_3}$ $\hat{J}_1 \hat{Z}_3 = \hat{J}_2 \hat{Z}_3 + \hat{U}_1 - \hat{J}_1 \hat{Z}_1$

$\boxed{\hat{U}_1 = \hat{J}_1 \hat{Z}_1 + \hat{J}_1 \hat{Z}_3 - \hat{J}_2 \hat{Z}_3}$

$\cancel{\hat{J}_1 \hat{Z}_1} + \hat{J}_1 \hat{Z}_3 - \hat{J}_2 \hat{Z}_3 - \hat{U}_2 = \cancel{\hat{J}_1 \hat{Z}_1} + \hat{J}_2 \hat{Z}_2$

$\hat{J}_1 \hat{Z}_3 = \hat{U}_2 + \hat{J}_2 \hat{Z}_3 + \hat{J}_2 \hat{Z}_2$

$\hat{J}_1 = \frac{\hat{U}_2}{\hat{Z}_3} + \frac{\hat{J}_2(\hat{Z}_3 + \hat{Z}_2)}{\hat{Z}_3}$; $\hat{J}_1 = \frac{\hat{U}_2 \hat{S}_0}{\hat{S}_0 \hat{Z}_3} + \hat{J}_2\left(1 + \frac{\hat{Z}_2}{\hat{Z}_3}\right)$

$\hat{J}_1 = \frac{\hat{U}_2 \hat{S}_0}{\frac{\hat{Z}_3 + \hat{Z}_1 \frac{\hat{Z}_3}{\hat{Z}_1}}{\hat{Z}_1}} + \hat{J}_2 \hat{S}_z$

c). $\boxed{\hat{J}_1 = \frac{\hat{U}_2 \hat{S}_0}{\hat{Z}_0} + \hat{J}_2 \hat{S}_z}$

$\boxed{\hat{U}_1 = \hat{U}_2 \hat{S}_0 + \hat{J}_2 \hat{S}_z \hat{Z}_z}$

$\hat{U}_1 = \hat{U}_2 + \frac{\hat{U}_2}{\hat{Z}_3} \hat{Z}_1 + \hat{J}_2 \hat{S}_z \hat{Z}_1 + \hat{J}_2 \hat{Z}_2$

$\hat{U}_1 = \hat{U}_2\left(1 + \frac{\hat{Z}_1}{\hat{Z}_3}\right) - \hat{J}_2(\hat{S}_z \hat{Z}_1 + \hat{Z}_2)$

$\hat{J}_2 \hat{S}_z \left(\hat{Z}_1 + \frac{\hat{Z}_2}{1 + \frac{\hat{Z}_2}{\hat{Z}_3}}\right) = \hat{J}_2 \hat{S}_z\left(\hat{Z}_1 + \frac{\hat{Z}_2 \hat{Z}_3}{\hat{Z}_2 + \hat{Z}_3}\right) = \hat{J}_2 \hat{S}_z \hat{Z}_z$

$\hat{U}_1 = \hat{U}_2 \hat{S}_0 + \hat{J}_2 \hat{S}_z\left(\hat{Z}_1 + \frac{\hat{Z}_2}{\hat{S}_z}\right) = \hat{U}_2 \hat{S}_0 + \hat{J}_2 \hat{S}_z \hat{Z}_z$

$\boxed{\hat{U}_1 = \hat{U}_{10} + \hat{U}_{1z}}$ Przypomina linię długą

$\boxed{\hat{J}_1 = \hat{J}_{10} + \hat{J}_{1z}}$ $\hat{S}_0 = 1 + \frac{\hat{Z}_1}{\hat{Z}_2}$; $\hat{S}_z = 1 + \frac{\hat{Z}_2}{\hat{Z}_3}$

Czwórnik symetryczny. $\underline{\hat{Z}_1 = \hat{Z}_2}$ $\quad \hat{S}_0 = \hat{S}_z = \hat{s}$

$\hat{U}_{10} = \hat{U}_2 \hat{S}$; $\frac{\hat{U}_{10}}{\hat{U}_2} = \hat{s}$ Przypomina linię długą (współcz. liniowy)

$\hat{J}_{1z} = \hat{J}_2 \hat{S}$; $\frac{\hat{J}_{1z}}{\hat{J}_2} = \hat{S}$ Czyli linia długa jest układem symetrycz

Czwórnikiem możemy zastąpić linię długą

8. I 1949 r.

$$\hat{S}_0 \hat{S}_z = \frac{(\hat{Z}_1+\hat{Z}_3)(\hat{Z}_2+\hat{Z}_3)}{\hat{Z}_3^{\,3}} = \frac{\hat{Z}_0}{\frac{\hat{Z}_3^2}{\hat{Z}_2+\hat{Z}_3}} = \frac{\hat{Z}_0}{\frac{\hat{Z}_3^2+\hat{Z}_2\hat{Z}_3-\hat{Z}_2\hat{Z}_3}{\hat{Z}_2+\hat{Z}_3}} = \frac{\hat{Z}_0}{\frac{\hat{Z}_3(\hat{Z}_2+\hat{Z}_3)-\hat{Z}_2\hat{Z}_3}{\hat{Z}_2+\hat{Z}_3}}$$

$$\hat{S}_0 \hat{S}_z = \frac{\hat{Z}_0}{\hat{Z}_3 - \frac{\hat{Z}_2\hat{Z}_3}{\hat{Z}_2+\hat{Z}_3}} = \frac{\hat{Z}_0}{\hat{Z}_3+\hat{Z}_1-\hat{Z}_1-\frac{\hat{Z}_2\hat{Z}_3}{\hat{Z}_2+\hat{Z}_3}} = \frac{\hat{Z}_0}{\hat{Z}_0 - \underbrace{\left(\hat{Z}_1+\frac{\hat{Z}_2\hat{Z}_3}{\hat{Z}_2+\hat{Z}_3}\right)}_{\hat{Z}_z}}$$

$$\boxed{\hat{S}_0 \hat{S}_z = \frac{\hat{Z}_0}{\hat{Z}_0 - \hat{Z}_z}}$$

Dla czwórnika symetrycznego:

$$\boxed{s^2 = \frac{\hat{Z}_0}{\hat{Z}_0 - \hat{Z}_z}}$$

$$\hat{U}_1 = \hat{s}(\hat{U}_2 + \hat{J}_2 \hat{Z}_2) ; \quad \hat{J}_1 = \hat{s}\left(\hat{J}_2 + \frac{\hat{U}_2}{\hat{Z}_0}\right)$$

warunek symetrii układu.

$$\hat{Z}_0 = \frac{\hat{U}_{10}}{\hat{J}_{10}} ; \quad \hat{Z}_z = \frac{\hat{U}_{1z}}{\hat{J}_{1z}}$$

dla linii długiej $\quad \hat{U}_1 = \hat{U}_2 \cos hip\, \hat{\varkappa} l + \hat{J}_2 \hat{Z} \sin hip\, \hat{\varkappa} l$

$$\hat{J}_1 \hat{Z}_1 = \hat{J}_2 \hat{Z} \cos h\, \hat{\varkappa} l + \hat{U}_2 \sin h\, \hat{\varkappa} l$$

$$\hat{Z} = \sqrt{\frac{R+j\omega L}{G+j\omega C}} \qquad \varkappa = a + jb$$

a). $\left.\begin{array}{l}\hat{U}_{10} = \hat{U}_2 \cos h\, \hat{\varkappa} l \\ \hat{J}_{10}\hat{Z} = \hat{U}_2 \sin h\, \hat{\varkappa} l\end{array}\right\}$ $\quad$ b). $\left.\begin{array}{l}\hat{U}_{1z} = \hat{J}_2 \hat{Z} \sin h\, \hat{\varkappa} l \\ \hat{J}_{1z}\hat{Z} = \hat{J}_2 \hat{J} \cos h\, \hat{\varkappa} l\end{array}\right\}$

$$\hat{U}_1 = \hat{U}_{10} + \hat{U}_{1z} ; \quad \hat{J}_1 = \hat{J}_{10} + \hat{J}_{1z}$$

$$\cos h\, \hat{\varkappa} l = \hat{S} ; \quad \frac{\hat{U}_{10}}{\hat{J}_{10}} = \hat{Z}_{10} ; \quad \frac{\hat{U}_{1z}}{\hat{J}_{1z}} = \hat{Z}_{1z}$$

$$\hat{U}_{10} = \hat{J}_{10} \hat{Z}_{10} \qquad \hat{U}_{1z} = \hat{J}_{1z} \hat{Z}_{1z}$$

$\hat{U}_1 = \hat{U}_2 \hat{S} + J_{12} Z_{12} = \hat{U}_2 \hat{S} + \hat{J}_2 \hat{S} \hat{Z}_{12}$

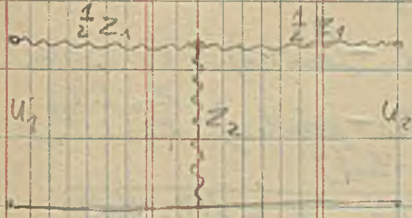
$J = \frac{\hat{U}_{10}}{\hat{Z}_{10}} + \hat{J}_2 \hat{S} = \frac{\hat{U}_2 \hat{S}}{\hat{Z}_{10}} + \hat{J}_2 \hat{S}$

$\left.\begin{array}{l} \hat{U}_1 = \hat{S}(\hat{U}_2 + \hat{J}_2 \hat{Z}_{12}) \\ \hat{J}_1 = \hat{S}\left(\hat{J}_2 + \frac{U_2}{\hat{Z}_{10}}\right) \end{array}\right\}$ stąd wniosek, że linia długa jest układem symetrycznym

jeżeli: $\overset{\text{czwórnik}}{\hat{S}} = \overset{\text{linia długa}}{\hat{S}}$

$\left.\begin{array}{l} \hat{Z}_0 = \hat{Z}_{10} \\ \hat{Z}_2 = \hat{Z}_{12} \end{array}\right\}$ wtedy ten czwórnik jest równoważny z linią długą

Czwórnik taki jest układem zastępczym dla linii długiej t.zw. sztuczna linia długa.



$\hat{S} = 1 + \frac{\frac{1}{2}\hat{Z}_1}{\hat{Z}_2}$; $\hat{S} = 1 + \frac{\hat{Z}_1}{2\hat{Z}_2}$ dla czwórnika

$\hat{S} = \cosh \hat{\gamma} l$ dla linii długiej

$S = \sqrt{\frac{1}{2}(\cosh 2al + \cos 2bl)}$; $\operatorname{tg} \gamma = \operatorname{tgh} al \operatorname{tg} bl$

$\hat{S} = S(\cos \gamma + j \sin \gamma)$

$\hat{S} = 1 + \frac{\hat{Z}_1}{2\hat{Z}_2}$; $\hat{Z}_{10} = \frac{1}{2}\hat{Z}_1 + \hat{Z}_2$ mając S i $\hat{Z}_{10}$ można znaleźć $\hat{Z}_2$

$2\hat{Z}_2 \hat{S} = 2\hat{Z}_2 + \hat{Z}_1$

$\hat{Z}_1 = 2\hat{Z}_2 \hat{S} - 2\hat{Z}_2 = 2\hat{Z}_2(\hat{S} - 1)$

$\hat{Z}_{10} = \hat{Z}_2(\hat{S} - 1) + \hat{Z}_2$; $\hat{Z}_{10} = \hat{S}\hat{Z}_2$; $\boxed{\hat{Z}_2 = \frac{\hat{Z}_{10}}{\hat{S}}}$

$\hat{Z}_1 = 2\frac{\hat{Z}_{10}}{\hat{S}}(\hat{S} - 1) = 2\hat{Z}_{10}\left(1 - \frac{1}{\hat{S}}\right)$

$\boxed{\hat{Z}_1 = 2\hat{Z}_{10}\left(1 - \frac{1}{\hat{S}}\right)}$ cewki o takich Z zastąpią daną linię długą

$\left.\begin{array}{l} \hat{U}_{10} = \hat{U}_2 \hat{S} \\ \hat{J}_{12} = \hat{J}_2 \hat{S} \end{array}\right\}$ $\frac{U_{10}}{U_2} = \frac{J_{12}}{J_2} = \hat{S} = \cosh \hat{\gamma} l$

$\frac{U_{10}}{J_{10}} = \hat{Z}_{10}$; $\frac{U_2 \hat{S}}{J_{10}} = \hat{Z}_{10}$

$$\left.\begin{aligned}\hat U_{10} &= \hat U_2 \cos h\, \hat\varkappa l\\ \hat I_{10}\hat Z &= \hat U_2 \sin h\, \hat\varkappa l\end{aligned}\right\} \frac{\hat U_{10}}{\hat I_{10}} = \hat Z_{10} = \hat Z\, \mathrm{ctg}\, h\, \hat\varkappa l$$

$$\left.\begin{aligned}\hat U_{1z} &= \hat I_2 \hat Z \sin h\, \hat\varkappa l\\ \hat I_{1z}\hat Z &= \hat U_2 \cos h\, \hat\varkappa l\end{aligned}\right\} \frac{\hat U_{1z}}{\hat I_{1z}} = \hat Z_{1z} = \hat Z\, \mathrm{tg}\, h\, \hat\varkappa l$$

$$\hat Z_{10}\hat Z_{1z} = \hat Z^2 \underbrace{\mathrm{ctg}\, h\,\hat\varkappa l \cdot \mathrm{tg}\, h\,\hat\varkappa l}_{=1} ; \quad \hat Z = \sqrt{\hat Z_{10}\hat Z_{1z}} \quad ; \cos h\alpha = \frac{1}{\sqrt{1-\mathrm{tg}^2 h\alpha}}$$

$$\hat S = \cos h\, \hat\varkappa l = \frac{1}{\sqrt{1-\mathrm{tg}^2 h\hat\varkappa l}} \qquad \hat Z_{1z} = \hat Z\, \mathrm{tg}\, h\,\hat\varkappa l \; : \; \mathrm{tg}\, h\,\hat\varkappa l = \frac{\hat Z_{1z}}{\hat Z}$$

$$\boxed{\mathrm{tg}\, h\,\hat\varkappa l = \frac{\hat Z_{1z}}{\sqrt{\hat Z_{10}\hat Z_{1z}}} = \sqrt{\frac{\hat Z_{1z}}{\hat Z_{10}}}} \qquad \boxed{\hat S = \sqrt{\frac{\hat Z_{10}}{\hat Z_{10} - \hat Z_{1z}}}}$$

tak samo dla linii długiej jak i czwórnika.

$$\hat S^2 = \frac{\hat Z_{10}}{\hat Z_{10} - \hat Z_{1z}} = \frac{Z_{10}e^{-j\varphi_{10}}}{Z_{10}e^{-j\varphi_{10}} - Z_{1z}e^{-j\varphi_{1z}}}$$

$$\hat Z_{10} = Z_{10}(\cos\varphi_{10} + j\sin\varphi_{10})$$
$$\hat Z_{1z} = Z_{1z}(\cos\varphi_{1z} + j\sin\varphi_{1z})$$

z pomiarów Z_{10} i φ_{10}, Z_{1z} i φ_{1z}

$$\hat S^2 = \frac{Z_{10}}{Z_{10} - Z_{1z}e^{-j(\varphi_{1z}-\varphi_{10})}} = \frac{Z_{10}[Z_{10} - Z_{1z}e^{j(\varphi_{1z}-\varphi_{10})}]}{[Z_{10} - Z_{1z}e^{-j(\varphi_{1z}-\varphi_{10})}][Z_{10} - Z_{1z}e^{j(\varphi_{1z}-\varphi_{10})}]}$$

$$\hat S^2 = \frac{Z_{10}[Z_{10} - Z_{1z}e^{j(\varphi_{1z}-\varphi_{10})}]}{Z_{10}^2 + Z_{1z}^2 - Z_{10}Z_{1z}e^{j(\varphi_{1z}-\varphi_{10})} - Z_{10}Z_{1z}e^{-j(\varphi_{1z}-\varphi_{10})}}$$

$$\hat S^2 = \frac{Z_{10}[Z_{10} - Z_{1z}e^{j(\varphi_{1z}-\varphi_{10})}]}{Z_{10}^2 + Z_{1z}^2 - Z_{10}Z_{1z}2\cos(\varphi_{1z}-\varphi_{10})}$$

$$\hat S^2 = \frac{Z_{10}[Z_{10} - Z_{1z}\cos(\varphi_{1z}-\varphi_{10}) + jZ_{1z}\sin(\varphi_{1z}-\varphi_{10})]}{Z_{10}^2 + Z_{1z}^2 - 2Z_{10}Z_{1z}\cos(\varphi_{1z}-\varphi_{10})}$$

$$Z_{10}^2 + Z_{1z}^2 - Z_{10}Z_{1z}e^{-j(\varphi_{1z}-\varphi_{10})} - Z_{10}Z_{1z}e^{j(\varphi_{1z}-\varphi_{10})} = Z_{10}^2 + Z_{1z}^2 - Z_{10}Z_{1z}[e^{j(\varphi_{1z}-\varphi_{10})} + e^{-j(\varphi_{1z}}$$

$$\hat S^2 = \frac{Z_{10}[Z_{10} - Z_{1z}\cos(\varphi_{1z}-\varphi_{10})]}{Z_{10}^2 + Z_{1z}^2 - 2Z_{10}Z_{1z}\cos(\varphi_{1z}-\varphi_{10})} - j\frac{Z_{10}Z_{1z}\sin(\varphi_{1z}-\varphi_{10})}{Z_{10}^2 + Z_{1z}^2 - 2Z_{10}Z_{1z}\cos(\varphi_{1z}-\varphi_{10})}$$

$$S^4 = \frac{Z_{10}^2[Z_{10} - Z_{1z}\cos(\varphi_{1z}-\varphi_{10})]^2 + Z_{10}^2Z_{1z}^2\sin^2(\varphi_{1z}-\varphi_{10})}{[Z_{10}^2 + Z_{1z}^2 - 2Z_{10}Z_{1z}\cos(\varphi_{1z}-\varphi_{10})]^2}$$

$$S^4 = \frac{Z_{10}^2[Z_{10}^2 - 2Z_{10}Z_{1z}\cos(\varphi_{1z}-\varphi_{10}) + Z_{1z}^2\cos^2(\varphi_{1z}-\varphi_{10}) + Z_{1z}^2\sin^2(\varphi_{1z}-\varphi_{10})]}{[Z_{10}^2 + Z_{1z}^2 - 2Z_{10}Z_{1z}\cos(\varphi_{1z}-\varphi_{10})]^2}$$

64 $$S^4 = \frac{Z_{10}^2[Z_{10}^2 - 2Z_{10}Z_{1z}\cos(\varphi_{1z}-\varphi_{10}) + Z_{1z}^2}{[Z_{10}^2 + Z_{1z}^2 - 2Z_{10}Z_{1z}\cos(\varphi_{1z}-\varphi_{10})]^2} = \frac{Z_{10}^2}{Z_{10}^2 + Z_{1z}^2 - 2Z_{10}Z_{1z}\cos(\varphi_{1z}-\varphi_{10})}$$

$$\boxed{S^2 = \frac{Z_{10}}{\sqrt{Z_{10}^2 + Z_{1z}^2 - 2 Z_{10} Z_{1z} \cos(\varphi_{1z} - \varphi_{10})}}}$$

$$\boxed{\operatorname{tg} 2\gamma = -\frac{Z_{1z} \sin(\varphi_{1z} - \varphi_{10})}{Z_{10} - Z_{1z} \cos(\varphi_{1z} - \varphi_{10})}}$$

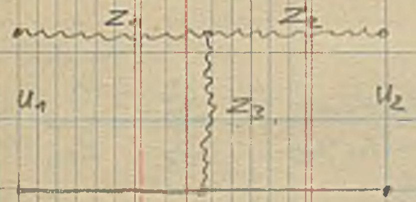
przy linii $S = \sqrt{\frac{1}{2}(\cosh 2al + \cos 2bl)}$; $\operatorname{tg} \gamma = \operatorname{tgh} al \operatorname{tg} bl$

$-\varphi_{1z} - (-\varphi_{10}) = \varphi_{10} - \varphi_{1z} = -(\varphi_{1z} - \varphi_{10})$

$$\overline{AB} = \sqrt{Z_{10}^2 + Z_{1z}^2 - 2 Z_{10} Z_{1z} \cos(\varphi_{1z} - \varphi_{10})}$$

$$\operatorname{tg} \alpha = \frac{BC}{CA} = -\frac{Z_{1z} \sin(\varphi_{1z} - \varphi_{10})}{OA - OC}$$

$$\operatorname{tg} \alpha = -\frac{Z_{1z} \sin(\varphi_{1z} - \varphi_{10})}{Z_{10} - Z_{1z} \cos(\varphi_{1z} - \varphi_{10})} \quad ; \quad \boxed{\alpha = 2\gamma}$$



$\hat{S}_0, \hat{S}_z$

$U_1, J_{10}, J_{1z}, P_{10}, P_{1z}$

$P_{10} = U_1 J_{10} \cos \varphi_{10}$; $\cos \varphi_{10} = \frac{P_{10}}{U_1 J_{10}}$

znajdujemy φ_{10} i φ_{1z} Z_{10} i Z_{1z}

$\frac{U_1}{J_{10}} = Z_{10}$; $\hat{Z}_{10} = Z_{10}(\cos \varphi_{10} - j \sin \varphi_{10})$; $\hat{Z}_{1z} = \dots$

Znaleźliśmy $\hat{Z}_{10}$ i $\hat{Z}_{1z}$ szukamy $\hat{S}_0, \hat{S}_z$

$$\hat{S}_0 \hat{S}_z = \frac{\hat{Z}_{10}}{\hat{Z}_{10} - \hat{Z}_{1z}} = \frac{\frac{\hat{U}_1}{\hat{J}_{10}}}{\frac{U_1}{\hat{J}_{10}} - \frac{U_1}{\hat{J}_{1z}}} \quad ; \quad |\hat{S}_0 \hat{S}_z| = \frac{\frac{1}{\hat{J}_{10}}}{\frac{1}{\hat{J}_{10}} - \frac{1}{\hat{J}_{1z}}} = \frac{\frac{1}{\hat{J}_{10}}}{\frac{\hat{J}_{1z} - \hat{J}_{10}}{\hat{J}_{10} \hat{J}_{1z}}} = \frac{|\hat{J}_{1z}|}{|\hat{J}_{1z} - \hat{J}_{10}|}$$

$$S_0 S_z = \frac{J_{1z}}{\sqrt{(J_{1z} \cos \varphi_{1z} - J_{10} \cos \varphi_{10})^2 + (J_{1z} \sin \varphi_{1z} - J_{10} \sin \varphi_{10})^2}}$$

$\hat{U}_1 = U_1$

$\hat{J}_{10} = J_{10}(\cos \varphi_{10} - j \sin \varphi_{10})$

$\hat{J}_{1z} = J_{1z}(\cos \varphi_{1z} - j \sin \varphi_{1z})$

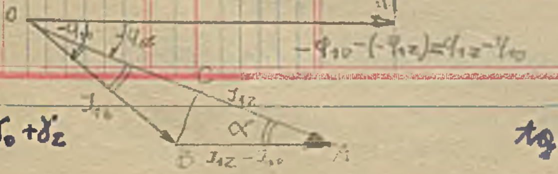
$J_{1z}^2 \cos^2 \varphi_{1z} + J_{10}^2 \cos^2 \varphi_{10} - 2 J_{1z} J_{10} \cos \varphi_{1z} \cos \varphi_{10} + J_{1z}^2 \sin^2 \varphi_{1z} +$

$+ J_{10}^2 \sin^2 \varphi_{10} - 2 J_{1z} J_{10} \sin \varphi_{1z} \sin \varphi_{10} = J_{1z}^2 + J_{10}^2 - 2 J_{1z} J_{10} \cos(\varphi_{1z} - \varphi_{10})$

$$\boxed{S_0 S_z = \frac{J_{1z}}{\sqrt{J_{1z}^2 + J_{10}^2 - 2 J_{1z} J_{10} \cos(\varphi_{1z} - \varphi_{10})}}}$$

sposobem uproszczonym 1. równanie

$\sphericalangle(\gamma_0 + \gamma_z) = \sphericalangle[J_{1z} ; (J_{1z} - J_{10})]$ względnie



$\alpha = \gamma_0 + \gamma_z$

$-\varphi_{10} - (-\varphi_{1z}) = \varphi_{1z} - \varphi_{10}$

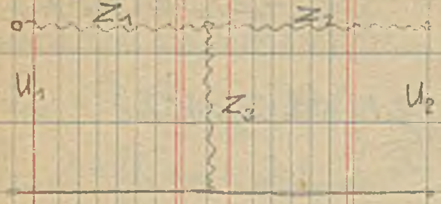
$$\operatorname{tg} \alpha = \frac{BC}{CA} = \frac{J_{10} \sin(\varphi_{1z} - \varphi_{10})}{OA - OC} = \frac{J_{10} \sin(\varphi_{1z} - \varphi_{10})}{J_{1z} - J_{10} \cos(\varphi_{1z} - \varphi_{10})}$$

$$\operatorname{tg} \alpha = \operatorname{tg}(\gamma_0 + \gamma_z)$$

$$\boxed{\operatorname{tg}(\gamma_0 + \gamma_z) = \frac{J_{10} \sin(\varphi_{1z} - \varphi_{10})}{J_{1z} - J_{10} \cos(\varphi_{1z} - \varphi_{10})}}$$

2-ie równanie

Pomiar czwórnika z drugiej strony.



$\hat Z_{20}$ i $\hat Z_{2z}$ $\qquad \varphi_{20}$ i φ_{2z}

$$\hat Z_{10}=\hat Z_1+\hat Z_3=\hat Z_3\left(1+\frac{\hat Z_1}{\hat Z_3}\right)=\hat Z_3\hat S_0$$

$$\hat Z_{20}=\hat Z_2+\hat Z_3=\hat Z_3\left(1+\frac{\hat Z_2}{\hat Z_3}\right)=\hat Z_3\hat S_z$$

$$\frac{|\hat S_0|}{|\hat S_z|}=\frac{|Z_{10}|}{|Z_{20}|} \qquad \boxed{\frac{S_0}{S_z}=\frac{Z_{10}}{Z_{20}}}$$ 3-e równanie

szukamy czwartego równania.

$$\frac{\hat S_0}{\hat S_z}=\frac{\hat Z_{10}}{\hat Z_{20}} \;;\; \frac{S_0e^{j\gamma_0}}{S_ze^{+\gamma_z}}=\frac{Z_{10}e^{j\varphi_{10}}}{Z_{20}e^{+\varphi_{20}}} \qquad \boxed{\gamma_0-\gamma_z=\varphi_{10}-\varphi_{20}}$$ 4-e równanie.

w układzie symetrycznym:

$$\hat S_0=\hat S_z \;;\; S_0=S_z=S \;;\quad \gamma_0=\gamma_z=\gamma$$

$$S^2=\frac{J_{1z}}{\sqrt{J_{1z}^2+J_{10}^2-2J_{1z}J_{10}\cos(\varphi_{1z}-\varphi_{10})}} \;;\; tg\,2\gamma=\frac{J_{10}\sin(\varphi_{1z}-\varphi_{10})}{J_{1z}-J_{10}\cos(\varphi_{1z}-\varphi_{10})} \;;\; \left.\frac{\hat S_0}{\hat S_z}=\frac{\hat Z_{20}}{\hat Z_{2z}}\right\}\text{wyprowadzić}$$

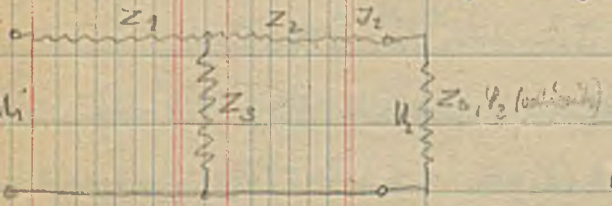
11. I. 1948 r.

$$\hat Z_{1z}=\hat Z_1+\frac{\hat Z_2\hat Z_3}{\hat Z_2+\hat Z_3}=\hat Z_1+\frac{\hat Z_2}{\hat S_z}=\frac{\hat S_z\hat Z_1+Z_2}{\hat S_z}$$

$$\hat Z_{2z}=\hat Z_2+\frac{\hat Z_1\hat Z_3}{\hat Z_1+\hat Z_3}=\hat Z_2+\frac{\hat Z_1}{\hat S_0}=\frac{\hat S_0\hat Z_2+\hat Z_1}{\hat S_0}$$

$$\frac{\hat Z_{1z}}{\hat Z_{2z}}=\frac{\hat S_0}{\hat S_z}\cdot\underbrace{\frac{\hat S_z\hat Z_1+\hat Z_3}{\hat S_0\hat Z_2+\hat Z_1}}_{=1} \;;\; \frac{\frac{\hat Z_2+\hat Z_3}{\hat Z_3}\hat Z_1+\hat Z_2}{\frac{\hat Z_1+\hat Z_3}{\hat Z_3}\hat Z_2+\hat Z_1}=\frac{\frac{\hat Z_1\hat Z_2+\hat Z_3\hat Z_1}{\hat Z_3}+\hat Z_2}{\frac{\hat Z_1\hat Z_2+\hat Z_3\hat Z_2}{\hat Z_3}+Z_1}=\frac{\hat Z_1\hat Z_3+\hat Z_2\hat Z_1+\hat Z_2\hat Z_3}{\hat Z_1\hat Z_2+\hat Z_2\hat Z_3+\hat Z_1\hat Z_3}=1$$

Każda linia ma maksymalną zdolność przesyłową.



$$P_2=U_2J_2\cos\varphi_2 \qquad P_2\sim|U_2J_2|$$

$$\left.\begin{aligned}\hat U_{10}&=\hat U_2\hat S_0\\ U_{1z}&=\hat J_2\hat S_z\hat Z_{1z}\end{aligned}\right\} \quad \left.\begin{aligned}\hat U_2&=\frac{\hat U_{10}}{\hat S_0}\\ J_2&=\frac{\hat U_{1z}}{\hat S_z\hat Z_{1z}}\end{aligned}\right\} \quad \hat U_2\hat J_2=\frac{\hat U_{10}\hat U_{1z}}{\hat S_0\hat S_z\hat Z_{1z}}$$

$\hat S_0, \hat S_z, \hat Z_{1z}=$ const. dla danego systemu przesyłowego

$$\hat U_2\hat J_2\sim\hat U_{10}\hat U_{1z}$$

$$\hat P_2\sim|\hat U_{10}\hat U_{1z}|$$

$\hat{U}_1 = U_1$ kierunek podstawowy; istnieją kąty: $\sphericalangle(\hat{U}_1, \hat{U}_{10}) = \psi_0$

$U_{10}e^{j\psi_0} = U_2e^{j\psi}S_0e^{j\gamma_0}$; $\psi_0 = \psi + \gamma_0$ $\qquad \sphericalangle(\hat{U}_1, \hat{U}_{1z}) = \psi_z$

$\hat{U}_{1z} = \frac{\hat{U}_2}{\hat{Z}_0}\hat{S}_z\hat{Z}_{1z}$; $\psi_z = \psi + \varphi_2 + \gamma_z \mp \varphi_{1z}$; $\sphericalangle(\hat{U}_1, \hat{U}_2) = \psi.$

$\hat{Z}_0 = Z_0(\cos\varphi_2 - j\sin\varphi_2) = Z_0e^{-j\varphi_2}$

$\sphericalangle(\hat{U}_{10}, \hat{U}_{1z}) = \alpha = \psi_0 - \psi_z = \psi + \gamma_0 - \psi - \varphi_2 - \gamma_z \mp \varphi_{1z}$

$\alpha = \varphi_{1z} - \varphi_2 + \gamma_0 - \gamma_z = const$

Kąt między U_{1z} i U_{10} niezależny jest od obciążenia czyli od U_2 i I_2.

$\hat{U}_1 = \hat{U}_{10} + \hat{U}_{1z}$; $\hat{U}_1 = U_1$

$\alpha = \psi_0 - (-\psi_2) = \psi_0 + \psi_z$

$\alpha = \varphi_{1z} - \varphi_2 + \gamma_0 - \gamma_z = const.$

gdy iloczyn $U_{10} \cdot U_{1z}$ będzie mieć maximum to otrzyma maximum przesyłanej mocy.

$a^2 = x^2 + y^2 - 2xy\cos(180-\alpha)$

$a^2 = x^2 + y^2 + 2xy\cos\alpha$

$a^2 = x^2 + y^2 + 2xy\cos\alpha \qquad m = xy$

$\frac{dm}{dx} = x\frac{dy}{dx} + y$

$0 = 2x + 2y\frac{dy}{dx} + 2x\frac{dy}{dx}\cos\alpha + 2y\cos\alpha$

$\frac{dy}{dx}(y + x\cos\alpha) + x + y\cos\alpha = 0$

$\frac{dy}{dx} = -\frac{x + y\cos\alpha}{y + x\cos\alpha}$; $\frac{dm}{dx} = -x\frac{x + y\cos\alpha}{y + x\cos\alpha} + y = \frac{-x^2 - xy\cos\alpha + y^2 + xy\cos\alpha}{y + x\cos\alpha} = \frac{y^2 - x^2}{y + x}$

$x = y$

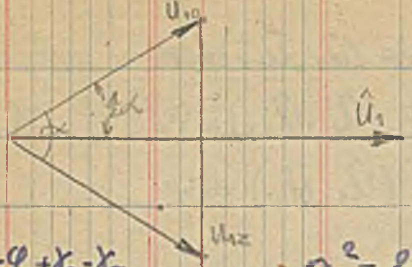
$\frac{d^2m}{dx^2} = x\frac{d^2y}{dx^2} + 2\frac{dy}{dx}$; $0 = 1 + y\frac{d^2y}{dx^2} + \frac{dy}{dx}\cdot\frac{dy}{dx} + \frac{d^2y}{dx^2}x\cos\alpha + \frac{dy}{dx}\cos\alpha + \frac{dy}{dx}\cos\alpha$

$0 = 2 + \frac{d^2y}{dx^2}(y + x\cos\alpha) + 2\cos\alpha$; $0 = 1 + y\frac{d^2y}{dx^2} + 1 + \frac{d^2y}{dx^2}x\cos\alpha + 2\cos\alpha$

$\frac{d^2y}{dx^2} - \left(x\frac{d^2y}{dx^2} + 2\right) = \frac{-2(1+\cos\alpha)}{y + x\cos\alpha}$

$\frac{d^2m}{dx^2} = x\frac{d^2y}{dx^2} + 2 = \frac{-2x(1+\cos\alpha)}{y + x\cos\alpha} - 2$

$\frac{d^2m}{dx^2} = \frac{-2x(1+\cos\alpha)}{x(1+\cos\alpha)} - 2$; $\frac{d^2m}{dx^2} < 0$



$P_2=U_2J_2\cos\varphi_2$

$\hat{U}_{10}=\hat{U}_2\hat{S}_0$; $\hat{U}_2=\frac{\hat{U}_{10}}{\hat{S}_0}$

$\hat{U}_{12}=\hat{J}_2\hat{S}_z\hat{Z}_{12}$; $\hat{J}_2=\frac{\hat{U}_{1z}}{\hat{S}_z\hat{Z}_{12}}$

$P_2=\frac{U_{10}^2\cos\varphi_2}{S_0S_zZ_{1z}}$

$U_1=2U_{10}\cos\frac{1}{2}\alpha$

$\alpha=\varphi_{12}-\varphi_2+\gamma_0-\gamma_Z$

$a^2=2x^2+2x^2\cos\alpha$; $x^2=\frac{a^2}{2(1+\cos\alpha)}$; $x=\frac{a}{\sqrt{2(1+\cos\alpha)}}$

$a^2=2x^2(1+\cos\alpha)$ $\quad \sqrt{2(1+\cos\alpha)}=2\cos\frac{1}{2}\alpha$

$U_{10}=\frac{U_1}{2\cos\frac{1}{2}\alpha}$

$$\boxed{P_{m2}=\frac{U_1^2\cos\varphi_2}{4S_0S_zZ_{1z}\cos^2\frac{1}{2}\alpha}}$$

maxymalna moc przesyłowa

$\alpha=\varphi_{z}-\varphi_2+\gamma_0-\gamma_2$

$$P_{m_2}=\frac{U_1^2\cos\varphi_2}{4S_0S_zZ_{1z}\cos^2\frac{1}{2}(\varphi_{12}-\varphi_2+\gamma_0-\gamma_2)}$$

W układzie symetrycznym $\gamma_0=\gamma_2$

$$P_{m_2}=\frac{U_1^2\cos\varphi_2}{4S_0S_zZ_{1z}\cos^2\frac{1}{2}(\varphi_{1z}-\varphi_2)}$$

$P_{2m}=f(\varphi_2)$ to znajdujemy przy jakim φ_2 będzie można przesłać max. moc.

$\frac{\cos\varphi_2}{\cos^2\frac{1}{2}(\varphi_{1z}-\varphi_2)}=m$; $m=f(\varphi_2)$

21. I. 1949 r.

$$P_{m_2}=\frac{U_1^2\cos\varphi_2}{2S_0S_zZ_{1z}[1+\cos(\varphi_{1z}-\varphi_2+\gamma_0-\gamma_z)]}=f(\varphi_2)$$

Przy jakim φ_2 musi być największa moc

$m=\frac{\cos\varphi_2}{1+\cos(\varphi_{1z}-\varphi_2+\gamma_0-\gamma_z)}$

$\frac{dm}{d\varphi_2}$; $d(x.z)=x\,dz+z\,dx$; $z=\frac{1}{y}$; $dz=\frac{1}{y^2}dy$

$d\left(\frac{x}{y}\right)=-x\frac{1}{y^2}dy+\frac{1}{y}dx=\frac{y\,dx-x\,dy}{y^2}$

$x=\cos\varphi_2$; $y=1+\cos(\varphi_{1z}-\varphi_2+\gamma_0-\gamma_z)$

$$\frac{dm}{d\varphi_2}=-\frac{[1+\cos(\varphi_{1z}-\varphi_2+\gamma_0-\gamma_z)]\sin\varphi_2-\cos\varphi_2\sin(\varphi_{1z}-\varphi_2+\gamma_0-\gamma_z)}{[1+\cos(\varphi_{1z}-\varphi_2+\gamma_0-\gamma_z)]^2}$$

$$-[1+\cos(\underbrace{\varphi_{1z}-\varphi_2+\gamma_0-\gamma_z}_{\alpha})]\sin\varphi_2-\cos\varphi_2\sin(\underbrace{\varphi_{1z}-\varphi_2+\gamma_0-\gamma_z}_{\alpha})=0$$

$-[1+\cos\alpha]\sin\varphi_2=\cos\varphi_2\sin\alpha$; $\operatorname{tg}\varphi_2=-\frac{\sin\alpha}{1+\cos\alpha}=-\frac{2\sin\frac{\alpha}{2}\cos\frac{\alpha}{2}}{2\cos^2\frac{1}{2}\alpha}$

$\operatorname{tg}\varphi_2=-\frac{\sin\frac{\alpha}{2}}{\cos\frac{\alpha}{2}}=-\operatorname{tg}\frac{\alpha}{2}$; $2\varphi_2=-[\varphi_{1z}-\varphi_2+\gamma_0-\gamma_z]=\varphi_2-\varphi_{1z}-\gamma_0+\gamma_z$

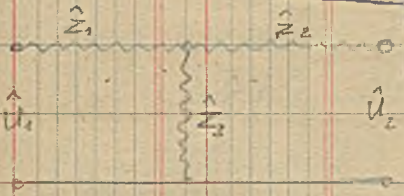
$\boxed{\varphi_2=-(\varphi_{1z}+\gamma_0-\gamma_z)}$ przy tym φ_2 P_{m2} będzie maxymalne.

$$P_{m_2}=\frac{U_1^2\cos\varphi_2}{2S_0S_zZ_{1z}[1+\cos(\varphi_{1z}-\varphi_2+\delta_0-\delta_z)}\ ;\quad \varphi_2=-\varphi_{1z};\quad S_0=S_z;\quad \delta_0=\delta_z$$

$$P_{m_2}=\frac{U_1^2\cos\varphi_{1z}}{2S^2Z_{1z}[1+\cos 2\varphi_{1z}]}\ ;\quad P_{2m}=\frac{U_1^2\cos\varphi_{1z}}{4SZ_{1z}\cos^2\varphi_{1z}}\ ;$$

$$\boxed{P_{2m}=\frac{U_1^2}{4SZ_{1z}\cos\varphi_{1z}}}$$

równania z klucza



druga

$\hat{U}_1$, $\hat{I}_0$ — prąd gdy przyłożone jest $\hat{U}_1$, niezależnie jakie na końcu będzie $\hat{U}_2$, otrzymamy nieprzepisowe $\hat{U}_2$. $\hat{U}_1$, $\hat{I}_z$ otrzymamy φ_0 i φ_z.

przy biegu luzem $\hat{U}_2$, $\hat{I}_2=0$ wtedy na początku $\hat{U}_{10}$, $\hat{I}_{10}$, φ_{10}

-"- -"- zwarcia $\hat{U}_2=0$, $\hat{I}_z$ kiedy -"- -"- $\hat{U}_{10}$, $\hat{I}_{1z}$, φ_{1z}

Chcemy znaleźć $\hat{I}_0$, $\hat{I}_1$, φ_0, φ_z.

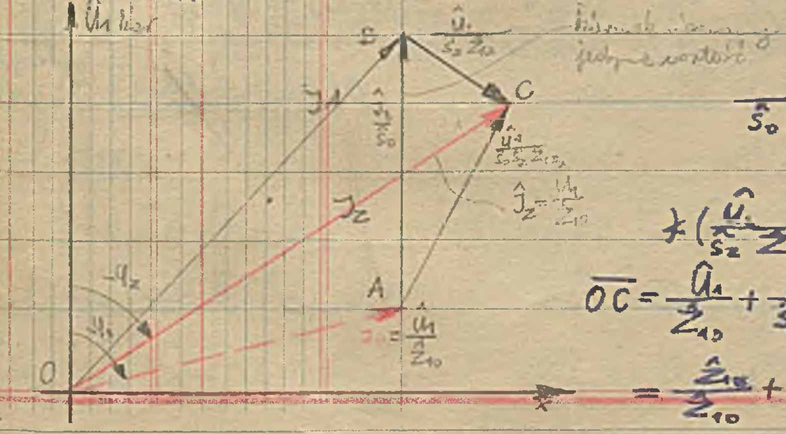
$$\left.\begin{aligned}\hat{U}_1&=\hat{U}_{10}+\hat{U}_{1z}=\hat{U}_2\hat{S}_0+\hat{I}_2\hat{S}_z\hat{Z}_{1z}\\ \hat{I}_1&=\hat{I}_{10}+\hat{I}_{1z}=\frac{\hat{U}_2\hat{S}_0}{\hat{Z}_{10}}+\hat{I}_2\hat{S}_z\end{aligned}\right\}\ \hat{S}_0\hat{S}_z=\frac{\hat{Z}_{10}}{\hat{Z}_{10}-\hat{Z}_{1z}}\ \Big|\qquad \hat{U}_2\hat{S}_0=\hat{U}_1-\hat{I}_2\hat{S}_z\hat{Z}_{1z}$$

$$\hat{I}_1=\frac{\hat{U}_1-\hat{I}_2\hat{S}_z\hat{Z}_{1z}}{\hat{Z}_{10}}+\hat{I}_2\hat{S}_z$$

$$\hat{I}_1=\frac{\hat{U}_1}{\hat{Z}_{10}}+I_2\left(\hat{S}_z-\frac{\hat{S}_z\hat{Z}_{1z}}{\hat{Z}_{10}}\right)=\frac{\hat{U}_1}{\hat{Z}_{10}}+\frac{I_2}{\hat{S}_0}\left(\hat{S}_0\hat{S}_z-\frac{\hat{S}_0\hat{S}_z\hat{Z}_{1z}}{\hat{Z}_{10}}\right)=$$

$$=\frac{\hat{Z}_{10}}{\hat{Z}_{10}-\hat{Z}_{1z}}-\frac{\hat{Z}_{10}}{\hat{Z}_{10}-\hat{Z}_{1z}}\cdot\frac{\hat{Z}_{1z}}{\hat{Z}_{10}}=\frac{\hat{Z}_{10}}{\hat{Z}_{10}-\hat{Z}_{1z}}-\frac{\hat{Z}_{1z}}{\hat{Z}_{10}-\hat{Z}_{1z}}=\frac{\hat{Z}_{10}-\hat{Z}_{1z}}{\hat{Z}_{10}-\hat{Z}_{1z}}=1$$

$$\hat{I}_1=\frac{\hat{U}_1}{\hat{Z}_{10}}+\frac{\hat{I}_2}{\hat{S}_0}\ ;\quad \frac{\hat{U}_1}{\hat{Z}_{10}}=\hat{I}_0\ ;\quad \hat{I}_1=\hat{I}_0+\frac{\hat{I}_2}{\hat{S}_0}$$



$$\hat{U}_{10}=\hat{U}_2\hat{S}_0$$

$$\frac{\hat{U}_1}{\hat{S}_0\hat{S}_z\hat{Z}_{1z}}=\frac{\hat{U}_{10}}{\hat{S}_0\hat{S}_z\hat{Z}_{1z}}+\frac{\hat{U}_{1z}}{\hat{S}_0\hat{S}_z\hat{Z}_{1z}}=$$

$$=\frac{\hat{U}_2}{\hat{S}_z\hat{Z}_{1z}}+\frac{\hat{I}_2}{\hat{S}_0}$$

$$\angle\left(\frac{\hat{U}_2}{\hat{S}_z\hat{Z}_{1z}};\frac{\hat{I}_2}{\hat{S}_0}\right)=\angle(\hat{U}_{10};\hat{U}_{1z})=\alpha=\text{const}$$

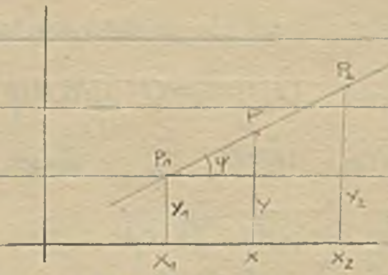
$$\overline{OC}=\frac{\hat{U}_1}{\hat{Z}_{10}}+\frac{\hat{U}_1}{\hat{S}_0\hat{S}_z\hat{Z}_{1z}}=\frac{\hat{U}_1}{\hat{Z}_{1z}}\left(\frac{\hat{Z}_{1z}}{\hat{Z}_{10}}+\frac{1}{\hat{S}_0\hat{S}_z}\right)=$$

$$=\frac{\hat{Z}_{1z}}{\hat{Z}_{10}}+\frac{\hat{Z}_{10}-\hat{Z}_{1z}}{\hat{Z}_{10}}=\frac{\hat{Z}_{1z}-\hat{Z}_{1z}+\hat{Z}_{10}}{\hat{Z}_{10}}=\frac{\hat{Z}_{10}}{\hat{Z}_{10}}=1$$

$$\overline{OC}=\frac{\hat{U}_1}{\hat{Z}_{1z}}=\hat{I}_z$$

Miejscem geometrycznym punktu B o α = const jest koło.

Przesuwamy środek osi współrzędnych



$$\frac{y-y_1}{x-x_1}=\frac{y_2-y_1}{x_2-x_1}=tg\,\psi$$

$$tg\,\psi=\frac{tg\,\psi_2-tg\,\psi_1}{1+tg\,\psi_1\,tg\,\psi_2}$$

$$A(0,0)\ ,\ C(a,0)\ ,\ B(x_B,y_a)$$

$$AB:\frac{y-0}{x-0}=\frac{y_a-0}{x_B-0}=\frac{y_a}{x_B}=tg\,\psi_1\ ;\quad BC:\frac{y-y_B}{x-x_B}=\frac{0-y_B}{a-x_B}=\frac{-y_B}{a-x_B}=tg\,\psi_2$$

$$tg\,\alpha=\frac{tg\,\psi_2-tg\,\psi_1}{1+tg\,\psi_2\,tg\,\psi_1}=\frac{\frac{y}{x-a}-\frac{y}{x}}{1+\frac{y^2}{x(x-a)}}=\frac{\frac{xy-y(x-a)}{x(x-a)}}{\frac{x(x-a)+y^2}{x(x-a)}}=\frac{xy-y(x-a)}{x^2-ax+y^2}$$

$$tg\,\alpha(x^2-ax+y^2)=ay$$

$$tg\,\alpha\left[x^2-ax+y^2-\frac{ay}{tg\alpha}\right]=0$$

$$x^2-ax+\frac{1}{4}a^2+y^2-\frac{ay}{tg\alpha}+\frac{1}{4}\frac{a^2}{tg^2\alpha}-\frac{1}{4}a^2-\frac{1}{4}\frac{a^2}{tg^2\alpha}=0$$

$$\left(x-\frac{1}{2}a\right)^2+\left(y-\frac{a}{2tg\alpha}\right)^2=\frac{1}{4}a^2+\frac{1}{4}\frac{a^2}{tg^2\alpha}$$

$$\left(x-\frac{1}{2}a\right)^2+\left(y-\frac{1}{2}\frac{a}{tg\alpha}\right)^2=\frac{1}{4}a^2\left(1+\frac{1}{tg^2\alpha}\right)=\frac{a^2}{4\sin^2\alpha}$$

$$x^2+y^2=r^2$$

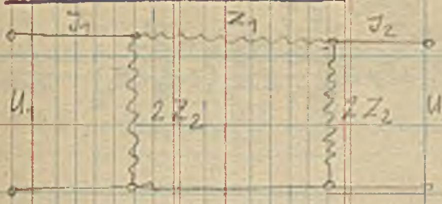
$$(x-x_1)^2+(y-y_1)^2=r^2$$

$$r=\frac{a}{2\sin\alpha}$$

$O\left(\frac{1}{2}a;\ \frac{1}{2}\frac{a}{tg\,\alpha}\right)$ współrzędne środka

Czwórnik Π

24.I.1949r.



$$\hat{U}_1-\hat{U}_2=\hat{Z}_1\left(\hat{J}_1-\frac{\hat{U}_1}{2\hat{Z}_2}\right)$$

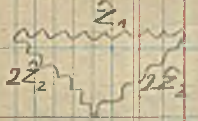
$$\hat{J}_1-\hat{J}_2=\frac{1}{2\hat{Z}_2}(\hat{U}_1+\hat{U}_2)$$

1). Bieg luzem: $\hat{U}_2,\ \hat{J}_2=0 \quad \hat{J}_{10},\ \hat{U}_{10}$

$$\left.\begin{aligned}\hat{U}_{10}-\hat{U}_2&=\hat{Z}_1\left(\hat{J}_{10}-\frac{\hat{U}_{10}}{2\hat{Z}_2}\right)\\ \hat{J}_{10}&=\frac{1}{2\hat{Z}_2}(\hat{U}_{10}+\hat{U}_2)\end{aligned}\right\}$$

$$\hat{U}_{10}-\hat{U}_2=\hat{Z}_1\left(\frac{\hat{U}_{10}}{2\hat{Z}_0}+\frac{\hat{U}_2}{2\hat{Z}_2}-\frac{\hat{U}_{10}}{2\hat{Z}_2}\right);$$

$$\hat{U}_{10}=\frac{\hat{U}\hat{Z}_1}{2\hat{Z}_2}+\hat{U}_2=\hat{U}_2\underbrace{\left(1+\frac{\hat{Z}_1}{2\hat{Z}_2}\right)}_{\hat{S}}$$



$$\boxed{\hat{Z}_{10}=\frac{(2\hat{Z}_2+\hat{Z}_1)2\hat{Z}_2}{2\hat{Z}_2+\hat{Z}_1+2\hat{Z}_2}}$$

$$\boxed{\hat{U}_{10}=\hat{U}_2\hat{S}}$$

$$\boxed{\hat{J}_{10}=\frac{\hat{U}_2\hat{S}}{\hat{Z}_{10}}}$$

2) Bieg zwarcia: $\hat{U}_2=0$; $\hat{J}_2$; $\hat{U}_{1z}$; $\hat{J}_{1z}$

$$\left.\begin{aligned}\hat{U}_{1z}&=\hat{Z}_1\left(\hat{J}_{1z}-\frac{\hat{U}_{1z}}{2\hat{Z}_2}\right)\\ \hat{J}_{1z}-\hat{J}_2&=\frac{\hat{U}_{1z}}{2\hat{Z}_2}\end{aligned}\right\}$$

$$\hat{U}_{1z}=2\hat{Z}_2(\hat{J}_{1z}-\hat{J}_2)$$

$$2\hat{Z}_2(\hat{J}_{1z}-\hat{J}_2)=\hat{Z}_1(\cancel{\hat{J}_{1z}}-\cancel{\hat{J}_{1z}}+\hat{J}_2)$$

$$2\hat{Z}_2\hat{J}_{1z}=\hat{J}_2\hat{Z}_1+2\hat{J}_2\hat{Z}_2$$

$$\hat{Z}_{1z}=\frac{2\hat{Z}_1\hat{Z}_2}{2\hat{Z}_2+\hat{Z}_1}$$

$$\boxed{\hat{Z}_{1z}=\frac{\hat{Z}_1}{\hat{S}}} \quad \boxed{\hat{J}_{1z}=\hat{J}_2\hat{S}} \quad \boxed{U_{1z}=J_2\hat{S}\hat{Z}_{1z}}$$

$$\hat{J}_{1z}=\hat{J}_2\left(\frac{2\hat{Z}_2+\hat{Z}_1}{2\hat{Z}_2}\right)=\hat{J}_2\underbrace{\left(1+\frac{\hat{Z}_1}{2\hat{Z}_2}\right)}_{\hat{S}}$$

3). Bieg normalny.

$$\hat{J}_1=\frac{1}{2\hat{Z}_1}\hat{U}_1+\frac{1}{2\hat{Z}_1}U_2+\hat{J}_2 ;$$

$$\hat{U}_1-\hat{U}_2=\hat{Z}_1\left[\frac{\hat{U}_1}{2\hat{Z}_2}+\frac{\hat{U}_2}{2\hat{Z}_2}+\hat{J}_2-\frac{\hat{U}_1}{2\hat{Z}_2}\right]$$

$$\hat{U}_1=\frac{\hat{Z}_1}{2\hat{Z}_2}\hat{U}_2+\hat{J}_2\hat{Z}_1+\hat{U}_2=\hat{U}_2\underbrace{\left(1+\frac{\hat{Z}_1}{2\hat{Z}_2}\right)}_{\hat{S}}+\hat{J}_2\hat{Z}_1$$

$$\hat{U}_1=\hat{U}_2\hat{S}+\hat{J}_2\hat{S}\hat{Z}_{1z}$$

$$\hat{Z}_1=\hat{Z}_{1z}\hat{S}$$

$$\boxed{\hat{U}_1=\hat{U}_{10}+\hat{U}_{1z}}$$

$$(\hat{J}_1-\hat{J}_2)2\hat{Z}_2=\hat{U}_1+\hat{U}_2 ; \quad \hat{U}_1=-\hat{U}_2+2\hat{Z}_2(\hat{J}_1-\hat{J}_2)$$

$$\left.\begin{aligned}\hat{U}_1+\hat{U}_2&=2\hat{Z}_2(\hat{J}_1-\hat{J}_2)\\ \hat{U}_1-\hat{U}_2&=\hat{Z}_1\left(\hat{J}_1-\frac{\hat{U}_1}{2\hat{Z}_2}\right)\end{aligned}\right\}$$

$$2\hat{U}_2=2\hat{Z}_2(\hat{J}_1-\hat{J}_2)-\hat{J}_1\hat{Z}_1+\frac{\hat{U}_1\hat{Z}_1}{2\hat{Z}_2}$$

$$2\hat{U}_2=2\hat{Z}_2(\hat{J}_1-\hat{J}_2)-\hat{Z}_1\hat{J}_1+(\hat{J}_1-\hat{J}_2)\hat{Z}_1-\frac{\hat{U}_2}{2\hat{Z}_2}\hat{Z}_1$$

$$2\hat{U}_2+\frac{\hat{U}_2\hat{Z}_1}{2\hat{Z}_2}=(\hat{J}_1-\hat{J}_2)(\hat{Z}_1+2\hat{Z}_2)-\hat{Z}_1\hat{J}_1$$

$$2\hat{U}_2+\frac{\hat{U}_2\hat{Z}_2}{2\hat{Z}_1}=\cancel{\hat{J}_1\hat{Z}_1}-\hat{J}_2\hat{Z}_1+2\hat{Z}_2\hat{J}_1-\hat{J}_2 2\hat{Z}_2-\cancel{\hat{Z}_1\hat{J}_1}=-\hat{J}_2(2\hat{Z}_2+\hat{Z}_1)+2\hat{Z}_2\hat{J}_1$$

[illegible]

$$\hat{J}_1=\frac{2\hat{U}_2}{2\hat{Z}_2}+\frac{\hat{U}_2\hat{Z}_1}{(2\hat{Z}_2)^2}+\frac{J_2(2\hat{Z}_2+\hat{Z}_1)}{2\hat{Z}_2}=\hat{J}_2\hat{S}+\hat{U}_2\hat{S}\left(\frac{1}{\hat{S}\hat{Z}_2}+\frac{\hat{Z}_1}{(2\hat{Z}_2)^2\hat{S}}\right)$$

$$\frac{1}{\hat{S}\hat{Z}_2}+\frac{\hat{Z}_1}{(2\hat{Z}_2)^2\hat{S}}=\frac{2\hat{Z}_2}{(2\hat{Z}_2+\hat{Z}_1)\hat{Z}_2}+\frac{\hat{Z}_1}{2\hat{Z}_2(2\hat{Z}_2+\hat{Z}_1)}=\frac{2}{2\hat{Z}_2+\hat{Z}_1}+\frac{\hat{Z}_1}{2\hat{Z}_2(2\hat{Z}_2+\hat{Z}_1)}=\frac{4\hat{Z}_2+\hat{Z}_1}{2\hat{Z}_2(2\hat{Z}_2+\hat{Z}_1)}=\frac{1}{\hat{Z}_{10}}$$

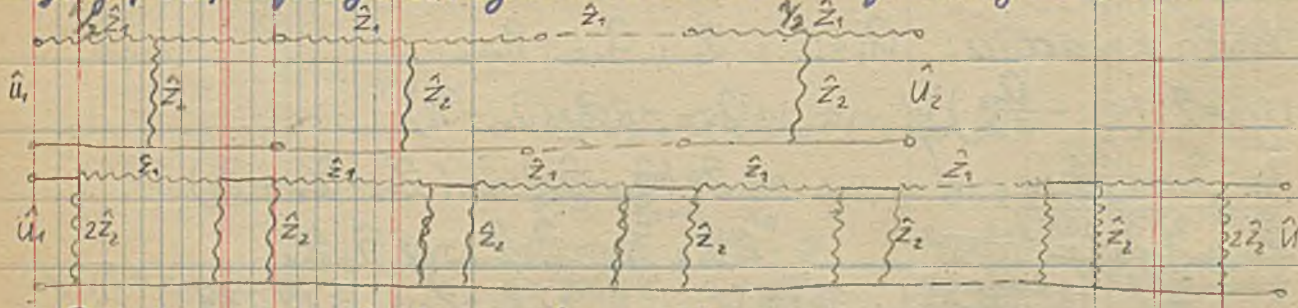
$$\boxed{\hat{J}_1=\hat{S}\left(\hat{J}_2+\frac{\hat{U}_2}{\hat{Z}_{10}}\right)}$$

$\hat{S}=\cosh\hat{\gamma}l=\hat{S}$ $\hat{Z}_{1z}=\hat{Z}_{12}$ dla linii długiej $\hat{S}=\frac{2\hat{Z}_2+\hat{Z}_1}{2\hat{Z}_2}$; $\frac{2\hat{Z}_1\hat{Z}_2}{2\hat{Z}_2+\hat{Z}_1}=\hat{Z}_{12}$; $\frac{\hat{Z}_1}{\hat{S}}=Z_{12}$

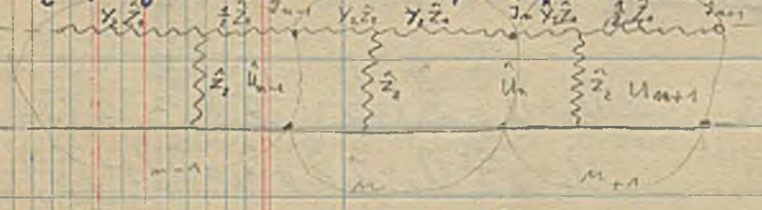
$$\boxed{\hat{Z}_1=Z_{1z}\hat{S}}$$ $2\hat{Z}_2+\hat{Z}_1=2\hat{Z}_2\hat{S}$; $2\hat{Z}_2+Z_{12}\hat{S}=2\hat{Z}_2\hat{S}$

$$\boxed{\hat{Z}_2=\frac{Z_{1z}\hat{S}}{2(\hat{S}-1)}}$$ $2\hat{Z}_2(1-\hat{S})=-Z_{1z}\hat{S}$; $\hat{Z}_2$ dla czwórnika Π gdy był równoważny z linią długą.

Gdy połączymy szereg czwórników otrzymamy linię łańcuchową.



Różnią się tylko końcami i początkami.



$$\left(\begin{array}{ll}\hat{U}_{n-1}-2\hat{S}U_n+\hat{U}_{n+1}=0 & \hat{S}=1+\frac{\hat{Z}_1}{2\hat{Z}_2}\\ \boxed{\hat{U}_n=B_1e^{\hat{\gamma}n}+B_2e^{-\hat{\gamma}n}} & \hat{J}_n=\frac{2}{\hat{Z}_1(\hat{S}+1)}=\end{array}\right)$$

1). $\hat{U}_{n-1}-\hat{U}_n=\frac{\hat{Z}_1}{2}(\hat{J}_{n-1}+\hat{J}_n)$

2). $\hat{J}_{n-1}-\hat{J}_n=\frac{1}{\hat{Z}_2}\left(\hat{U}_{n-1}-\frac{\hat{Z}_1 J_{n-1}}{2}\right)$

3). $\hat{U}_n-\hat{U}_{n+1}=\frac{\hat{Z}_1}{2}(\hat{J}_n+\hat{J}_{n+1})$

4). $\hat{J}_n-\hat{J}_{n+1}=\frac{1}{\hat{Z}_2}\left(\hat{U}_n-\frac{\hat{Z}_1}{2}J_n\right)$

$$\left.\begin{array}{l}\hat{J}_{n-1}+\hat{J}_n=\frac{2}{\hat{Z}_1}(\hat{U}_{n-1}-\hat{U}_n)\\ \hat{J}_{n-1}-\hat{J}_n=\frac{1}{\hat{Z}_2}\left(\hat{U}_{n-1}-\frac{\hat{Z}_1}{2}J_{n-1}\right)\end{array}\right\}$$

$$2\hat{J}_{n-1}=\hat{U}_{n-1}\left(\frac{2}{\hat{Z}_1}+\frac{1}{\hat{Z}_2}\right)-\frac{2}{\hat{Z}_1}\hat{U}_n-\frac{\hat{Z}_1}{2\hat{Z}_2}\hat{J}_{n-1}$$

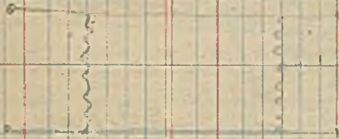
$$2\hat{J}_{n-1}+\frac{\hat{Z}_1}{2\hat{Z}_2}J_{n-1}=\frac{2}{\hat{Z}_1}\underbrace{\left(1+\frac{\hat{Z}_1}{2\hat{Z}_2}\right)}_{\hat{S}}U_{n-1}-\frac{2}{\hat{Z}_1}\hat{U}_n$$

$$\hat{J}_{n-1}\left(2+\frac{\hat{Z}_1}{2Z_2}\right)=\frac{2}{Z_1}\hat{U}_{n-1}\hat{s}-\frac{2}{Z_1}\hat{U}_n$$

$$J_{n-1}(1+\hat{s})=\frac{2}{Z_1}(\hat{U}_{n-1}\hat{s}-U_n)$$

$$\boxed{\hat{J}_{n-1}=\frac{2}{Z_1(1+\hat{s})}(\hat{U}_{n-1}\hat{s}-\hat{U}_n)}$$

25. I. 1949 r.



$$\hat{U}_1-\hat{U}_2=\hat{Z}_1\left(\hat{J}_1-\frac{1}{2Z_2}\hat{U}_1\right)$$

$$\hat{J}_1-\hat{J}_2=\frac{1}{2Z_2}(\hat{U}_1+\hat{U}_2)$$

$\hat{J}_2=\,?$ $\quad\hat{U}_1=\hat{Z}_1\hat{J}_1-\frac{\hat{Z}_1}{2Z_2}\hat{U}_1+\hat{U}_2$; $\quad\hat{U}_1+\frac{\hat{Z}_1}{2Z_2}\hat{U}_1=\hat{Z}_1\hat{J}_1+\hat{U}_2$

$\hat{U}_1\hat{s}=\hat{Z}_1\hat{J}_1+\hat{U}_2$; $\quad\hat{U}_1=\frac{\hat{U}_2}{\hat{s}}+\frac{\hat{Z}_1\hat{J}_1}{\hat{s}}$

$$\hat{J}_1=\frac{1}{2Z_2}\left(\frac{\hat{U}_2}{\hat{s}}+\frac{\hat{Z}_1\hat{J}_1}{\hat{s}}+\hat{U}_2\right)+\hat{J}_2=\frac{1}{2Z_2\hat{s}}\hat{U}_2+\frac{1}{2Z_2}\hat{U}_2+\frac{\hat{Z}_1\hat{J}_1}{2Z_2\hat{s}}+\hat{J}_2$$

$$\hat{J}_1\left(1-\frac{\hat{Z}_1}{2Z_2\hat{s}}\right)=\hat{J}_2+\hat{U}_2\left(\frac{1}{2Z_2\hat{s}}+\frac{1}{2Z_2}\right)$$

$\hat{J}_1=\hat{J}_2\frac{2\hat{Z}_2\hat{s}}{2Z_2\hat{s}-Z_1}+\hat{U}_2\left(\frac{1}{2Z_2\hat{s}}+\frac{1}{2Z_2}\right)\frac{2Z_2\hat{s}}{2Z_2\hat{s}-Z_1}$; $\quad\frac{2\hat{Z}_2}{2Z_2\hat{s}-Z_1}=\frac{2\hat{Z}_2}{2Z_2\left(\frac{2Z_2+Z_1}{2Z_2}-Z_1\right)}=\frac{2\hat{Z}_2}{2Z_2+Z_1-Z_1}=1$

$$\hat{J}_1=\hat{J}_2\hat{s}\underbrace{\frac{2\hat{Z}_2}{2Z_2\hat{s}-Z_1}}_{=1}+\hat{U}_2\hat{s}\left(\frac{1}{2Z_2\hat{s}}+\frac{1}{2Z_2}\right)\underbrace{\frac{2\hat{Z}_2}{2Z_2\hat{s}-Z_1}}_{=1}$$

$$\hat{J}_1=\hat{J}_2\hat{s}+\hat{U}_2\hat{s}\left(\frac{1}{2Z_2\hat{s}}+\frac{1}{2Z_2}\right)=\hat{J}_2\hat{s}+\hat{U}_2\hat{s}\left(\frac{2\hat{Z}_2}{(2Z_2+Z_1)2Z_2}+\frac{1}{2Z_2}\right)$$

$\hat{J}_1=\hat{J}_2\hat{s}+\hat{U}_2\hat{s}\underbrace{\frac{2\hat{Z}_2+\hat{Z}_1+2\hat{Z}_2}{(2Z_2+\hat{Z}_1)2\hat{Z}_2}}_{\frac{1}{Z_{10}}}$ $\qquad\hat{Z}_{10}=\frac{(2\hat{Z}_2-Z_1)2\hat{Z}_2}{2Z_2+Z_1+2\hat{Z}_2}$

$$\boxed{\hat{J}_1=\hat{J}_{12}+\hat{J}_{10}}$$

$$\hat{U}_1=\hat{U}_{10}+\hat{U}_{12}=\hat{U}_2\hat{s}+\hat{J}_2\hat{s}\hat{Z}_{12}$$

$\boxed{\hat{U}_1=\hat{s}(\hat{U}_2+\hat{J}_2\hat{Z}_{12})}$ $\boxed{\hat{J}_1=\hat{s}\left(\hat{J}_2+\frac{\hat{U}_2}{Z_{10}}\right)}$ Warunek symetrii czwórnika T i Π.

ciąg dalszy

$$\frac{2}{Z_1}\hat{U}_{n-1}-\frac{2}{Z_1}\hat{U}_n=\hat{J}_n+\frac{2}{Z_1(\hat{s}+1)}\hat{U}_{n-1}-\frac{2}{Z_1(\hat{s}+1)}\hat{U}_n$$

$$\hat{J}_n=\frac{2}{Z_1(\hat{s}+1)}\left[(\hat{s}\cancel{+1})\hat{U}_{n-1}-(\hat{s}\cancel{+1})\hat{U}_n-\hat{s}\cancel{\hat{U}_{n-1}}-\cancel{\hat{U}_n}\right]$$

$$\boxed{\hat{J}_n=\frac{2}{Z_1(\hat{s}+1)}[U_{n-1}-\hat{s}\hat{U}_n]}$$

$$\hat{J}_{n+1}=\hat{J}_n-\frac{1}{Z_2}\hat{U}_n+\frac{\hat{Z}_1}{2Z_2}\hat{J}_n=J_n\Big(\underbrace{1+\frac{\hat{Z}_1}{2Z_2}}_{\hat{S}}\Big)-\frac{1}{Z_2}\hat{U}_n=\hat{J}_n\hat{S}-\frac{1}{Z_2}\hat{U}_n$$

$$\boxed{J_{n+1}=\frac{2\hat{S}}{Z_1(\hat{S}+1)}\left[\hat{U}_{n-1}-\hat{S}\hat{U}_n\right]-\frac{1}{Z_2}U_n}$$ do równania wstawiam J_n i J_{n+1}

$$\hat{U}_n-\hat{U}_{n+1}=\frac{Z_1}{2}\left[\frac{2}{Z_1(\hat{S}+1)}\left(\hat{U}_{n-1}-\hat{S}\hat{U}_n+\hat{S}\hat{U}_{n-1}-\hat{S}^2\hat{U}_n\right)-\frac{1}{Z_2}\hat{U}_n\right]$$

$$=\frac{Z_1}{2}\left\{\frac{2}{Z_1(\hat{S}+1)}\left[\hat{U}_{n-1}(\hat{S}+1)-\hat{U}_n\hat{S}(\hat{S}+1)\right]-\frac{1}{Z_2}\hat{U}_n\right\}$$

$$=\hat{U}_{n-1}-\hat{U}_n\hat{S}-\frac{\hat{Z}_1}{2Z_2}U_n=\hat{U}_{n-1}-\hat{U}_n\hat{S}-(\hat{S}-1)\hat{U}_n=\hat{U}_{n-1}-2\hat{S}\hat{U}_n+\hat{U}_n$$

$$\boxed{\hat{U}_{n-1}+\hat{U}_{n+1}=2\hat{S}\hat{U}_n}$$

Kiedy linia łańcuchowa będzie równoważna linii długiej.

dla linii długiej: $\hat{U}_x=A_1e^{\hat{\varkappa}x}+A_2e^{-\hat{\varkappa}x}$ $\qquad A_1=\frac{1}{2}(\hat{U}_2-\hat{J}_2\hat{Z})$

$\hat{J}_x\hat{Z}=-A_1e^{\hat{\varkappa}x}+A_2e^{-\hat{\varkappa}x}$ $\qquad A_2=\frac{1}{2}($

$\hat{U}_n=B_1e^{\hat{\varkappa}'n}+B_2e^{-\hat{\varkappa}'n}$ szukamy stałych całkowania oraz $\hat{\varkappa}'$ dla linii łańcuchowej

$\hat{U}_{n-1}=B_1e^{\hat{\varkappa}'(n-1)}+B_2e^{-\hat{\varkappa}'(n-1)}$; $\hat{U}_{n+1}=B_1e^{\hat{\varkappa}'(n+1)}+B_2e^{-\hat{\varkappa}'(n+1)}$

$$\hat{U}_{n-1}-2\hat{S}\hat{U}_n+U_{n+1}=0$$

$$B_1e^{\hat{\varkappa}'(n-1)}+B_2e^{-\hat{\varkappa}'(n-1)}-2\hat{S}B_1e^{\hat{\varkappa}'n}-2\hat{S}B_2e^{-\hat{\varkappa}'n}+B_1e^{\hat{\varkappa}'(n+1)}+B_2e^{-\hat{\varkappa}'(n+1)}=0$$

$$B_1e^{\hat{\varkappa}'n}(e^{-\hat{\varkappa}'}-2\hat{S}+e^{\hat{\varkappa}'})+B_2e^{-\hat{\varkappa}'n}(e^{\hat{\varkappa}'}-2\hat{S}+e^{-\hat{\varkappa}'})=0$$

$$\underbrace{(Be^{\hat{\varkappa}'n}+B_2e^{-\hat{\varkappa}'n})}_{=U_n}\underbrace{(e^{\hat{\varkappa}'}-2\hat{S}+e^{-\hat{\varkappa}'})}_{=0\ \text{[illegible]}}=0$$

$$\boxed{\hat{S}=\frac{e^{\hat{\varkappa}'}-e^{-\hat{\varkappa}'}}{2}=\cos hip\,\hat{\varkappa}'}$$ wtedy będzie dane równanie równoważne linii długiej

$$\hat{S}=1+\frac{\hat{Z}_1}{2Z_2}=\cos hip\,\hat{\varkappa}'$$

$$\hat{J}_n=\frac{2}{Z_1(\hat{S}+1)}\left[\hat{U}_{n-1}-\hat{S}\hat{U}_n\right]=\frac{2}{Z_1(\hat{S}+1)}\left[B_1e^{\hat{\varkappa}'(n-1)}+B_2e^{-\hat{\varkappa}'(n-1)}-\hat{S}B_1e^{\hat{\varkappa}'n}-\hat{S}B_2e^{-\hat{\varkappa}'n}\right]$$

$$\hat{J}_n=\frac{2}{Z_1(\hat{S}+1)}\left[B_1e^{\hat{\varkappa}'n}(e^{-\hat{\varkappa}'}-\hat{S})+B_2e^{-\hat{\varkappa}'n}(e^{\hat{\varkappa}'}-\hat{S})\right]$$

$$e^{-\hat{\varkappa}'}-\hat{S}=\frac{2e^{-\hat{\varkappa}'}-e^{\hat{\varkappa}'}-e^{-\hat{\varkappa}'}}{2}=\frac{e^{-\hat{\varkappa}'}-e^{\hat{\varkappa}'}}{2}$$

$$e^{\hat{\varkappa}'}-\hat{S}=\frac{2e^{\hat{\varkappa}'}-e^{\hat{\varkappa}'}-e^{-\hat{\varkappa}'}}{2}=\frac{e^{\hat{\varkappa}'}-e^{\hat{\varkappa}'}}{2}$$

$$\hat{J}_n=\frac{2}{\hat{Z}_1(\hat{S}+1)}\cdot\frac{e^{\hat{\varkappa}'}-e^{-\hat{\varkappa}'}}{2}\left(-B_1e^{\hat{\varkappa}'_n}+B_2e^{-\hat{\varkappa}'_n}\right)$$

$$\hat{S}+1=\frac{e^{\hat{\varkappa}'}+e^{-\hat{\varkappa}'}+2}{2}=\underline{(e^{\frac{1}{2}\hat{\varkappa}'}+e^{-\frac{1}{2}\hat{\varkappa}'})^2}$$

$$\hat{J}_n=\frac{4^2(e^{\frac{1}{2}\hat{\varkappa}'}+e^{-\frac{1}{2}\hat{\varkappa}'})(e^{\frac{1}{2}\hat{\varkappa}'}-e^{-\frac{1}{2}\hat{\varkappa}'})}{2\hat{Z}_1(e^{\frac{1}{2}\hat{\varkappa}'}+e^{-\frac{1}{2}\hat{\varkappa}'})^2}=\underbrace{\frac{2}{\hat{Z}_1}\operatorname{tg}h\,\tfrac{1}{2}\hat{\varkappa}}_{\frac{1}{\hat{Z}}}\left(-B_1e^{\hat{\varkappa}'_n}+B_2e^{-\hat{\varkappa}'_n}\right)$$

$$\hat{J}_n\hat{Z}=-B_1e^{\hat{\varkappa}'_n}+B_2e^{-\hat{\varkappa}'_n}$$

$\boxed{\hat{Z}=\frac{\hat{Z}_1}{2}\operatorname{ctg}h\,\tfrac{1}{2}\hat{\varkappa}'}$ oporność falowa linii łańcuchowej.

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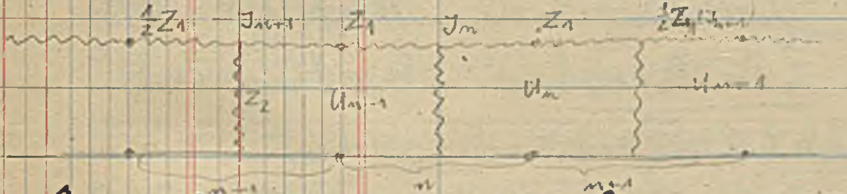


$$\hat{U}_{n-1}-2\hat{S}\hat{U}_n+\hat{U}_{n+1}=0 \qquad \hat{S}=1+\frac{\hat{Z}_1}{2\hat{Z}_2}$$

$$\boxed{\hat{U}_n=B_1e^{\hat{\varkappa}n}+B_2e^{-\hat{\varkappa}n}} \qquad \hat{J}_n=\frac{2}{\hat{Z}_1(\hat{S}+1)}\left[\hat{U}_{n-1}-\hat{S}\hat{U}_n\right]$$

$$\boxed{\hat{J}_n\hat{Z}=-B_1e^{\hat{\varkappa}n}+B_2e^{-\hat{\varkappa}n}} \qquad \hat{Z}=\frac{\hat{Z}_1}{2\operatorname{tg}h\frac{1}{2}\hat{\varkappa}}$$

$$\hat{S}=\cosh\hat{\varkappa}=1+2\sinh^2\tfrac{1}{2}\hat{\varkappa}=1+\frac{Z_1}{2Z_2} \qquad \sinh^2\tfrac{1}{2}\hat{\varkappa}=\frac{\hat{Z}_1}{4\hat{Z}_2}$$

$$\hat{Z}=\frac{\hat{Z}_1\cosh\frac{1}{2}\hat{\varkappa}}{2\sinh\frac{1}{2}\hat{\varkappa}}=\frac{\hat{Z}_1\cosh\frac{1}{2}\hat{\varkappa}}{2\sqrt{\frac{Z_1}{4Z_2}}}=\frac{\hat{Z}_1}{\sqrt{\frac{Z_1}{Z_2}}}\cosh\tfrac{1}{2}\hat{\varkappa}=\sqrt{Z_1Z_2}\cosh\tfrac{1}{2}\hat{\varkappa}$$

$$\boxed{\hat{Z}=\sqrt{Z_1Z_2}\cosh\tfrac{1}{2}\hat{\varkappa}}$$

$$\hat{\varkappa}=a+jb;\quad \cosh\hat{\varkappa}=\cosh(a+jb)$$

$$\cosh\hat{\varkappa}=\frac{e^{(a+jb)}+e^{-(a+jb)}}{2}=\frac{1}{2}e^{a}\cdot e^{jb}+\frac{1}{2}e^{-a}\cdot e^{-jb}=\frac{1}{2}e^{a}(\cos b+j\sin b)+$$

$$+\frac{1}{2}e^{-a}(\cos b-j\sin b)=\frac{e^a+e^{-a}}{2}\cos b+j\frac{e^a-a^{-a}}{2}\sin b$$

$$p=\cosh a\cos b \qquad p^2=\cosh^2a\cos^2b$$

$$q=\sinh a\sin b \qquad q^2=\sinh^2a\sin^2b \qquad \sinh^2a=\frac{q^2}{\sin^2b} \qquad \sin^2b=\frac{q^2}{\sinh^2a}$$

$$p^2=(1+\sinh^2a)(1-\sin^2b)=1+\sinh^2a-\sin^2b-\underbrace{\sinh^2a\sin^2b}_{q^2}$$

$$p^2=1+\sinh^2a-\frac{q^2}{\sinh^2a}-q^2;\quad \sinh^2a\,p^2=\sinh^2a+\sinh^4a-q^2-\sinh^2a\,q^2$$

$$\sinh^4a+\sinh^2a(1-p^2-q^2)-q^2=0;\quad \underline{\sinh^2a=-\frac{1}{2}(1-p^2-q^2)+\sqrt{\frac{1}{4}(1-p^2-q^2)^2+q^2}}$$

$$p^2=1+\sinh^2a-\frac{q^2}{\sinh^2a}-q^2=1+\frac{q^2}{\sin^2b}-\sin^2b-q^2$$

$$\sin^2b\,p^2=\sin^2b+q^2-\sin^4b-\sin b\,q^2$$

$$\sin^4b-(1-p^2-q^2)\sin^2b-q^2=0$$

$$\underline{\sin^2b=\frac{1}{2}(1-p^2-q^2)+\sqrt{\frac{1}{4}(1-p^2-q^2)^2+q^2}}$$

$$\hat{S} = \cosh \hat{\varkappa} = 1 + \frac{\hat{Z}_1}{2\hat{Z}_2} = 1 + \frac{z_1 e^{j\varphi_1}}{2z_2 e^{j\varphi_2}} = 1 + \frac{z_1 e^{j(\varphi_1-\varphi_2)}}{2z_2} = 1 + \frac{z_1}{2z_2}\left[\cos(\varphi_1-\varphi_2) + j\sin(\varphi_1-\varphi_2)\right]$$

$$\cosh \hat{\varkappa} = 1 + \underbrace{\frac{z_1}{2z_2}\cos(\varphi_1-\varphi_2)}_{p} + j\underbrace{\frac{z_1}{2z_2}\sin(\varphi_1-\varphi_2)}_{q}$$

$p = 1 + \frac{z_1}{2z_2}\cos(\varphi_1-\varphi_2)$; $q = \frac{z_1}{2z_2}\sin(\varphi_1-\varphi_2)$ w ten sposób znajdujemy $\hat{\varkappa}$ dla linii łańcuchowej.

$$\hat{U}_n = B_1 e^{\hat{\varkappa}n} + B_2 e^{-\hat{\varkappa}n}$$
$$\hat{U}_0 = B_1 e^{\hat{\varkappa}0} + B_2 e^{-\hat{\varkappa}0}$$

$$\left.\begin{aligned}\hat{U}_0 &= \hat{B}_1 + \hat{B}_2\\ \hat{J}_0\hat{Z} &= -\hat{B}_1 + B_2\end{aligned}\right\}$$

$$B_1 = \tfrac{1}{2}(\hat{U}_0 - \hat{J}_0\hat{Z})$$
$$B_2 = \tfrac{1}{2}(\hat{U}_0 + \hat{J}_0\hat{Z})$$

$$\left.\begin{aligned}\hat{U}_n &= \tfrac{1}{2}(\hat{U}_0 - \hat{J}_0\hat{Z})e^{\hat{\varkappa}n} + \tfrac{1}{2}(\hat{U}_0 + \hat{J}_0\hat{Z})e^{-\hat{\varkappa}n}\\ \hat{J}_n\hat{Z} &= \underbrace{-\tfrac{1}{2}(\hat{U}_0 - \hat{J}_0\hat{Z})e^{\hat{\varkappa}n}}_{\text{fala odbita}} + \underbrace{\tfrac{1}{2}(\hat{U}_0 + \hat{J}_0\hat{Z})e^{-\hat{\varkappa}n}}_{\text{fala główna}}\end{aligned}\right\}$$

$\hat{U}_c$ = napięcie na końcu

$$\left(\hat{U}_c = \hat{U}_2 = \tfrac{1}{2}(U_0 - \hat{J}_0\hat{Z})e^{\hat{\varkappa}c} + \tfrac{1}{2}(\hat{U}_2 + \hat{J}_2\hat{Z})e^{-\hat{\varkappa}c}\right)$$

$$\left.\begin{aligned}\hat{U}_c = \hat{U}_2 &= \hat{B}_1 e^{\hat{\varkappa}c} + \hat{B}_2 e^{-\hat{\varkappa}c}\\ \hat{J}_2\hat{Z} &= -\hat{B}_1 e^{\hat{\varkappa}c} + \hat{B}_2 e^{-\hat{\varkappa}c}\end{aligned}\right\}$$

$$\hat{U}_2 - \hat{J}_2\hat{Z} = 2\hat{B}_1 e^{\hat{\varkappa}c};$$
$$U_2 + \hat{J}_2 Z = 2\hat{B}_2 e^{-\hat{\varkappa}c};$$

$$\left.\begin{aligned}\hat{B}_1 &= \tfrac{1}{2}(\hat{U}_2 - \hat{J}_2\hat{Z})e^{-\hat{\varkappa}c}\\ \hat{B}_2 &= \tfrac{1}{2}(\hat{U}_2 + \hat{J}_2\hat{Z})e^{\hat{\varkappa}c}\end{aligned}\right\}$$

$$\hat{U}_n = \tfrac{1}{2}(\hat{U}_2 - \hat{J}_2\hat{Z})e^{-\hat{\varkappa}c}e^{\hat{\varkappa}n} + \tfrac{1}{2}(\hat{U}_2 + \hat{J}_2\hat{Z})e^{\hat{\varkappa}c}e^{-\hat{\varkappa}n}$$

$$\left.\begin{aligned}\hat{U}_n &= \tfrac{1}{2}(\hat{U}_2 - \hat{J}_2\hat{Z})e^{-\hat{\varkappa}(c-n)} + \tfrac{1}{2}(\hat{U}_2 + \hat{J}_2\hat{Z})e^{\hat{\varkappa}(c-n)}\\ \hat{J}_n\hat{Z} &= -\tfrac{1}{2}(\hat{U}_2 - \hat{J}_2\hat{Z})e^{-\hat{\varkappa}(c-n)} + \tfrac{1}{2}(\hat{U}_2 + \hat{J}_2\hat{Z})e^{\hat{\varkappa}(c-n)}\end{aligned}\right\}$$

$$\left.\begin{aligned}\hat{U}_n &= \hat{U}_2\cosh\hat{\varkappa}(c-n) + \hat{J}_2\hat{Z}\sinh\hat{\varkappa}(c-n)\\ \hat{J}_n\hat{Z} &= \hat{J}_2\hat{Z}\cosh\hat{\varkappa}(c-n) + \hat{U}_2\sinh\hat{\varkappa}(c-n)\end{aligned}\right\}$$

1). Bieg luzem: $\hat{U}_2$; $\hat{J}_2 = 0$, $\hat{U}_{00}$; $\hat{J}_{00}$; $\hat{U}_{n0}$, $\hat{J}_{n0}$ – na końcu jakiegoś dowolnego ogniwa

$$\left.\begin{aligned}\hat{U}_2 &= \hat{U}_{00}\cosh\hat{\varkappa}c - \hat{J}_{00}\hat{Z}\sinh\hat{\varkappa}c\\ \hat{J}_2\hat{Z} = 0 &= \hat{J}_{00}\hat{Z}\cosh\hat{\varkappa}c - \hat{U}_{00}\sinh\hat{\varkappa}c\end{aligned}\right\}$$

$$\hat{J}_{00}\hat{Z} = \hat{U}_{00}\,\mathrm{tgh}\,\hat{\varkappa}c$$

$$\hat{U}_2 = \hat{U}_{00}\cosh\hat{\varkappa}c - \hat{U}_{00}\sinh\hat{\varkappa}c\,\mathrm{tgh}\,\hat{\varkappa}c$$

$$\hat{U}_2 = \hat{U}_{00}\left(\cosh\hat{\varkappa}c - \frac{\sinh^2\hat{\varkappa}c}{\cosh\hat{\varkappa}c}\right) = \frac{U_{00}}{\cosh\hat{\varkappa}c}\left(\underbrace{\cosh^2\hat{\varkappa}c - \sinh^2\hat{\varkappa}c}_{=1}\right)$$

$$\boxed{\hat{U}_2 = \frac{\hat{U}_{00}}{\cosh\hat{\varkappa}c}}$$

$$\hat{J}_{x0}\hat{Z}=\hat{J}_{00}\hat{Z}\cosh\hat{\varkappa}x-\hat{U}_{00}\sinh\hat{\varkappa}x$$ 5.III.1949 r.

$$\hat{J}_{x0}\hat{Z}=\hat{J}_{00}\hat{Z}\cosh\hat{\varkappa}x-\hat{J}_{00}\hat{Z}\frac{\sinh\hat{\varkappa}x}{\operatorname{tgh}\hat{\varkappa}c}$$

$$\hat{J}_{x0}=\hat{J}_{00}\left[\cosh\hat{\varkappa}x-\frac{\sinh\hat{\varkappa}x}{\operatorname{tgh}\hat{\varkappa}c}\right]=\hat{J}_{00}\left[\frac{\cosh\hat{\varkappa}x\operatorname{tgh}\hat{\varkappa}c-\sinh\hat{\varkappa}x}{\operatorname{tgh}\hat{\varkappa}c}\right]=\hat{J}_{00}\left[\frac{\cosh\hat{\varkappa}x\sinh\hat{\varkappa}c-\sinh\hat{\varkappa}x\cosh\hat{\varkappa}}{\sinh\hat{\varkappa}c}\right.$$

$$\boxed{\hat{J}_{x0}=\hat{J}_{00}\frac{\sinh\hat{\varkappa}(c-x)}{\sinh\hat{\varkappa}c}}$$

2). __Bieg zwarcia.__ $\hat{U}_2=0$, $\hat{J}_2$, $\hat{U}_{0z}$, $\hat{J}_{0z}$, $\hat{U}_{xz}$, $\hat{J}_{xz}$

$$\left.\begin{aligned}\hat{U}_2=0&=\hat{U}_{0z}\cosh\hat{\varkappa}c-\hat{J}_{0z}\hat{Z}\sinh\hat{\varkappa}c\\ \hat{J}_2Z&=\hat{J}_{0z}\hat{Z}\cosh\hat{\varkappa}c-U_{0z}\sinh\hat{\varkappa}c\end{aligned}\right\}\hat{U}_{0z}=\hat{J}_{0z}\hat{Z}\operatorname{tgh}\hat{\varkappa}c$$

$$\hat{J}_2\hat{Z}=\hat{J}_{0z}\hat{Z}\cosh\varkappa c-\hat{J}_{00}\hat{Z}\sinh\hat{\varkappa}c\operatorname{tgh}\hat{\varkappa}c$$

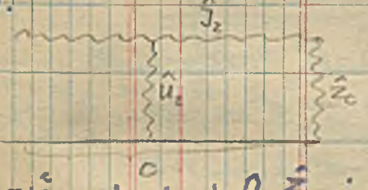
$$\hat{J}_2=\hat{J}_{0z}\left[\frac{\cosh^2\hat{\varkappa}c-\sinh^2\hat{\varkappa}c}{\cosh\hat{\varkappa}c}\right]\qquad\boxed{\hat{J}_2=\frac{\hat{J}_{0z}}{\cosh\varkappa c}}$$

$$\hat{U}_{xz}=\hat{U}_{0z}\cosh\varkappa x-\hat{J}_{0z}\hat{Z}\sinh\varkappa x$$

$$\hat{U}_{xz}=\hat{U}_{0z}\cosh\hat{\varkappa}x-\hat{U}_{0z}\frac{\sinh\varkappa x}{\operatorname{tgh}\varkappa c}=\hat{U}_{0z}\frac{\cosh\hat{\varkappa}x\sinh\varkappa c-\sinh\varkappa x\cosh\varkappa c}{\sinh\varkappa c}$$

$$\hat{U}_{xz}=\hat{U}_{0z}\left[\cosh\varkappa x-\frac{\sinh\varkappa x}{\operatorname{tg}h\varkappa c}\right]\qquad\boxed{\hat{U}_{xz}=\hat{U}_{0z}\frac{\sinh\varkappa(c-x)}{\sinh\varkappa c}}$$

3).



$$\hat{U}_x=\hat{U}_2\cosh\varkappa(c-x)+\hat{J}_2\hat{Z}\sinh\varkappa(c-x)$$

$$\hat{J}_x\hat{Z}=\hat{J}_2\hat{Z}\cosh\varkappa(c-x)+\hat{U}_2\sinh\varkappa(c-x)$$

$$\hat{U}_x=\hat{U}_2\cosh\varkappa(c-x)+\hat{U}_2\frac{\hat{Z}}{\hat{Z}_c}\sinh\varkappa(c-x)$$

$\hat{Z}_c=\hat{Z}$ zamykamy koniec linii opornością falowej linii.

$$\hat{U}_x=\hat{U}_2\left[\cosh\varkappa(c-x)+\frac{\hat{Z}}{\hat{Z}_c}\sinh\varkappa(c-x)\right]\qquad\hat{U}_2=\hat{J}_2\hat{Z}_c$$

$$\frac{e^a+e^{-a}}{2}+\frac{e^a-e^{-a}}{2}=e^a$$

$$\hat{J}_x\hat{Z}=\hat{J}_2\hat{Z}\cosh\varkappa(c-x)+J_2\hat{Z}_c\sinh\varkappa(c-x)$$

$$\hat{J}_x=\hat{J}_2\left[\cosh\varkappa(c-x)+\frac{\hat{Z}_c}{Z}\sinh\varkappa(c-x)\right]=\hat{J}_2\left[\cosh\varkappa(c-x)+\sinh\varkappa(c-x)\right]=\hat{J}_2e^{\varkappa(c-x)}$$

$$\hat{U}_x=\hat{U}_2\left[\cosh\varkappa(c-x)+\sinh\varkappa(c-x)\right]=\hat{U}_2e^{-\varkappa(c-x)}$$

$$\hat{U}_0=\hat{U}_2e^{\varkappa c};\quad\hat{J}_0=\hat{J}_2e^{\varkappa c};\quad B_1=\frac{1}{2}(\hat{U}_2-\hat{J}_2\hat{Z})e^{-\varkappa c}$$

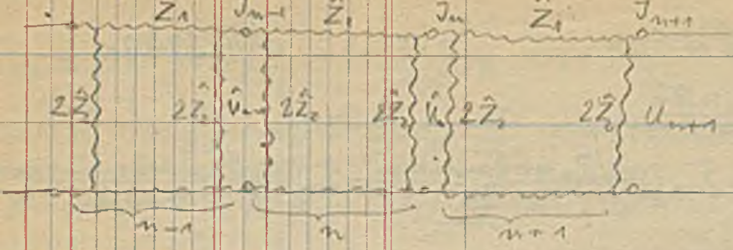
Dla nieskończenie długiego łańcucha (linii) $B_1=0$ linia taka nie ma fali odbitej.

$\hat{Z}_{00}$ - oporność pozorna na pocz. przy biegu luzem

$$\hat{Z}_{00}=\frac{\hat{U}_{00}}{\hat{J}_{00}};\quad\hat{J}_{00}\hat{Z}=\hat{U}_{00}\operatorname{tg}h\varkappa c$$

$$\hat{Z}_{00}=\frac{\hat{J}_{00}\hat{Z}}{\hat{J}_{00}\operatorname{tg}h\varkappa c}\qquad\boxed{\hat{Z}_T=\hat{Z}_{00}\operatorname{tg}h\varkappa c}$$

Dla linii zakończonej opornością falową nie istnieje fala odbita, zachowuje się tak jak linia nieskończenie długa $B_1=0$; $\hat{Z}=\hat{Z}_c$. Oporność pozorna [illegible] równa oporności falowej (echo).

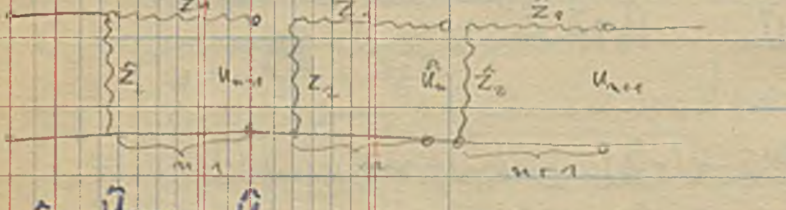


$$\hat{U}_{n-1} = 2\hat{S}U_n + \hat{U}_{n+1} = 0$$
$$\hat{U}_n = B_1 e^{\varkappa n} + B_2 e^{-\varkappa n}$$
$$\hat{S} = \cosh \varkappa = 1 + \frac{\hat{Z}_1}{2\hat{Z}_2}$$

$$\hat{U}_{n-1} - \hat{U}_n = \hat{Z}_1\left(\hat{J}_{n-1} - \frac{\hat{U}_{n-1}}{2\hat{Z}_2}\right) \quad ; \quad \hat{J}_{n-1} = \frac{\hat{U}_{n-1}}{\hat{Z}_1} - \frac{\hat{U}_n}{\hat{Z}_1} + \frac{\hat{U}_{n-1}}{2\hat{Z}_2}$$
$$\hat{J}_{n-1} - \hat{J}_n = \frac{1}{2\hat{Z}_2}(\hat{U}_{n-1} + \hat{U}_n) \quad ; \quad \hat{J}_n = \frac{\hat{U}_n}{\hat{Z}_1} - \frac{\hat{U}_{n+1}}{\hat{Z}_1} + \frac{\hat{U}_n}{2\hat{Z}_2}$$
$$\hat{U}_n - \hat{U}_{n-1} = \hat{Z}_1\left(\hat{J}_n - \frac{\hat{U}_n}{2\hat{Z}_2}\right) \quad ; \quad \hat{J}_{n-1} - \hat{J}_n = \frac{\hat{U}_{n-1}}{\hat{Z}_1} - \frac{\hat{U}_n}{\hat{Z}_1} + \frac{\hat{U}_{n-1}}{2\hat{Z}_2} - \frac{\hat{U}_n}{\hat{Z}_1} + \frac{\hat{U}_{n+1}}{\hat{Z}_1} - \frac{\hat{U}_n}{2\hat{Z}_2}$$
$$J_n - \hat{J}_{n+1} = \frac{1}{2\hat{Z}_2}(\hat{U}_n + \hat{U}_{n+1}) \quad ; \quad \hat{J}_{n-1} - \hat{J}_n = \frac{\hat{U}_{n-1}}{2\hat{Z}_2} + \frac{\hat{U}_n}{2Z_2}$$
$$\hat{U}_{n-1} + \hat{U}_{n+1} - 2\hat{U}_n - 2U_n\frac{\hat{Z}_1}{2\hat{Z}_2} = 0$$

Napięcie na końcu pomnożone przez $2\hat{S}$ równa się sumie 2 ogniw sąsiednich

$$\hat{U}_{n-1} + \hat{U}_{n+1} - 2\hat{U}_n\underbrace{\left(1 + \frac{\hat{Z}_1}{2\hat{Z}_2}\right)}_{\hat{S}} = 0 \qquad \hat{U}_{n-1} - 2\hat{S}\hat{U}_n + \hat{U}_{n+1} = 0$$
$$\hat{J}_n\hat{Z}_1 = \hat{U}_n + \hat{U}_n\frac{\hat{Z}_1}{2\hat{Z}_2} - \hat{U}_{n+1} = \hat{U}_n\left(1 + \frac{\hat{Z}_1}{2\hat{Z}_2}\right) - \hat{U}_{n+1} = \hat{U}_n\hat{S} - \hat{U}_{n+1}$$
$$\hat{J}_n\hat{Z}_1 = \hat{B}_1 e^{\varkappa n}\cdot\hat{S} + \hat{B}_2 e^{-\varkappa n}\hat{S} - \hat{B}_1 e^{\varkappa(n+1)} - \hat{B}_2 e^{-\varkappa(n+1)} = \hat{B}_1 e^{\varkappa n}\cdot(\hat{S} - e^{\varkappa}) + \hat{B}_2 e^{-\varkappa n}(\hat{S} - e^{\varkappa})$$
$$\hat{J}_n\hat{Z}_1 = -\hat{B}_1 e^{\varkappa n}\sinh\varkappa + B_2 e^{-\varkappa n}\sinh\varkappa \qquad \hat{S} - e^{\varkappa} = \frac{e^{\varkappa} + e^{-\varkappa} - 2e^{\varkappa}}{2} = -\frac{e^{\varkappa} - e^{-\varkappa}}{2}$$
$$\hat{J}_n\frac{Z_1}{\sinh\varkappa} = -\hat{B}_1 e^{\varkappa n} + B_2 e^{-\varkappa n} \quad ; \quad \hat{Z}_m = \frac{\hat{Z}_1}{\sinh\varkappa} \; ; \quad \hat{S} - e^{\varkappa} = -\sinh\varkappa$$
$$\hat{U}_n = \hat{B}_1 e^{\varkappa n} + \hat{B}_2 e^{-\varkappa n} \quad ; \quad Z_T = \frac{\hat{Z}_1}{2\operatorname{tgh}\frac{1}{2}\varkappa} \; ; \quad \hat{S} - e^{-\varkappa} = \frac{e^{\varkappa} + e^{-\varkappa} - 2e^{-\varkappa}}{2} = \frac{e^{\varkappa} - e^{-\varkappa}}{2}$$
$$\hat{S} - e^{-\varkappa} = \sinh\varkappa$$
$$\hat{J}_n\hat{Z} = -Be^{\varkappa n} + B_2 e^{-\varkappa n}$$



$$\hat{U}_{n-1} - \hat{U}_n = \hat{Z}_1\hat{J}_n$$
$$\hat{J}_{n-1} - \hat{J}_n = \frac{\hat{U}_{n-1}}{\hat{Z}_2}$$
$$\hat{U}_n - \hat{U}_{n+1} = \hat{Z}_1\hat{J}_{n+1}$$
$$\hat{J}_n - \hat{J}_{n+1} = \frac{\hat{U}_n}{Z_2}$$
$$\hat{J}_n = \frac{\hat{U}_{n-1}}{Z_1} - \frac{\hat{U}_n}{\hat{Z}_1}$$
$$\hat{J}_{n+1} = \frac{\hat{U}_n}{\hat{Z}_1} - \frac{\hat{U}_{n+1}}{\hat{Z}_1} \; ; \quad \hat{J}_n - \hat{J}_{n+1} = \frac{\hat{U}_{n-1}}{\hat{Z}_1} - \frac{\hat{U}_n}{\hat{Z}_1} - \frac{\hat{U}_n}{\hat{Z}_1} + \frac{\hat{U}_{n+1}}{\hat{Z}_1} = \frac{\hat{U}_n}{\hat{Z}_2}$$
$$\hat{U}_{n-1} - 2\hat{U}_n + \hat{U}_{n+1} - U_n\frac{\hat{Z}_1}{\hat{Z}_2} = 0$$
$$\hat{U}_{n-1} - 2U_n\left(1 + \frac{Z_1}{2Z_2}\right) + \hat{U}_{n+1} = 0$$
$$\hat{U}_{n-1} - 2\hat{S}\hat{U}_n + \hat{U}_{n+1} = 0$$

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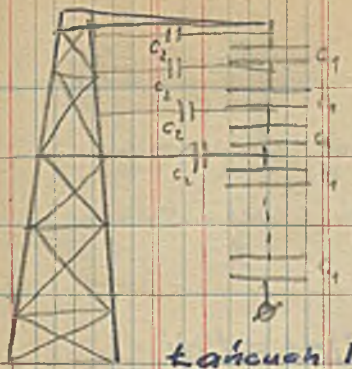
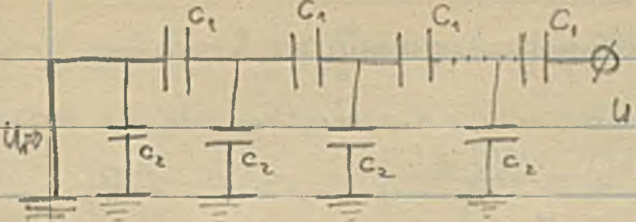
Izolatory linii przesyłowych jako szereg kondensatorów.





Łańcuch Γ

$\hat{S} = S = \cosh \varkappa \qquad \hat{\varkappa} = \varkappa$

$$\hat{Z}_1 = -j\frac{1}{\omega C_1} = \frac{1}{j\omega C_1} \quad ; \quad \hat{Z}_2 = \frac{1}{j\omega C_2}$$

$$\boxed{S = \frac{2(1+C_2)}{2C_1}}$$

$$\hat{S} = 1 + \frac{\hat{Z}_1}{2\hat{Z}_2} = 1 + \frac{j\omega C_2}{2j\omega C_1} = 1 + \frac{C_2}{2C_1}$$

$$\hat{U}_m = \hat{B}_1 e^{\varkappa m} + \hat{B}_2 e^{-\varkappa m}$$

$$0 = \hat{U}_0 = \hat{B}_1 + \hat{B}_2 = 0, \quad B_1 = -B_2$$

$$\hat{U}_2 = B_1(e^{\varkappa c} - e^{-\varkappa c}) = 2B_1 \sinh \varkappa c \quad ; \quad B_1 = \frac{\hat{U}_2}{2\sinh \varkappa c}$$

$$U_m = \frac{U_2}{2\sinh \varkappa c}(e^{\varkappa m} - e^{-\varkappa m}) \qquad \boxed{U_m = U_2 \frac{\sinh \varkappa m}{\sinh \varkappa c}}$$

Stany nieustalone.

Teoria linii długiej uwzględnia zależność od częstotliwości.

$$\hat{\varkappa} = a + jb \qquad \cosh \varkappa = \hat{S} = 1 + \frac{\hat{Z}_1}{2\hat{Z}_2} = p + jq$$

$$p = 1 + \frac{\hat{Z}_1}{2\hat{Z}_2}\cos(\varphi_1 - \varphi_2) \qquad \hat{Z}_2 = R_2 + jX_2$$

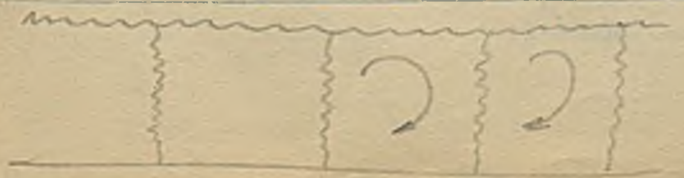
$$q = \frac{Z_1}{2Z_2}\sin(\varphi_1 - \varphi_2) \qquad X = \omega L - \frac{1}{\omega C}$$

$$\sinh^2 a = -\frac{1}{2}(1-p^2-q^2) + \sqrt{\frac{1}{4}(1-p-q^2) + q^2}$$

$$\sinh^2 b =$$

$$\left.\begin{aligned} a &= f(\omega) \\ b &= f(\omega) \end{aligned}\right\} \qquad \omega = 2\pi f$$

a i b jako funkcje częstotliwości



Każdy obwód zamknięty ma częstotliwość drgań własnych.

I. a) Powstawanie prądu w obwodach R,L.

Włączam w czasie co krótkim. Prąd nie wzrasta odrazu i na początku płynie prąd i nieustalony.

$$i = \frac{U+e'}{R} \; ; \quad e' = -L\frac{di}{dt}$$

$$iR = U - L\frac{di}{dt} \; ; \quad iRdt = Udt - Ldi \; ; \quad (U-iR)dt = Ldi$$

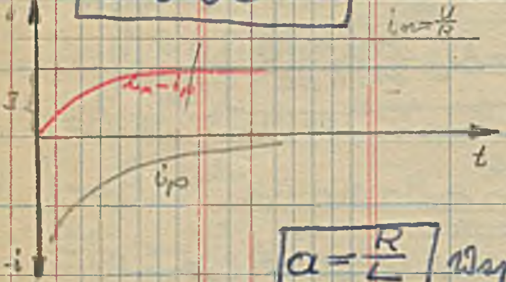
$$\int_0^i \frac{di}{U-iR} = \int_0^t \frac{dt}{L} \qquad \text{dla } t=0 \; ; \; i=0$$
$$\text{-"-} \; t=t \; ; \; i=i$$

$$-\frac{1}{R}\ln(U-iR)\Big|_0^i = \frac{t}{L}\Big|_0^t \; ; \quad \ln(U-iR) - \ln U = -\frac{R}{L}t$$

$$\ln\frac{U-iR}{U} = -\frac{R}{L}t \; ; \quad \frac{U-iR}{U} = e^{-\frac{R}{L}t}$$

$$U - iR = Ue^{-\frac{R}{L}t} \; ; \quad -iR = -U + Ue^{-\frac{R}{L}t} \; ; \quad i = \frac{U}{R} - \frac{U}{R}e^{-\frac{R}{L}t} \; ; \quad J = \frac{U}{R}$$

$$\boxed{i = J - Je^{-\frac{R}{L}t}} \qquad \underline{i = f(t)} \qquad J = i \text{ ustalone} \; ; \; Je^{-\frac{R}{L}t} = i \text{ przejściowe}$$



Teoretycznie osiąga prąd swą maksymalną wartość w czasie nieskończenie długim. Praktycznie przyjmuje się prędzej.

$\boxed{a = \frac{R}{L}}$ Współczynnik tłumienia (nie mylić z linią długą)

I. obwód R,L 1). źródło stałe –; 2). źródło zmienne ~ a).b).
a). powstawanie b). zanikanie

II. -"- R,C 1). $\overline{a,b}$; 2). $\widetilde{a,b}$

III. -"- L,C 1). $\overline{a,b}$; 2). $\widetilde{a,b}$

$$e^{-\frac{R}{L}t} = e^{-at} \qquad \boxed{\frac{1}{a} = \tau} \text{ stała czasu}$$

$$e^{\frac{t}{\tau}} \; ; \quad [\tau] = \left[\frac{L}{R}\right] \text{ wymiarowo } [\tau] = [sec]$$

I. b) Zanikanie prądu w obwodach R,L.

gdy w obwodzie stan ustalony czyli płynie prąd J, wyłączam 1. i przełączam na 2. czyli zamykam obwód.

$$i = \frac{e'}{R} = -L\frac{\frac{di}{dt}}{R} \; ; \quad iR = -L\frac{di}{dt} \; ; \quad iRdt = -Ldi$$

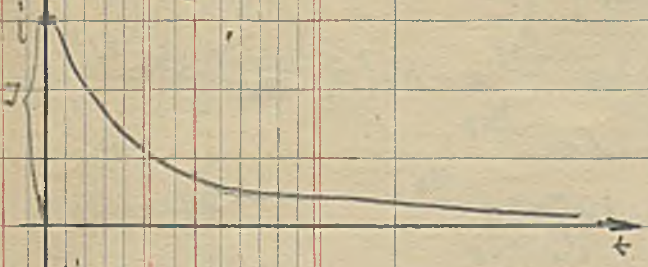
$$\int_J^i \frac{di}{iR} = -\int_0^t \frac{1}{L}dt \; ; \quad \frac{1}{R}\ln i\Big|_J^i = -\frac{1}{L}t\Big|_0^t \; ; \quad \ln\frac{i}{J} = -\frac{R}{L}t \; ; \quad \frac{i}{J} = e^{-\frac{R}{L}t}$$

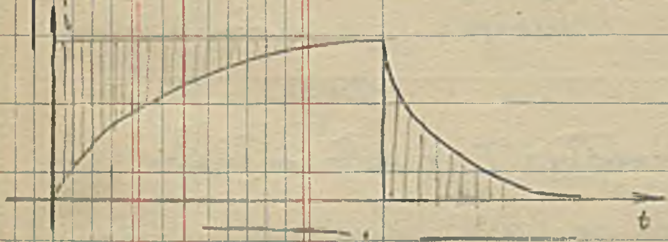
$$i = Je^{-\frac{R}{L}t}$$

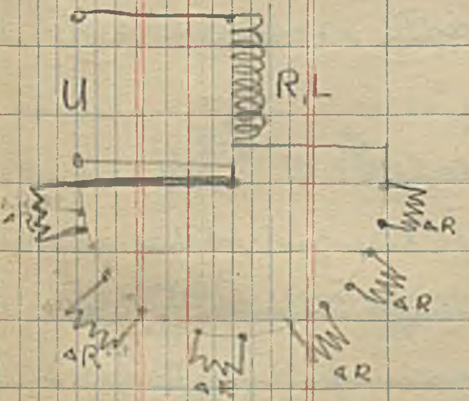
$$\boxed{i = Je^{-\alpha t}}$$

$$i = f(t)$$

$$i_u = 0 \; ; \quad i_p = Je^{-\alpha t}$$



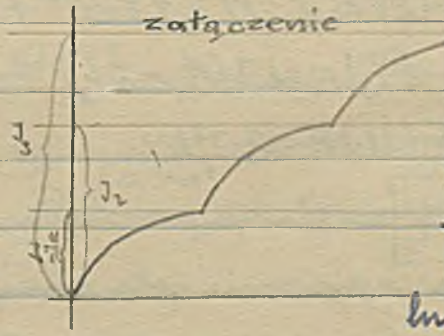




$$J_1 = \frac{U}{R+6\Delta R} = \frac{U}{R_1} \; ; \quad J_2 = \frac{U}{R+5\Delta R} = \frac{U}{R_2}$$

przy pierwszym załączeniu

$$\int_0^i \frac{di}{U-iR_2} = \frac{t}{L} \; ; \quad \boxed{i = J_1 - J_1 e^{-\frac{R_1}{L}t}}$$



$$\int_{J_1}^i \frac{di}{U-iR_2} = \frac{t}{L}$$

$$-\frac{1}{R_2}\ln(U-iR_2)\Big|_{J_1}^i = \frac{t}{L}$$

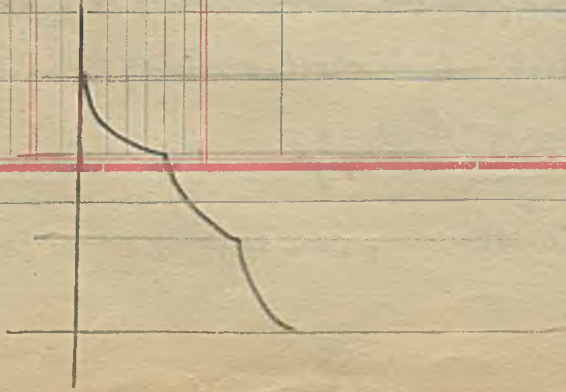
$$\ln\frac{U-iR_2}{U-J_1R_2} = -\frac{R}{L}(t_2-t_1)$$

$$\frac{U-iR_2}{U-J_1R_2} = e^{-\frac{R}{L}t}$$

$$U-iR_2 = (U-J_1R_2)e^{-\frac{R_2}{L}t}\;;$$

$$i = \frac{U}{R_2} - \left(\frac{U}{R_2} - J_1\right)e^{-\frac{R}{L}t}$$

$$i = J_2 - (J_2 - J_1)e^{-\frac{R}{L}t}$$



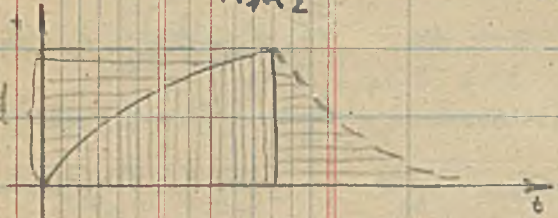
18.III.1949r.

$R_1 = R + 5\Delta R$; $R_2 = R + 4\Delta R$; $i = J_2 - (J_2 - J_1) e^{-\frac{R_2}{L}t}$

$$e' = -L\frac{di}{dt} = -L\frac{d}{dt}\left[J_2 - (J_2 - J_1) e^{-\frac{R_2}{L}t}\right]$$

$$e' = -L\frac{R_2}{L}(J_2 - J_1) e^{-\frac{R_2}{L}t} = -R_2 (J_2 - J_1) e^{-\frac{R_2}{L}t} = -R_2\left(\frac{U}{R_2} - \frac{U}{R_1}\right) e^{-\frac{R_2}{L}t}$$

$$e' = -R_2 U \frac{R_1 - R_2}{R_1 R_2} e^{-\frac{R_2}{L}t} = \frac{\Delta R}{R_1} U e^{-\frac{R_2}{L}t}$$



$i = J - J e^{-\frac{R}{L}t}$

i_u i_p

$i = i_u + i_p$

Wydzielanie energii przez źródło : $\tau = \frac{L}{R}$ stała czasowa

$$A_1 = \int_0^t iU\,dt = UJ\int_0^t (1 - e^{-\frac{t}{\tau}})\,dt$$

$A_1 = UJ(t + \tau)$ energia pobrana ze źródła w czasie t

Energia zamieniona na ciepło.

$$A_1' = \int_0^t i^2 R\,dt = J^2 R \int_0^t (1 - e^{-\frac{t}{\tau}})^2 dt = J^2 R \int_0^t \left[1 - 2e^{-\frac{t}{\tau}} + e^{-\frac{2t}{\tau}}\right] dt$$

$$A_1' = J^2 R\left[t - 2\tau e^{-\frac{t}{\tau}} - 2\tau - \frac{1}{2}\tau e^{-\frac{2t}{\tau}} + \frac{1}{2}\tau\right]$$

$$A_1' = J^2 R\left(t - \frac{3}{2}\tau\right) = UJ\left(t - \frac{3}{2}\tau\right)$$

Energia oddana przez źródło powinna być równa energii zamienionej na ciepło.

$$A_1 - A_1' = \frac{1}{2}UJ\tau = \frac{1}{2}UJ\frac{L}{R} = \frac{1}{2}J^2 L$$

$\boxed{A_1 - A_1' = \frac{1}{2}J^2 L}$ ta różnica powstała w polu magnetycznym została użyta na wytworzenie pola magnetycznego.

$\frac{mv^2}{2}$

L odpowiada bezwładności masy. Jest to niemożliwość nagłego wzrostu prądu. J jako ruch ładunków odpowiada prędkości.

$$A_{11} = \int_0^t e' i\,dt = -L\int_0^t i\frac{di}{dt}dt = -L\int_J^0 i\,di = \frac{LJ^2}{2}$$ przy wyłączaniu.

$\boxed{A_{11} = \frac{LJ^2}{2}}$ Energia oddawana przez cewkę przy wyłączaniu

Przepływający ładunek

$Q_1=\int_0^t i\,dt=J\int_0^t(1-e^{-\frac{t}{\tau}})dt=J(t-\tau)$ przy włączaniu

$Q_2=\int_0^t i\,dt=\int_0 Je^{-\frac{t}{\tau}}dt=J[-\tau e^{-\frac{t}{\tau}}+\tau]=J\tau$

$\boxed{Q_1+Q_2=Jt}$

$f=m\frac{dv}{dt}$ f_s ; $f\frac{ds}{dt}$

$fv=m\frac{dv}{dt}v=\frac{d}{dt}(\frac{mv^2}{2})$; $e'i=-L\frac{di}{dt}i=\frac{d}{dt}(\frac{LJ^2}{2})$

$i=\frac{U+e'}{R}=\frac{U-L\frac{di}{dt}}{R}$; $iR=U-L\frac{di}{dt}$

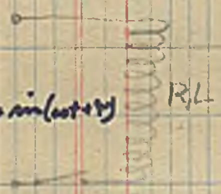
$iRdt=Udt-Ldi$; $\int_0^t Uidt-\int_0^t i^2Rdt=\int_0^J Lidi=\frac{LJ^2}{2}$ Energia wewn. pola magnetycznego

I. R, L, 1). U=const a). powstawania prądu, b). wygasania prądu.

I. R, L 2). $u=U_m\sin(\omega t+\psi)$ a). powstawanie prądu b). wygaszanie prądu

$i=i_u'+i_p$

$U=U_m\sin(\omega t+\psi)$



Szukamy i_u oraz i_p

$i=\frac{U+e'}{R}=\frac{U_m\sin(\omega t+\psi)-L\frac{di}{dt}}{R}$ wyprowadzić $a=\frac{R}{L}$

$i=\frac{U_m}{\sqrt{R^2+\omega^2L^2}}\left[\frac{a}{\sqrt{a^2+\omega^2}}\sin(\omega t+\psi)-\frac{\omega}{\sqrt{a^2+\omega^2}}\cos(\omega t+\psi)\right]+Ae^{-at}$

$i=\underbrace{J_m\sin(\omega t+\psi-\varphi)}_{i_u}+\underbrace{Ae^{-at}}_{i_p}$ $tg\,\varphi=\frac{\omega}{a}=\frac{\omega}{\frac{R}{L}}=\frac{\omega L}{R}$

$J_m=\frac{U_m}{\sqrt{R^2+\omega^2L^2}}$; $i=i_u+i_p$

$i_p=Ae^{-\frac{R}{L}t}$ szukamy stałej całkowania A

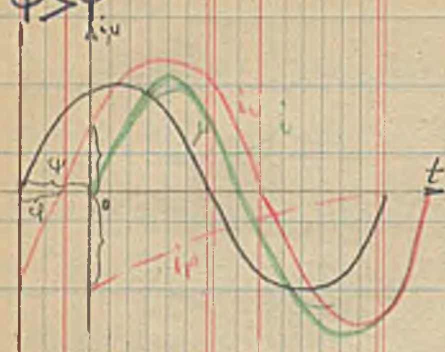
dla $t=0$; $i=0$; $0=i=J_m\sin(\psi-\varphi)+A$

$A=-J_m\sin(\psi-\varphi)$

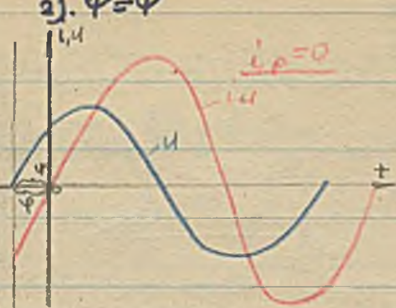
$i=J_m\sin(\omega t+\psi-\varphi)-J_m\sin(\psi-\varphi)e^{-\frac{R}{L}t}$

$i=\quad i_u\quad-\quad i_p$

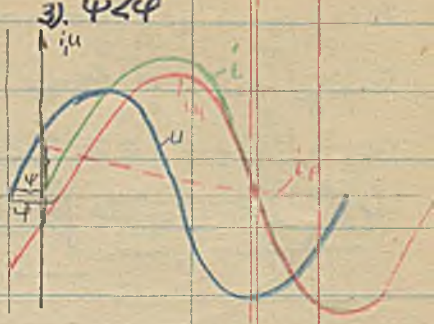
1). $\psi > \varphi$



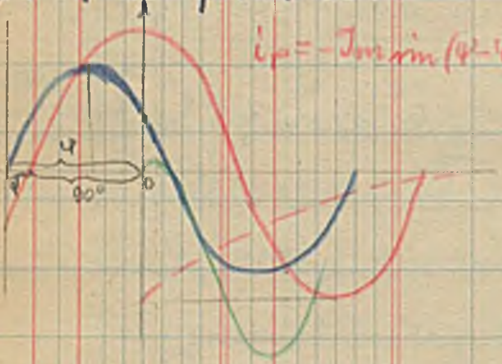
2). $\psi = \varphi$



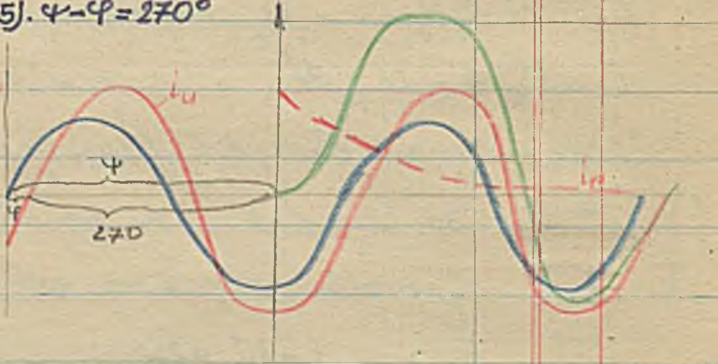
3). $\psi < \varphi$



4). $\psi - \varphi = 90°$; $\psi = \varphi + 90°$



5). $\psi - \varphi = 270°$



22.III.1949r.

$$i = J_m \sin(\omega t + \psi - \varphi) - J_m \sin(\psi - \varphi) e^{-at}$$

$i - f(\psi, t)$ $\quad \operatorname{tg}(\psi_1 - \varphi) = -\operatorname{tg}\varphi$; $\cos(\omega t_1 - \varphi) = \cos\varphi \, e^{-at}$

$\cos(\omega t_1 - \varphi) \geqslant 0$; $\cos \omega t_1 < 0$

$$\frac{\partial^2 i}{\partial t^2} = J_m \frac{\omega^2}{\sin\varphi} \cos \omega t_1 \text{ dla } \psi_1 = 0$$

$$\frac{\partial^2 i}{\partial t^2} = -J_m \frac{\omega^2}{\sin\varphi} \cos \omega t_1 \text{ dla } \psi_1 = 180°$$

$$\frac{\partial^2 i}{\partial \psi^2} = -\frac{J_m}{\cos\varphi} \sin \omega t_1 \text{ dla } \psi_1 = 0$$

$$\frac{\partial^2 i}{\partial \psi^2} = \frac{J_m}{\cos\varphi} \sin \omega t_1 \text{ dla } \psi_1 = 180°$$

$$\frac{\partial^2 i}{\partial t \partial \psi} = J_m \frac{\omega}{\sin\varphi} \sin \omega t_1 \text{ dla } \psi_1 = 0$$

$$\frac{\partial^2 i}{\partial t \partial \psi} = -J_m \frac{\omega}{\sin\varphi} \cos \omega t_1 \text{ dla } \psi_1 = 180$$

$$\frac{\partial^2 i}{\partial t^2} \cdot \frac{\partial^2 i}{\partial \psi^2} - \left(\frac{\partial^2 i}{\partial t \partial \psi}\right)^2 > 0 \quad = -J_m^2 \omega^2 \cos \omega t_1 \frac{\cos(\omega t_1 - \varphi)}{\sin^2\varphi \cos\varphi}$$

aby to było > 0
to: $\cos \omega t_1 < 0$
dla $\psi_1 = 0$, $\psi_1 = 180$

Poszczególne przypadki:

II. ćwiartka napięcia:

$$\Psi_1=0 \quad \frac{\partial^2 i}{\partial t^2}=I_m\frac{\omega^2}{\sin\varphi}\cos\omega t_1<0 \qquad \frac{\partial^2 i}{\partial \Psi^2}=-\frac{I_m}{\cos\varphi}\sin\omega t_1<0$$
spełnia warunki na max.

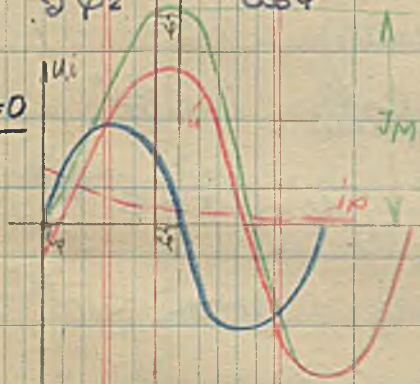
$$\Psi_1=180 \quad \frac{\partial^2 i}{\partial t^2}=-I_m\frac{\omega^2}{\sin\varphi}\cos\omega t_1>0 \qquad \frac{\partial^2 i}{\partial \Psi_2}=I_m\frac{1}{\cos\varphi}\sin\omega t_1>0$$
minimum

III ćwiartka napięcia:

$$\Psi_1=0 \quad \frac{\partial^2 i}{\partial t^2}=I_m\frac{\omega^2}{\sin\varphi}\cos\omega t_1<0 \qquad \frac{\partial^2 i}{\partial \Psi^2}=-\frac{I_m}{\cos\varphi}\sin\omega t_1>0$$
nie ma extremum

$$\Psi_1=180 \quad \frac{\partial^2 i}{\partial t^2}=-I_m\frac{\omega^2}{\sin\varphi}\cos\omega t_1>0 \qquad \frac{\partial^2 i}{\partial \varphi^2}=I_m\frac{1}{\cos\varphi}\sin\omega t_1<0$$
nie ma extremum

$\Psi_1=0$



I_M - maksymalna wartość prądu w obrębie kąta φ

$$i=\underbrace{I_m\sin(\omega t+\Psi-\varphi)}_{i_u}-\underbrace{I_m\sin(\Psi-\varphi)e^{-\alpha t}}_{i_p}$$

$$\Psi_1=0\ ;\quad i=I_m\sin(\omega t-\varphi)+I_m\sin\varphi\, e^{-\alpha t}$$

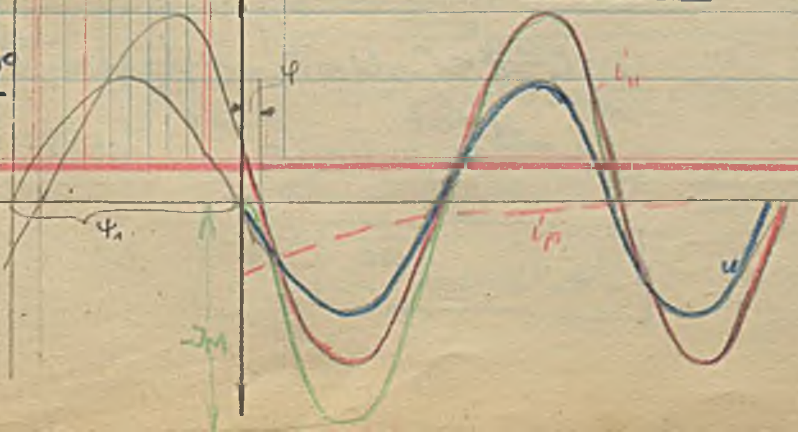
$$i=I_m[\underbrace{\sin(\omega t-\varphi)+\sin\varphi\, e^{-\alpha t}}_{<2}]$$ gdy t_1 to:

$$J_M=I_m[\sin(\omega t_1-\varphi)+\sin\varphi\, e^{-\alpha t_1}]$$ maximum ~~minimum~~ nie przekracza 2I

$I_M < 2I_m$

$\Psi_1=180°$

w II ćw. nap.



$i = Jm\, \sin(\omega t + 180 - \varphi) - Jm\, \sin(180 - \varphi) e^{-at}$

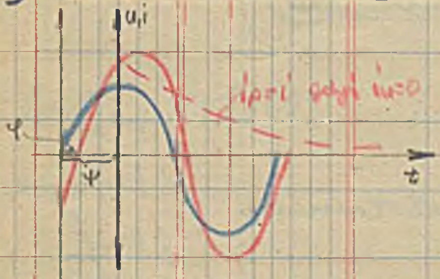
$i = -Jm\, \sin(\omega t - \varphi) - Jm\, \sin \varphi e^{-at}$

$J_M = -Jm[\underbrace{\sin(\omega t_1 - \varphi) + \sin \varphi e^{-at}}_{<2}]$

$\underline{J_M < -2 Jm}$

Poprzednio omawiane powstawanie prądu w obwodzie R, L.

I. b). Zanikanie prądu w obwodach R,L.



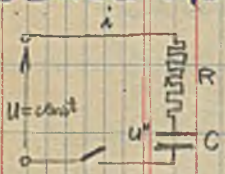
b). $u = U_m \sin(\omega t + \psi)$

2). przerywanie prądu

w danej chwili: $Jm\, \sin(\psi - \varphi) = i$

$i_p = Jm\, \sin(\psi - \varphi) e^{-\frac{R}{L}t}$

II. Obwód R,C.



1). U=const a). włączanie ładowania kondensatora

po naładowaniu $U = u''$

$i = \frac{U - u''}{R}$; $i = \frac{dq}{dt}$; $C = \frac{dq}{du''}$; $i = C\frac{du''}{dt}$

$RC\frac{du''}{dt} = U - u''$

$\int \frac{du''}{u'' - U} = -\int \frac{1}{CR} dt + \ln A$

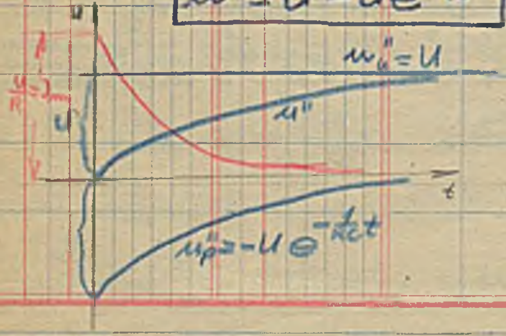
$\ln(u'' - U) = -\frac{t}{RC} + \ln A$;

$\frac{u'' - U}{A} = e^{-\frac{1}{RC}t}$; $u'' = U + A e^{-\frac{1}{RC}t}$

dla $t = 0$, $u'' = 0$

$0 = U + A$; $A = -U$ $U = u''_u$; $-Ue^{-\frac{1}{RC}t} = u''_p$

$$\boxed{u'' = U - Ue^{-\frac{1}{RC}t}}$$



$\frac{1}{RC} = \underline{a}$ $\frac{1}{a} = RC \underline{= \tau}$

$i = C\frac{du''}{dt} = CaUe^{-at} = C\frac{1}{RC}Ue^{-at}$

$i = \frac{U}{R}e^{-at} = Jme^{-at}$

$$\boxed{i = Jme^{-at}}$$

II. R.C.

1). $U = const$ b). wyładowanie kondensatora

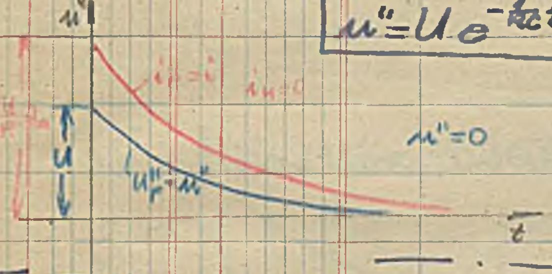
przerywanie

$$i = \frac{u''}{R} \ ; \quad i = -\frac{dq}{dt} \quad i = -C\frac{du''}{dt}$$

$$-RC\frac{du''}{dt} = u'' \ ; \quad \int_U^{u''}\frac{du''}{u''} = -\int_0^t \frac{1}{RC}dt \ ; \quad \ln u''\Big|_U^{u''} = -\frac{1}{RC}t$$

$$\boxed{u'' = Ue^{-\frac{1}{RC}t}}$$

$$\ln\frac{u''}{U} = -\frac{1}{RC}t$$



$u'' = 0$

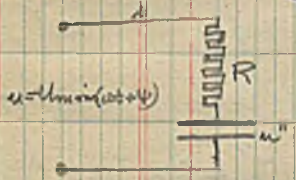
$$-C\frac{du''}{dt} = C\frac{1}{CR}\cdot Ue^{-\frac{1}{RC}t}$$

$$\boxed{i = \frac{U}{R}e^{-\frac{1}{RC}t}}$$ prąd wyładowania

II. R.C.

2). $u = U_m \sin(\omega t + \psi)$ 1). powstawania prądu ~RC

załączania



$$i = \frac{U_m \sin(\omega t + \psi) - u''}{R} \qquad e'' = u''$$

$$i = \frac{dq}{dt} \ ; \quad C\frac{dq}{du''} \ ; \quad i = C\frac{du''}{dt} \ ; \quad RC\frac{du''}{dt} + u'' = U_m \sin(\omega t + \psi)$$

$$\frac{du''}{dt} + \frac{u''}{RC} = \frac{U_m}{RC}\sin(\omega t + \psi)$$

$u'' = u''_u + u''_p$

całka rozwiązania

przedstawia u''_p

$$\frac{du''_p}{dt} + \frac{u''_p}{RC} = 0$$

$$\int\frac{du''_p}{dt_p} = -\int\frac{1}{RC}dt + \ln A \qquad \ln\frac{u''_p}{A} = -\frac{1}{RC}t \qquad \ln u''_p - \ln A = -\frac{t}{RC}$$

$$\boxed{u''_p = Ae^{-\frac{1}{RC}t}}$$

$$u''_u = -\frac{I_m}{\omega C}\cos(\omega t + \psi + \varphi) = -\frac{U_m}{Z\omega C}\cos(\omega t + \psi + \varphi)$$

$$u'' = -\frac{U_m}{Z\omega C}\cos(\omega t + \psi + \varphi) + Ae^{-\frac{1}{RC}t}$$

$$\frac{1}{RC} = a \qquad RC = \tau = \frac{1}{a} \qquad u'' = u''_u + u''_p$$

1.IV. 1949 r.

$t = 0 \quad u'' = 0$

$$\boxed{u'' = -\frac{U_m}{Z\omega C}\cos(\omega t + \psi + \varphi) + \frac{U_m}{Z\omega C}\cos(\psi + \varphi)e^{-at}}$$

$$0 = -\frac{U_m}{Z\omega C}\cos(\psi + \varphi) + A$$

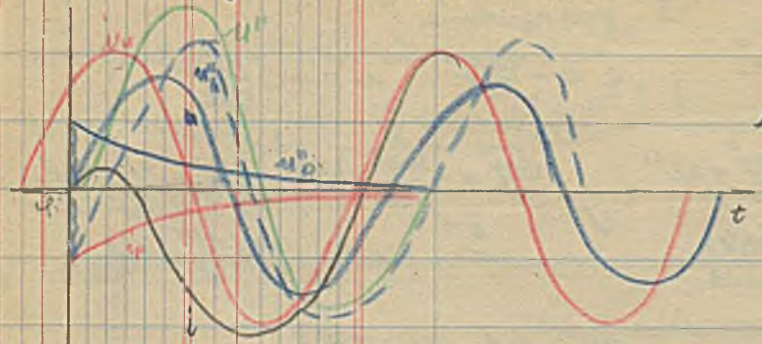
$$i = C\frac{du''}{dt} = C\frac{U_m}{Z\omega C}\omega \sin(\omega t + \psi + \varphi) - C\frac{U_m}{Z\omega C}a\cos(\psi + \varphi)e$$

$$A = \frac{U_m}{Z\omega C}\cos(\psi + \varphi)$$

$$i = I_m \sin(\omega t + \psi + \varphi) - I_m\frac{1}{R\omega C}\cos(\psi + \varphi)e^{-at}$$

$$\boxed{i = I_m \sin(\omega t + \psi + \varphi) - I_m \operatorname{tg}\varphi \cos(\psi + \varphi)e^{-at}}$$

1). $\psi=0$ przyłożone napięcie przechodzi przez 0

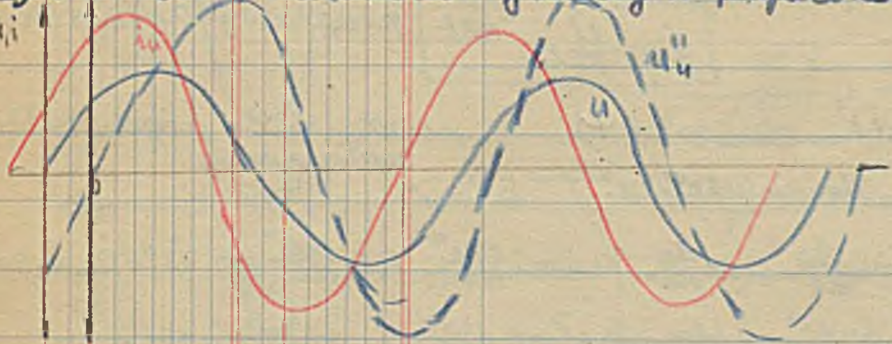


$$u''_u=\frac{U_m}{Z\omega C}\sin(\omega t+\psi+\varphi-\frac{\pi}{2})$$

$$u''_p=\frac{U_m}{Z\omega C}\cos\varphi e^{-at}=\frac{U_m}{Z\omega C}\sin(\varphi+\frac{\pi}{2})e^{-at}$$

$$i_p=-J_m \operatorname{tg}\varphi\cos\varphi e^{-at}$$

2). $\psi+\varphi=90°$ włączamy kiedy u przyłożone przechodzi przez 0



$$u''=\frac{U_m}{Z\omega C}\left[\cos(\psi+\varphi)e^{-at}-\cos(\omega t+\varphi+\varphi)\right]$$

≤ 2

Ten nawias przyjmuje maksymalną wartość ≤ 2

Nawias przyjmuje maksymalną wartość ≤ 2

$$\underline{u''\leq 2\frac{U_m}{Z\omega C}} \qquad \underline{U_M\leq\frac{2U_m}{Z\omega C}}$$

$$u''_p=\frac{U_m}{Z\omega C}\cos\frac{\pi}{2}e^{-at}=0 \quad ; \quad i_p=-J_m \operatorname{tg}\varphi\cos(\psi+\varphi)e^{-at}=0$$

Nie ma prądu przejściowego przy włączeniu kiedy prąd przechodzi przez wartość maksymalną.

3). $\psi=90°$

$$u''_p=\frac{U_m}{Z\omega C}\cos(90+\varphi)e^{-at}=-\frac{U_m}{Z\omega C}\sin\varphi e^{-at}$$

$i_p=J_m \operatorname{tg}\varphi\sin\varphi e^{-at}$ będzie zmieniać się w zależności od φ.

4) $\Psi + \varphi = 0$

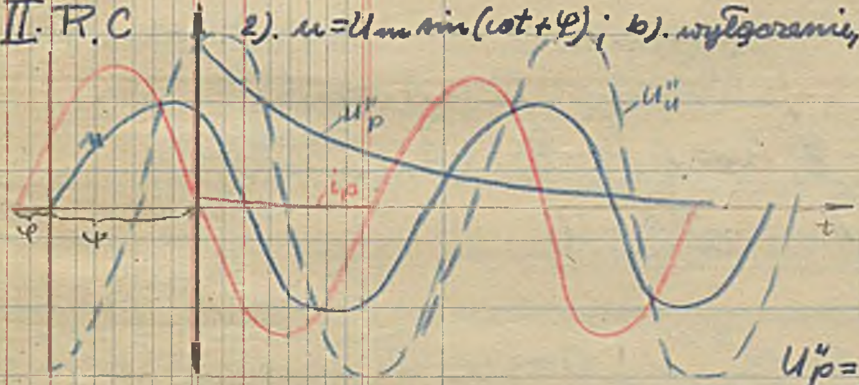
$$u''_p = \frac{U_m}{Z\omega C}\cos(\underbrace{\psi+\varphi}_{=0})e^{-at} = \frac{U_m}{Z\omega C}e^{-at}$$

$$i_p = -J_m e^{-at}$$

Gdy i_u przechodzi przez 0 to i_p posiada maximum.

II. R.C

2). $u = U_m \sin(\omega t + \varphi)$; b). wyłączanie, zanikanie 4. IV. 1948 r.



$$u = U e^{-at}; \quad a = \frac{1}{RC}$$

$$u''_u = \underbrace{\frac{U_m}{Z\omega C}\sin(\omega t+\varphi+\psi-90)}_{U''}$$

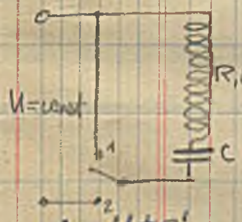
$$U'' = \frac{U_m}{Z\omega C}\sin(\psi+\varphi-90)$$

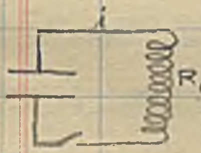
$$U''_p = \frac{U_m}{Z\omega C}\sin\left(\psi+\varphi-\frac{\pi}{2}\right)e^{-at}; \quad i_p = J_m e^{-at}$$

$$\underline{J''} = \underline{J_m \sin(\psi+\varphi)}; \quad i_p = J_m \sin(\psi+\varphi)e^{-at}$$

III. R.L.C

1). $U = const$ b). przerywanie, ~~zanikanie~~, wyładowanie ~~kondensatora~~





wyładowanie kondensatora przez indukcyjność.

$$i = \frac{U+e'}{R}; \quad e' = -L\frac{di}{dt}; \quad i = -\frac{dq}{dt}; \quad C = \frac{dq}{du}; \quad i = -C\frac{du}{dt}; \quad e' = CL\frac{d^2u}{dt^2}$$

$$-CR\frac{du}{dt} = u + CL\frac{d^2u}{dt^2}$$

$$y'' + py' + qy = 0; \quad p = \frac{R}{L}; \quad q = \frac{1}{LC}$$

$$CL\frac{d^2u}{dt^2} + CR\frac{du}{dt} + u = 0$$

$$k^2 + pk + q = 0; \quad k_1 = -\frac{p}{2} + \sqrt{\frac{p^2}{4} - q}$$

$$\frac{d^2u}{dt^2} + \frac{R}{L}\frac{du}{dt} + \frac{u}{CL} = 0$$

$$k_2 = -\frac{p}{2} - \sqrt{\frac{p^2}{4} - q}$$

1). $k_1 \neq k_2$ (rzeczywiste) $+\frac{p^2}{4} > q$

2). $k_1 = k_2$ - " - $\frac{p^2}{4} = q$ $k_1 = -\alpha + j\beta$

3). $k_1 \neq k_2$ (urojone) $\frac{p^2}{4} < q$ $k_2 = -\alpha - j\beta$

1). $y = A_1e^{k_1x} + A_2e^{k_2x}$ $\frac{R^2}{4L^2} > \frac{1}{LC}$ $| R > 2\sqrt{\frac{L}{C}}$

2). $y = e^{k_1x}(A_1 + A_2x)$ $| R = 2\sqrt{\frac{L}{C}}$

3). $y = e^{\alpha x}(A_1\sin\beta x + A_2\cos\beta x)$ $| R < 2\sqrt{\frac{L}{C}}$

$u = A_1e^{-a_1t} + A_2e^{-a_2t}$ $|k_1| = |a_1|$; $|k_2| = |a_2|$; $a_2 > a_1$

1). $u = A_1e^{-a_1t} + A_2e^{-a_2t}$ $u = f(t)$

$i = -C\frac{du}{dt} = -C[-a_1A_1e^{-a_1t} - a_2A_2e^{-a_2t}]$ $i_1 = f(t)$

szukamy stałych całkowania

dla $t = 0$, $i = 0$, $u = U$; $U = A_1 + A_2$; $0 = a_1A_1 + a_2A_2$

$A_2 = -\frac{a_1}{a_2}A_1$; $U = A_1 - \frac{a_1}{a_2}A_1 = A_1(1 - \frac{a_1}{a_2}) = A_1(\frac{a_2 - a_1}{a_2})$

$\boxed{A_1 = \frac{a_2}{a_1 - a_2}U}$ $\boxed{A_2 = -\frac{a_1}{a_2 - a_1}U}$

$\boxed{u = \frac{U}{a_2 - a_1}(a_2e^{-a_1t} - a_1e^{-a_2t})}$ $u = f(t)$ nie będzie maximum ani mini— gdy nie można przyrównać do 0

$\boxed{i = CU\frac{a_1a_2}{a_2 - a_1}(e^{-a_1t} - e^{-a_2t})}$ $i = f(t)$

Szukamy, czy i ma maximum i minimum, pierwszą pochodną przyrównujemy do 0.

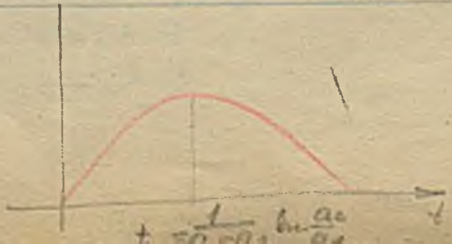
$\frac{di}{dt} = CU\frac{a_1a_2}{a_2 - a_1}(-a_1e^{-a_1t} + a_2e^{-a_1t_1}) = 0$

$a_1e^{-a_1t_1} = a_2e^{-a_2t_1}$ może istnieć extremum

$\frac{a_2}{a_1} = e^{(a_2 - a_1)t_1}$; $(a_2 - a_1)t_1 = \ln\frac{a_2}{a_1}$; $t_1 = \frac{1}{a_2 - a_1}\ln\frac{a_2}{a_1}$

$\frac{d^2i}{dt^2} = CU\frac{a_1a_2}{a_2 - a_1}(a_1^2e^{-a_1t} - a_2^2e^{-a_2t}) = CU\frac{a_1a_2}{a_2 - a_1}a_1e^{-a_1t}(a_1 - a_2) < 0$

istnieje maximum



2) $R = 2\sqrt{\frac{L}{C}} \qquad a_1 = a_2 = a$

$$\begin{cases} u = e^{-at}(A_1 + A_2 t) \\ i = Ce^{-at}(aA_1 + aA_2 t - A_2) \end{cases} \quad ; \quad i = C\frac{du}{dt} = -C(-aA_1e^{-at} - aA_2te^{-at} + A_2e^{-at}) \quad i = f(t)$$

Szukamy stałych całkowania

$$t=0, \quad u=U, \quad i=0 \quad ; \quad \left.\begin{aligned} U &= A_1 \\ 0 &= aA_1 - A_2 \end{aligned}\right\} \begin{aligned} A_1 &= U \\ A_2 &= aA_1 = aU \end{aligned}$$

$$\left.\begin{aligned} u &= Ue^{-at}(1+at) \ldots\ldots \quad u = f(t) \\ i &= CUe^{-at}(a + a^2t - a) = CUa^2te^{-at} \end{aligned}\right\} i = f(t)$$

$$\boxed{u = Ue^{-at}(1+at)}$$

$$\boxed{i = CUa^2te^{-at}}$$

Sprawdzamy przebieg funkcji. Pochodna napię nie może się równać zeru, nie ma więc max. ani mi

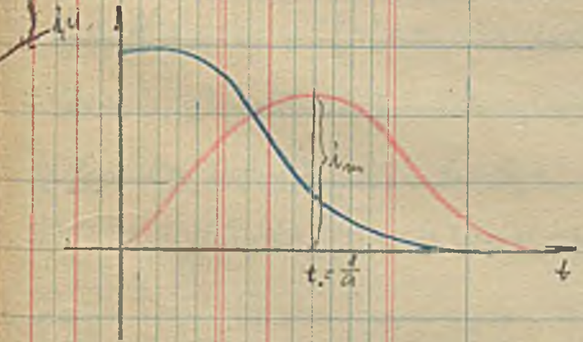
Sprawdzamy pochodną prądu.

$$\frac{di}{dt} = CUa^2(-ate^{-at} + e^{-at}) = CUa^2e^{-at}(1-at)$$

$1 - at_1 = 0 \quad ; \quad t_1 = \frac{1}{a}$ może być ekstremum

$$\frac{d^2i}{dt^2} = CUa^2(-ae^{-at} + a^2te^{-at} - ae^{-at}) = CUa^2e^{-at}(at-2) < 0$$

będzie istnieć maximum.



$i = CU \quad ; \quad k_1 = k_2 = k = -\frac{\rho}{2}$

$a = \frac{\rho}{2} \;;\; a = \frac{R}{2L} \;;\; a^2 = \frac{1}{LC} \;;\; a = \frac{1}{\sqrt{LC}}$

$q = \frac{1}{LC} \;;\; \frac{\rho^2}{4} = q$

$$i = CU\frac{1}{LC}te^{-at} = \frac{U}{L}te^{-at}$$

$$i_m = \frac{U}{L}\frac{1}{a}e^{-1} = \frac{U}{L}\sqrt{LC}\,e^{-1} = \frac{U}{L}LC\frac{1}{\sqrt{LC}}e^{-1}$$

$$\boxed{i_m = UCae^{-1}}$$

4.IV.1949r

3). $R<2\sqrt{\frac{L}{C}}$ $\quad k_1=-\frac{p}{2}+\sqrt{\frac{p^2}{4}-q}\ ;\ k_2=-\frac{p}{2}-\sqrt{\frac{p^2}{4}-q}$

$$k_1=-\frac{R}{2L}+\sqrt{\frac{R^2}{4L^2}-\frac{1}{LC}}=-\frac{R}{2L}+j\sqrt{\frac{1}{LC}-\frac{R^2}{4C^2}}$$

$$k_2=-\frac{R}{2L}-j\sqrt{\frac{1}{2C}-\frac{R^2}{4L^2}}$$

$$\left.\begin{array}{l}k_1=-\alpha+j\beta\\k_2=-\alpha-j\beta\end{array}\right\}\begin{array}{l}\alpha=\frac{R}{2L}\\\beta=\sqrt{\frac{1}{LC}-\alpha^2}\end{array}\qquad \alpha^2+\beta^2=\frac{1}{LC}$$

$$u=e^{-\alpha t}(A_1\sin\beta t+A_2\cos\beta t)$$

$$i=-C\frac{du}{dt}=-C[-\alpha A\sin\beta t\,e^{-\alpha t}+\beta A_1\cos\beta t\,e^{-\alpha t}-\alpha A_2\cos\beta t\,e^{-\alpha t}-\beta A_2\sin\beta t\,e^{-\alpha t}]$$

$t=0\ ;\ u=U\ ;\ i=0\ ;\ U=A_2\ ;\ 0=\alpha A_2-\beta A_1$

$\alpha U=\beta A_1 \qquad A_1=\frac{\alpha}{\beta}U$

$$u=e^{-\alpha t}\left(U\frac{\alpha}{\beta}\sin\beta t+U\cos\beta t\right)=\frac{U}{\beta}e^{-\alpha t}\underbrace{(\alpha\sin\beta t+\beta\cos\beta t)}_{\sqrt{\alpha^2+\beta^2}\sin(\beta t+\gamma)}$$

$$\boxed{u=\frac{U}{\beta\sqrt{LC}}e^{-\alpha t}\sin(\beta t+\gamma)}$$

$\sqrt{\alpha^2+\beta^2}=\frac{1}{\sqrt{LC}} \qquad \operatorname{tg}\gamma=\frac{\beta}{\alpha}$

$$i=-C\frac{du}{dt}=C\frac{U}{\beta\sqrt{LC}}[-\alpha\sin(\beta t+\gamma)e^{-\alpha t}+\beta\cos(\beta t+\gamma)e^{-\alpha t}]$$

$$i=C\frac{U}{\beta\sqrt{LC}}e^{-\alpha t}[\alpha\sin(\beta t+\gamma)-\beta\cos(\beta t+\gamma)]=C\frac{U}{\beta\sqrt{LC}}e^{-\alpha t}\cdot\sqrt{\alpha^2+\beta^2}\sin[(\beta t+\gamma)-\gamma]$$

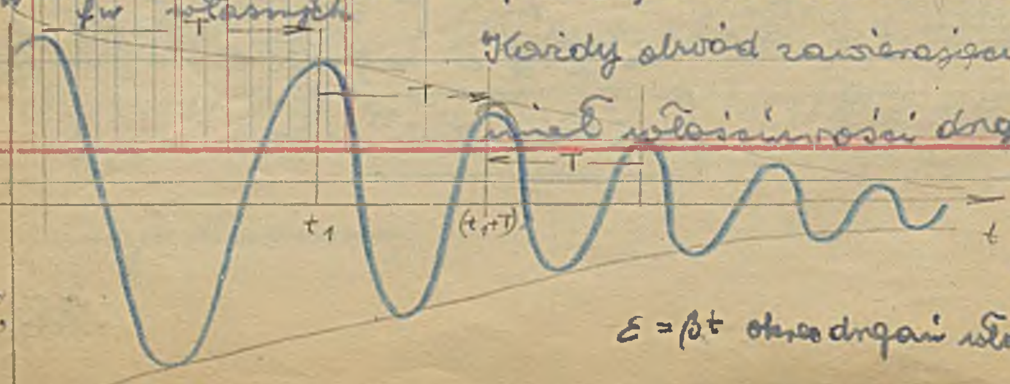
$i=C\frac{U}{\beta\sqrt{LC}}\quad e^{-\alpha t}\sin\beta t$ $\qquad$ i i u dwie sinusoidy z amplitudami

$\boxed{i=\frac{U}{\beta L}e^{-\alpha t}\sin\beta t}$ $\qquad$ zmieniającymi się w czasie i prze-

$\beta=2\pi f_{w}=\sqrt{\frac{1}{LC}-\frac{R^2}{4L^2}}$ $\qquad$ suniętymi w fazie. zanikające sinusoidy.

$T_w=\frac{1}{f_w}$ - okres drgań własnych $\qquad$ f_w = częstotliwość drgań własnych.

Każdy obwód zawierający R, L, C będzie miał właściwości drgań własnych.



Zachodzą gdy $R<2\sqrt{\frac{L}{C}}$ wyładowania oscylacyjne kondensatora.

$\varepsilon=\beta t$ okres drgań własnych.

$u_{m_1} = \frac{U}{\beta\sqrt{LC}} e^{-\alpha t_1}$; $u_{m_2} = \frac{U}{\beta\sqrt{LC}} e^{-\alpha(t_1+T_w)}$; $\frac{u_{m_1}}{u_{m_2}} = e^{-\alpha t_1+\alpha t_1+\alpha T_w} = e^{\alpha T_w}$

Krzywa przechodzi (odpowiadająca napięcie) przechodzi przez zero gdy $\sin(\beta t+\gamma)=0$ $\alpha T_w = \ln\frac{u_{m_1}}{u_{m_2}} = \lambda$ dekrement logarytmiczny $\alpha = \frac{\lambda}{T_w}$

1). gdzie przechodzi przez 0 $\sin(\beta t+\gamma)=0$

$\beta t+\gamma = 0, \pi, 2\pi \ldots =$; $\beta t = -\gamma, \pi-\gamma, 2\pi-\gamma \ldots$

$t = -\frac{\gamma}{\beta}$; $\frac{\pi-\gamma}{\beta}$; $\frac{2\pi-\gamma}{\beta}$ wtedy przechodzi przez zero

2). kiedy ta krzywa ma extremum

$$\frac{du}{dt} = \frac{U}{\beta\sqrt{LC}}\left(-\alpha\sin(\beta t+\gamma)e^{-\alpha t} + \beta\cos(\beta t+\gamma)e^{-\alpha t}\right]$$

$\frac{du}{dt} = -\frac{U}{\beta LC} e^{-\alpha t}\sin\beta t$ przyrównujemy do 0 ; $\sin\beta t = 0$; $\beta t = 0, \pi, 2\pi \ldots$

$$\frac{d^2u}{dt^2} = \mp\frac{U}{\beta LC}\underbrace{\left(\mp\alpha e^{-\alpha t}\sin\beta t \pm \beta\cos\beta t\, e^{-\alpha t}\right)}_{e^{-\alpha t}\sqrt{\alpha^2+\beta^2}\sin(\beta t+\gamma)}$$

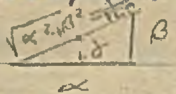
$$\frac{d^2u}{dt^2} = \frac{U}{\beta LC} e^{-\alpha t}(\alpha\sin\beta t - \beta\cos\beta t)$$

a). $\frac{d^2u}{dt^2} < 0$ wtedy $\beta t = 0, 2\pi, 4\pi$ maximum

b). $\frac{d^2u}{dt^2} > 0$ wtedy $\beta t = \pi, 3\pi, 5\pi$ minimum

dla $t=0$ $u = \frac{U}{\beta\sqrt{LC}}\sin\gamma$, $\sin\gamma = \beta\sqrt{CL}$, $\cos\gamma = \alpha\sqrt{LC}$

$u = \frac{U}{\beta\sqrt{LC}}\beta\sqrt{LC} = U$ na początku $u=U$



W rzeczywistości γ bardzo zbliżone do 90°

N.p.: $\operatorname{tg}\gamma = \frac{\beta}{\alpha}$; $C = 1\mu F$; $L = 90mH$, $R = 90\Omega$

$\frac{u_m}{i_m} = \frac{\beta L}{\beta\sqrt{LC}} = \sqrt{\frac{L}{C}} = Z_w$ oporność pozorna własna obwodu

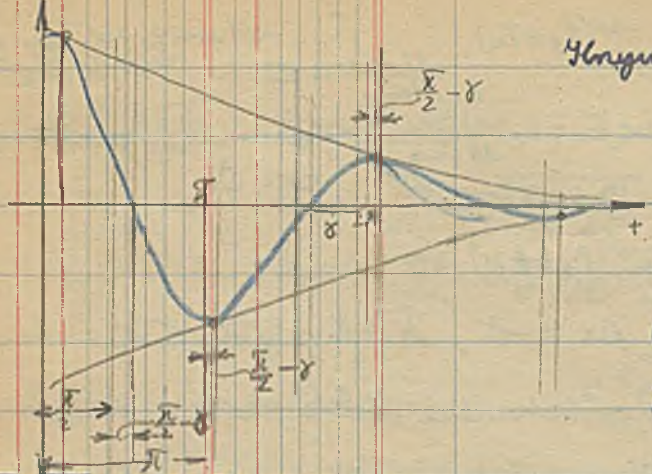
$\sqrt{\frac{L}{C}} = \sqrt{\frac{90\cdot10^{-3}}{10^{-6}}} = \sqrt{90\cdot10^3} = 3\cdot10^2 = 300\Omega$

$2\cdot300 = 600\Omega$
$90 < 600$
$2\sqrt{\frac{L}{C}} > R$

$\alpha = \frac{R}{2L} = \frac{90}{2\cdot90\cdot10^{-3}} = \frac{10^3}{2} = 500$ neper

$\beta = \sqrt{\frac{1}{LC} - \alpha^2} = \sqrt{\frac{10^6}{90\cdot10^{-3}} - 500^2} = \sqrt{\frac{10^9}{90} - 25\cdot10^4} = \sqrt{\frac{10^7}{9} - 25\cdot10^4} \cong 3300$

$\operatorname{tg}\gamma = \frac{3300}{500} = 6{,}6$ $\gamma = 81°$



Krzywa otrzymana nie jest sinusoidą.

$$\frac{U}{\beta\sqrt{LC}}e^{-\alpha t}=\frac{U}{\beta\sqrt{LC}}e^{-\alpha t}\sin(\beta t+\gamma)$$

styczność gdy $\underline{\beta t = 90-\gamma}$

wtedy będą miały wspólny punkt czyli będą styczne. Zachodzi to przy $\sin(\beta t+\gamma)=1$

5.IV.1949r.

$i=\frac{U}{\beta L}e^{-\alpha t}\sin\beta t$ 1) kiedy $=0$; $\sin\beta t=0$ $\beta t=0,\pi,2\pi$

2). extremum?

$$\frac{di}{dt}=\frac{U}{\beta L}(-\alpha\sin\beta t\, e^{-\alpha t}+\beta\cos\beta t\, e^{-\alpha t})=-\frac{U}{\beta L}\sqrt{\alpha^2+\beta^2}\,e^{-\alpha t}\underbrace{\sin(\beta t-\gamma)}_{=0}=0$$

$$\frac{di}{dt}=0\ ;\ \sin(\beta t-\gamma)=0\ ;\ \beta t-\gamma=0,\pi,2\pi\ldots\ ;\ \beta t=\gamma;\ \pi+\gamma,\ 4\pi+\gamma$$

$$\frac{d^2i}{dt^2}=\frac{U}{\beta L}\sqrt{\alpha^2+\beta^2}\,e^{-\alpha t}[\alpha\sin(\beta t-\gamma)-\beta\cos(\beta t-\gamma)]$$

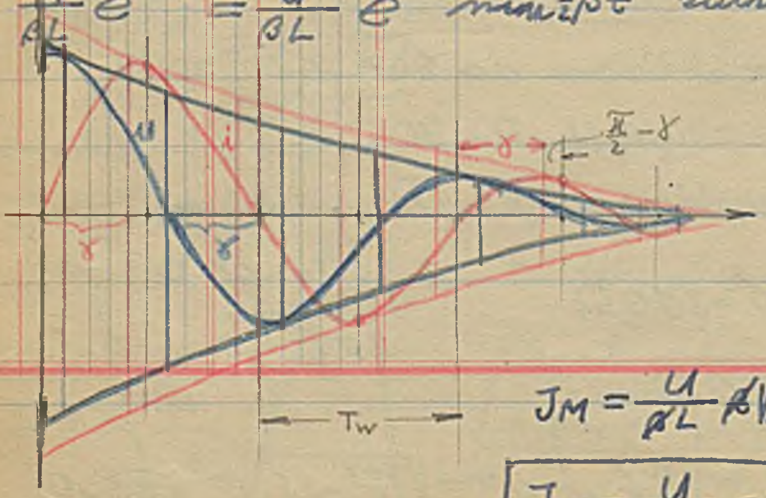
$\frac{d^2i}{dt^2}<0$; $\beta t-\gamma=0,2\pi,4\pi\ldots\ldots$ maximum

$\frac{d^2i}{dt^2}>0$; $\beta t-\gamma=\pi,3\pi,5\pi$ minimum

Punkty styczności krzywej wykładniczej i oscylacyjnej będą gdy

$\frac{U}{\beta L}e^{-\alpha t}=\frac{U}{\beta L}e^{-\alpha t}\sin\frac{1}{2}\beta t$ zaistnieje gdy $\sin\beta t=1$

$\beta t=\frac{\pi}{2},\ \frac{3}{2}\pi\ldots\ldots$



$i=\frac{U}{\beta L}e^{-\alpha t}\sin\beta t$

$J_M=\frac{U}{\beta L}e^{-\alpha t_1}\sin\gamma$

$\beta t_1=\gamma$, $\underline{t_1=\frac{\gamma}{\beta}}$

$J_M=\frac{U}{\beta L}e^{-\frac{\alpha}{\beta}\gamma}\sin\gamma$

$\sin\gamma=\beta\sqrt{LC}$

$$J_M=\frac{U}{\beta L}\beta\sqrt{LC}\,e^{-\frac{\alpha}{\beta}\gamma}=\frac{U}{\sqrt{\frac{C}{L}}}e^{-\frac{\alpha}{\beta}\gamma}$$

$$\boxed{J_M=\frac{U}{Z\omega}e^{-\frac{\alpha}{\beta}\gamma}}$$ maximum maximorum prądu

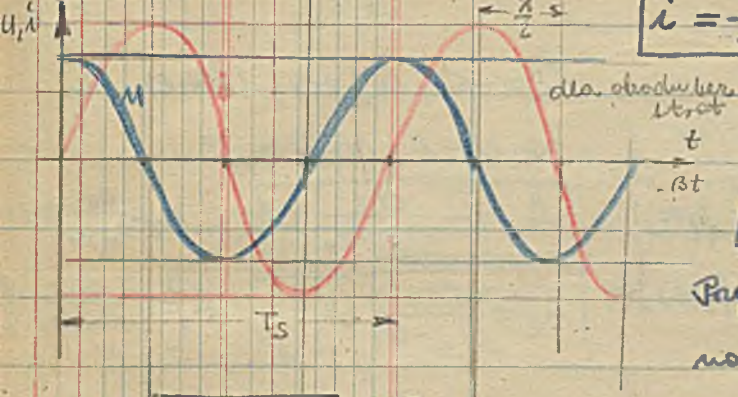
gdy $R \approx 0$ to $\alpha = \frac{R}{2L} = 0$. $\beta = \sqrt{\frac{1}{LC} - \alpha^2} = \frac{1}{\sqrt{LC}}$

$\operatorname{tg} \gamma = \frac{\beta}{\alpha} = \infty$; $\gamma = 90°$; $e^{-\alpha t} = 1$; $u = \frac{U}{\beta\sqrt{LC}} \sin(\beta t + 90°)$

$$\boxed{u = U \sin\left(\beta t + \frac{\pi}{2}\right)}$$

$i = \frac{U}{\beta L} \sin \beta t = U \frac{\sqrt{LC}}{L} \sin \beta t = \frac{U}{\sqrt{\frac{L}{C}}} \sin$

$$\boxed{i = \frac{U}{Z_{\omega}} \sin \beta t}$$



$\beta = 2\pi f_w$; $\beta = 2\pi f_s$

f - drgania swobodne

$$\boxed{T_s = 2\pi\sqrt{LC}}$$

wtedy $\beta = \frac{1}{\sqrt{LC}} = \frac{2\pi}{T_s}$

Przy tych drganiach powstaje rezonans napięć w tym obwodzie.

$\beta_w = \sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}} = 2\pi f_w$

$\beta_s = \frac{1}{\sqrt{LC}} = 2\pi f_s$

$$\boxed{f_s > f_w}$$

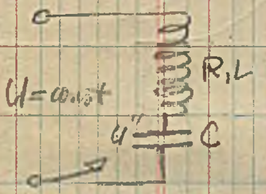
Gdy obwód nie ma oporu ohmowego ma t.zw. drgań swobodnych.

III. R.L.C.

1). U=const.

Okres [illegible] napięcia w tym obwodzie

a). zamykanie obwodu, ładowanie kondensatora.



$u''_u = U$ $i = \frac{U + e' - u''}{R}$; $i = \frac{dq}{dt}$; $C = \frac{dq}{du''}$

$e' = -L\frac{di}{dt} = -LC\frac{d^2u''}{dt^2}$ $i = C\frac{du''}{dt}$

$CR \frac{du''}{dt} = U - LC\frac{d^2u''}{dt^2} - u''$

$$\boxed{\frac{d^2u''}{dt^2} + \frac{R}{L}\frac{du''}{dt} + \frac{u''}{LC} = \frac{U}{LC}}$$

$u'' = u''_u + u''_p$ można znaleźć u''_p jako całkę szczególną.

$$\boxed{\frac{d^2u''_p}{dt^2} + \frac{R}{L}\frac{du''_p}{dt} + \frac{u''_p}{LC} = 0}$$ można znaleźć u''_p

Mamy 3 wypadki:

1). $R > 2\sqrt{\frac{L}{C}}$ ładowanie aperiodyczne

2). $R = 2\sqrt{\frac{L}{C}}$ wypadek graniczny między aperiod. i oscylacyjn.

3). $R < 2\sqrt{\frac{L}{C}}$ ładowanie oscylacyjne

ad 1: $R > 2\sqrt{\frac{L}{C}}$

$$u'' = \underbrace{A_1 e^{-a_1 t} + A_2 e^{-a_2 t}}_{u''_p} + \underbrace{U}_{u''_u}$$

$$a_1 = \frac{R}{2L} - \sqrt{\frac{R^2}{4L^2} - \frac{1}{LC}}$$
$$a_2 = \frac{R}{2L} + \sqrt{\frac{R^2}{4L^2} - \frac{1}{LC}}$$
$a_2 > a_1$

$$i = C\frac{du''}{dt} = C[-a_1 A_1 e^{-a_1 t} - a_2 A_2 e^{-a_2 t}] = -C(a_1 A_1 e^{-a_1 t} + a_2 A_2 e^{-a_2 t})$$

dla $t = 0$; $i = 0$; $u'' = 0$

$0 = A_1 + A_2 + U$

$0 = +a_1 A_1 + a_2 A_2$ } $A_2 = -U - A_1$

$0 = -a_1 A_1 + a_2 U + a_2 A_1$

$-a_2 U = A_1 (a_2 - a_1]$

$$\boxed{u'' = -\frac{U}{a_2 - a_1}(a_2 e^{-a_1 t} - a_1 e^{-a_2 t}) + U} \quad f_1(t)$$
$$\boxed{i'' = CU\frac{a_1 a_2}{a_2 - a_1}(e^{-a_1 t} - e^{-a_2 t})} \quad f_2(t)$$

$$\boxed{A_1 = -\frac{a_2}{a_2 - a_1} U}$$
$$\boxed{A_2 = \frac{a_1}{a_2 - a_1} U}$$

$A_2 = \frac{a_2}{a_2 - a_1} U - U$

$A_2 = U(\frac{a_2}{(a_2 - a_1} -1$

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ad 2) $a_1 = a_2 = u$

$u = e^{-at}(A_1 + A_2 t)$

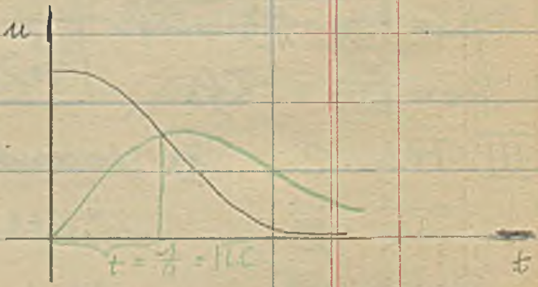
$u = U e^{-at}(1 + at)$

$i = CUa^2 t e^{-at}$

$a = \frac{R}{2L} = \frac{1}{\sqrt{LC}}$

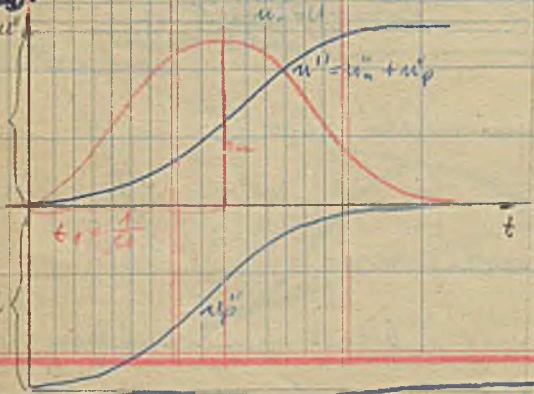
$i = \frac{U}{L} t e^{-at}$

gdy $t_1 = \frac{1}{a_1}$



ad 3). $u = e^{-\alpha t}(A_1 \sin\beta t + A_2 \cos\beta t)$

ad 2).



ad 2). $R = 2\sqrt{\frac{L}{C}}$; $u'' = e^{-at}(A_1 + A_2 t) + U$

$i = C[-aA_1 e^{at} - aA_2 t e^{-at} + A_2 e^{-at}]$

$i = -Ce^{-at}(aA_1 + atA_2 - A_2)$

$t = 0$, $u'' = 0$; $i = 0$

$0 = A_1 + U$ $\quad A_1 = -U$

$0 = aA_1 - A_2$ $\quad A_2 = -aU$

$$\boxed{u = -Ue^{-at}(1 + at] + U}$$

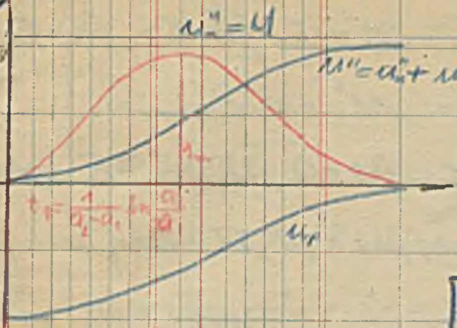
$i = CUe^{-at}(\cancel{a} + a^2 t - \cancel{a}) = CUe^{-at} a^2 t$

$= \frac{U}{L} t e^{-at}$ $\quad t = \frac{1}{a}$

$$i_m = \frac{U}{L} t_1 e^{-at_1} = \frac{U}{L}\frac{1}{a} e^{-a\frac{1}{a}}$$

LT $\quad i_m = \frac{U}{L}\sqrt{LC}\, e^{-1} = \frac{U}{\sqrt{\frac{L}{C}}} e^{-1}$

ad 1)



$i = CU\frac{a_1a_2}{a_2-a_1}(e^{-a_1t} - e^{-a_2t})$; $t_1 = \frac{1}{a_1-a_2}\ln\frac{a_2}{a_1}$

$e^{-a_1t} = \frac{a_2}{a_1}e^{-a_2t}$; $\frac{a_2}{a_1} = e^{(a_2-a_1)t_1}$.

$i_m = CU\frac{a_1a_2}{a_2-a_1}(e^{-a_1t_1} - \frac{a_1}{a_2}e^{-a_1t})$; $a_1e^{-a_1t_1} = a_2e^{-a_2t_1}$

$i_m = CUe^{-a_1t_1}\cdot\frac{a_1a_2}{a_2-a_1}(1-\frac{a_1}{a_2}) = CUe^{-a_1t_1}\cdot\frac{a_1a_2}{a_2-a_1}\cdot\frac{a_2-a_1}{a_2}$

$$\boxed{i_m = CUe^{-a_1t_1}a_1 = CUa_1e^{-\frac{a_1}{a_2-a_1}\ln\frac{a_2}{a_1}}}$$

ad 3). $u'' = e^{-\alpha t}\underbrace{(A_1\sin\beta t + A_2\cos\beta t)}_{u''_p} + \underbrace{U}_{u''_u}$

$i = C\frac{du}{dt} = C[-\alpha A_1\sin\beta t\, e^{-\alpha t} + \beta A_1\cos\beta t\, e^{-\alpha t} - \alpha A_2\cos\beta t\, e^{-\alpha t} - \beta A_2\sin\beta t\, e^{-\alpha t})$

$i = -Ce^{-\alpha t}(\alpha A_1\sin\beta t - \beta A_1\cos\beta t + \alpha A_2\cos\beta t + \beta A_2\sin\beta t)$

$t=0$; $i=0$; $u''=0$. $u'' = -Ue^{-\alpha t}(\frac{\alpha}{\beta}\sin\beta t + \cos\beta t) + U$

$0 = A_2 + U$ $A_2 = -U$ | $u'' = -\frac{U}{\beta}e^{-\alpha t}\underbrace{(\alpha\sin\beta t + \beta\cos\beta t)}_{\underbrace{\sqrt{\alpha^2+\beta^2}}_{\frac{1}{\sqrt{LC}}}\sin(\beta t+\gamma)} + U$ $\operatorname{tg}\gamma = \frac{\beta}{\alpha}$

$0 = \beta A_1 - \alpha A_2$ $A_1 = -\frac{\alpha}{\beta}U$

$$\boxed{u'' = \underbrace{-\frac{U}{\beta\sqrt{LC}}e^{-\alpha t}\sin(\beta t+\gamma)}_{u''_p} + \underbrace{U}_{u''_u}}$$

$i = C\frac{du''}{dt} = -\frac{U}{\beta\sqrt{LC}}e^{-\alpha t}[-\alpha\sin(\beta t+\gamma) + \beta\cos(\beta t+\gamma)]$

$i = \frac{CUe^{-\alpha t}}{\beta\sqrt{LC}}[\alpha\sin(\beta t+\beta) - \beta\cos(\beta+\gamma)] = \frac{CUe^{-\alpha t}}{\beta\sqrt{LC}}[\underbrace{\sqrt{\alpha^2+\beta^2}}_{\frac{1}{\sqrt{LC}}}\sin[(\beta+\gamma)-\gamma]$

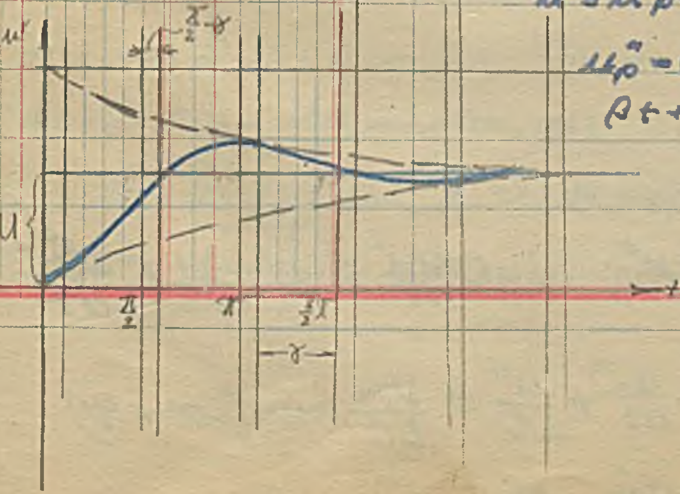
$$\boxed{i = \frac{U}{\beta L}e^{-\alpha t}\sin\beta t}$$

ale $t=0$; $u''_p = -\frac{U}{\beta\sqrt{LC}}\sin\gamma = -U$; $\sin\gamma = \beta\sqrt{LC}$

$u'' = u''_p + u''_u = -U + U = 0$

$u''_p = 0$ gdy $\sin(\beta t+\gamma) = 0$

$\beta t + \gamma = 0 \ldots\ldots 180$; $\beta t = -\gamma = 180 - \gamma$



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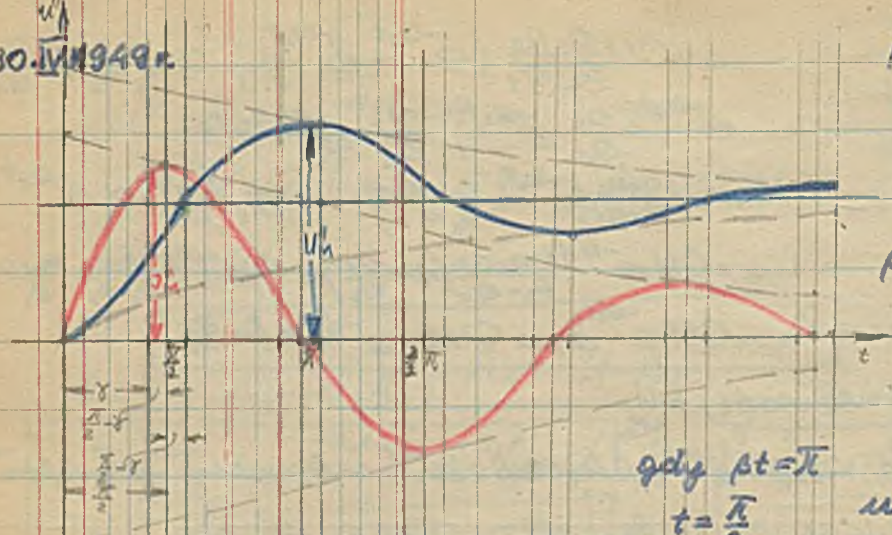


$\beta t = -\gamma \; ; \; \pi - \gamma \; ; \; 2\pi - \gamma$

$u''_p = u''_{p\,m}$

$\beta t = 0, 2\pi, 4\pi, 6\pi$ wtedy u_{min}

$\beta t = \pi, 3\pi, 5\pi$ wtedy u_{max}

gdy $\beta t = \pi$

$t = \frac{\pi}{\beta}$

$$u'' = -\frac{U}{\beta\sqrt{LC}} e^{-\frac{\alpha}{\beta}\pi} \sin(180+\gamma) + U$$

$$u''_M = \frac{U}{\beta\sqrt{LC}} e^{-\frac{\alpha}{\beta}\pi} \sin\gamma + U$$

$\sin\gamma = \beta\sqrt{LC}$; $\boxed{u''_M = U e^{-\frac{\alpha}{\beta}\pi} + U}$ amplituda napięcia max. [illegible]

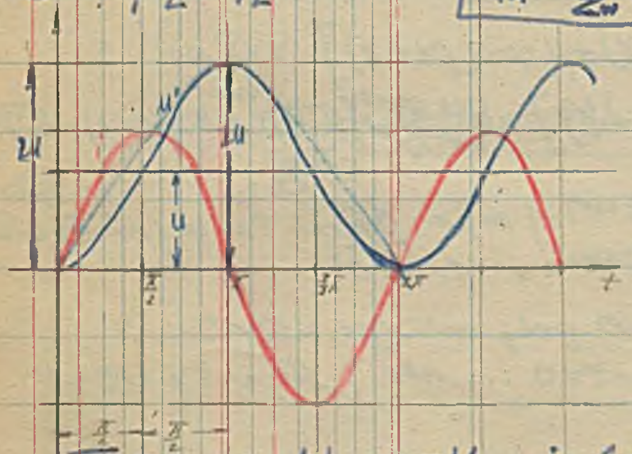
$\frac{1}{e^{\frac{\alpha}{\beta}\pi}} < 1$ $\quad u''_M < 2U$ Przepięcie nie przekroczy dwukrotnego napięcia przyłożonego.

gdy $\beta t = \gamma$ $\quad i = \frac{U}{\beta L} e^{-\alpha t} \sin\beta t$

$$J_M = \frac{U}{\beta L} e^{-\frac{\alpha}{\beta}\gamma} \sin\gamma = \frac{U\sqrt{LC}\beta}{\beta L} e^{-\frac{\alpha}{\beta}\gamma} = \frac{U}{\sqrt{\frac{L}{C}}} e^{-\frac{\alpha}{\beta}\gamma}$$

$\sqrt{\frac{L}{C}} = Z_w$; $\frac{\sqrt{LC}}{L} = \sqrt{\frac{C}{L}}$

$$\boxed{J_M = \frac{U}{Z_w} e^{-\frac{\alpha}{\beta}\gamma}}$$

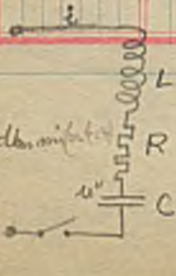


o ile pomijamy straty; $R \approx 0$; $\alpha \approx 0$

$\beta = \sqrt{\frac{1}{Lc} - \alpha^2} = \sqrt{\frac{1}{LC}}$; $\operatorname{tg}\gamma = \frac{\beta}{\alpha} = \infty$; $\underline{\gamma = 90^\circ}$

$J_M = \frac{U}{Z_w}$; $u''_M = 2U$

III R,L,C i b). $u = U_m \sin(\omega t + \varphi)$



$i = \frac{u + e' - u''}{R}$; $i = C\frac{du''}{dt}$; $e' = -LC\frac{d^2u''}{dt^2}$

$$\frac{d^2u''}{dt^2} + \frac{R}{L}\frac{du''}{dt} + \frac{u''}{LC} = \frac{U_m}{LC}\sin(\omega t + \varphi)$$

$$\frac{d^2u''_p}{dt^2} + \frac{R}{L}\frac{du''_p}{dt} + \frac{u''_p}{LC} = 0$$

1). $R > 2\sqrt{\frac{L}{C}}$; 2). $R = 2\sqrt{\frac{L}{C}}$; 3). $R < 2\sqrt{\frac{L}{C}}$

uzupełnić rozpatrywanie przypadku 1). i 2).

ad 3). $R < 2\sqrt{\frac{L}{C}}$ $\qquad u''_p = e^{-\alpha t}(A_1 \sin\beta t + A_2 \cos\beta t)$

$$u''_u = \frac{1}{C}\int i_u\,dt = \frac{1}{C} I_m \int \sin(\omega t + \psi - \varphi)\,dt$$

$i_u = I_m \sin(\omega t + \psi - \varphi)$

$u''_u = -\frac{1}{\omega C} I_m \cos(\omega t + \psi - \varphi)$; $u'' = u''_p + u''_u$

$$u'' = \underbrace{-\frac{U_m}{Z\omega C}\cos(\omega t + \psi - \varphi)}_{u''_u} + \underbrace{e^{-\alpha t}(A_1 \sin\beta t + A_2 \cos\beta t)}_{u''_p}$$

$$A_1 \sin\beta t + A_2 \cos\beta t = \underbrace{\sqrt{A_1^2 + A_2^2}}_{A} \sin(\beta t + \delta)$$

$$u'' = \underbrace{-\frac{U_m}{Z\omega C}\cos(\omega t + \psi - \varphi)}_{u''_u} + \underbrace{e^{-\alpha t} A \sin(\beta t + \delta)}_{u''_p}$$

wyznaczamy prąd i, a następnie stałe całkowania A i δ.

$$i_p = C\frac{du''_p}{dt} = CAe^{-\alpha t}[-\alpha \sin(\beta t + \delta) + \beta \cos(\beta t + \delta)]$$

$$i = i_u + i_p = \frac{U_m}{Z}\sin(\omega t + \psi - \varphi) - CAe^{-\alpha t}[\alpha \sin(\beta t + \delta) - \beta \cos(\beta t + \delta)]$$

dla $t = 0$; $u'' = 0$; $i = 0$

$$0 = -\frac{U_m}{Z\omega C}\cos(\psi - \varphi) + A \sin\delta$$

$$0 = \frac{U_m}{Z}\sin(\psi - \varphi) - CA[\alpha \sin\delta - \beta \cos\delta]$$

$$A \sin\delta = \frac{U_m}{Z\omega C}\cos(\psi - \varphi)$$

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$$CA\alpha \sin\delta - \beta CA \cos\delta = \frac{U_m}{Z}\sin(\psi - \varphi)$$

$$C\beta A \cos\delta = -\frac{U_m}{Z}\sin(\psi - \varphi) + CA\alpha \sin\delta$$

$$A \cos\delta = -\frac{U_m}{ZC\beta}\sin(\psi - \varphi) + \frac{C\alpha}{C\beta}\frac{U_m}{Z\omega C}\cos(\psi - \varphi)$$

$$A \cos\delta = \frac{U_m}{Z\omega C\beta L}[\alpha \cos(\psi - \varphi) - \omega \sin(\psi - \varphi)]$$

$$A\sin(\beta t+\delta)=A\cos\delta\sin\beta t+A\sin\delta\cos\beta t=$$

$$=\frac{U_m}{Z\omega C\beta}\left\{\left[\alpha\cos(\psi-\varphi)-\omega\sin(\psi-\varphi)\right]\sin\beta t+\left[\beta\cos(\psi-\varphi)\right]\cos\beta t\right\}=$$

$$=\frac{U_m}{Z\omega C\beta}\left[\alpha\cos(\psi-\varphi)\sin\beta t-\omega\sin(\psi-\varphi)\sin\beta+\beta\cos(\psi-\varphi)\cos\beta t\right]$$

$$A\sin(\beta t+\delta)=\frac{U_m}{Z\omega C\beta}\left[\sqrt{\alpha^2+\beta^2}\cos(\psi-\varphi)\sin(\beta t+\gamma)-\omega\sin(\psi-\varphi)\sin\beta t\right]$$

$$=\frac{U_m}{Z\omega C\beta}\left[\frac{1}{\sqrt{LC}}\cos(\psi-\varphi)\sin(\beta t+\gamma)-\omega\sin(\psi-\varphi)\sin\beta t\right]$$

$$A\cos(\beta t+\delta)=A\cos\delta\cos\beta t-A\sin\delta\sin\beta t$$

$$=\frac{U_m}{Z\omega C\beta}\left[\alpha\cos(\psi-\varphi)-\omega\sin(\psi-\varphi)\right]\cos\beta t-\frac{U}{Z\omega C}\left[\cos(\psi-\varphi)\right]\sin\beta t=$$

$$=\frac{U_m}{Z\omega C\beta}\left\{\left[\alpha\cos(\psi-\varphi)-\omega\sin(\psi-\varphi)\right]\cos\beta t-\left[\beta\cos(\psi-\varphi)\right]\sin\beta t\right\}=$$

$$=\frac{U_m}{Z\omega C\beta}\left[\alpha\cos(\psi-\varphi)\cos\beta t-\omega\sin(\psi-\varphi)\cos\beta t-\beta\cos(\psi-\varphi)\sin\beta t\right]$$

$$=\sqrt{\alpha^2+\beta^2}\cos(\psi-\varphi)\sin(\beta t-\gamma')\;;\quad \operatorname{tg}\gamma'=\frac{\alpha}{\beta}=\frac{1}{\operatorname{tg}\gamma}=\operatorname{ctg}\gamma$$

$$\gamma'=90-\gamma$$

$$A\cos(\beta t+\delta)=\left[\frac{1}{\sqrt{LC}}\cos(\psi-\varphi)\sin(\beta t-\gamma')-\omega\sin(\psi-\varphi)\cos\beta t\right.$$

3.V.1949r. $\operatorname{ctg}\gamma=\operatorname{tg}(90-\gamma)$ $\gamma'=90-\gamma$

$$\sin(\beta t-\gamma')=\sin(\beta t-90+\gamma)=-\sin\left[90-(\beta t+\gamma)\right]=-\cos(\beta t+\gamma)$$

$$A\cos(\beta t+\delta)=\frac{U_m}{Z\omega C\beta}\left[\cancel{\frac{U_m}{Z\omega C\beta}}\left[\underline{\frac{1}{\sqrt{LC}}}\cos\underline{(\psi-\varphi)}\cos(\beta t+\gamma)-\omega\sin(\psi-\varphi)\cos\beta t\right]\right.$$

$$u_p''=Ae^{-\alpha t}\sin(\beta t+\delta)$$

$$u_p''=\frac{U_m}{Z\omega C\beta}e^{-\alpha t}\left[\frac{1}{\sqrt{LC}}\cos(\psi-\varphi)\sin(\beta t+\gamma)-\omega\sin(\psi-\varphi)\sin\beta t\right]$$

$$i_p=CAe^{-\alpha t}\left[\alpha\sin(\beta t+\delta)+\beta\cos(\beta t+\delta)\right]$$

$$i_p=-Ce^{-\alpha t}\left[\alpha A\sin(\beta t+\delta)-\beta A\cos(\beta t+\delta)\right]$$

$$i_p=-\frac{U_m}{Z\omega\beta}e^{-\alpha t}\left[\frac{\alpha}{\sqrt{LC}}\cos(\psi-\varphi)\sin(\beta t+\gamma)-\alpha\omega\sin(\psi-\varphi)\sin\beta t-\frac{\beta}{\sqrt{LC}}\cos(\psi-\varphi)\cos(\beta t+\right.$$

$$\left.+\beta\omega\sin(\psi-\varphi)\cos\beta t\right]$$

$$i_p=-\frac{U_m}{Z\omega\beta}e^{-\alpha t}\left[\frac{1}{LC}\cos(\psi-\varphi)\sin\beta t-\frac{\omega}{\sqrt{LC}}\sin(\psi-\varphi)\sin(\beta t-\gamma)\right]$$

$$\boxed{i_p=\frac{U_m}{Z\beta\sqrt{LC}}e^{-\alpha t}\left[\sin(\psi-\varphi)\sin(\beta t-\gamma)-\frac{1}{\sqrt{LC}}\cos(\psi-\varphi)\sin\beta t\right]}$$

$$\boxed{u''_p = -\frac{U_m}{ZC\beta}\left[\sin(\psi-\varphi)\sin\beta t - \frac{1}{\omega\sqrt{LC}}\cos(\psi-\varphi)\sin(\beta t+\gamma)\right]}$$

$$u''_p = -\frac{U_m e^{-\alpha t}}{ZC\beta}\left[\sin(\psi-\varphi)\sin\beta t - \frac{1}{\omega\sqrt{LC}}\cos(\psi-\varphi)\sin\beta t\cos\gamma - \frac{1}{\omega\sqrt{LC}}\cos(\psi-\varphi)\cos\beta t\sin\gamma\right]$$

$$u''_p = -\frac{U_m}{ZC\beta}e^{-\alpha t}\left\{\underbrace{\left[\sin(\psi-\varphi) - \frac{1}{\omega\sqrt{LC}}\cos(\psi-\varphi)\cos\gamma\right]}_{m}\sin\beta t - \underbrace{\left[\frac{1}{\omega\sqrt{LC}}\cos(\psi-\varphi)\sin\gamma\right]}_{n}\cos\beta t\right\}$$

$$u''_p = -\frac{U_m}{ZC\beta}e^{-\alpha t}N\sin(\beta t-\varepsilon) \qquad N=\sqrt{m^2+n^2} \;;\; \operatorname{tg}\varepsilon \frac{n}{m}$$

$$(m^2+n^2) = \sin^2(\psi-\varphi) - \frac{2}{\omega\sqrt{LC}}\sin(\psi-\varphi)\cos(\psi-\varphi)\cos\gamma + \frac{1}{\omega^2LC}\cos^2(\psi-\varphi)\cos^2\gamma + \frac{1}{\omega^2LC}\cos^2(\psi-\varphi)\sin$$

$$m^2+n^2) = \sin^2(\psi-\varphi) - \frac{1}{\omega\sqrt{LC}}\cos\gamma\sin 2(\psi-\varphi) + \frac{1}{\omega^2LC}\cos^2(\psi-\varphi) + \cos^2(\psi-\varphi) - \cos^2(\psi-\varphi)$$

$$(m^2+n^2) = 1 - \cos^2(\psi-\varphi)\left(1-\frac{1}{\omega^2LC}\right) - \frac{\alpha}{\omega}\sin 2(\psi-\varphi)$$

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$$u'' = \frac{U_m}{Z\omega C}\cos(\omega t+\psi-\varphi) - \frac{U_m}{ZC\beta}e^{-\alpha t}\left[\sin(\psi-\varphi)\sin\beta t - \frac{1}{\omega\sqrt{LC}}\cos(\psi-\varphi)\sin(\beta t+\gamma)\right]$$

$$i = \frac{U_m}{Z}\sin(\omega t+\psi-\varphi) + \frac{U_m}{Z\beta\sqrt{LC}}e^{-\alpha t}\left[\sin(\psi-\varphi)\sin(\beta t-\gamma) - \frac{1}{\omega\sqrt{LC}}\cos(\psi-\varphi)\sin\beta t\right]$$

$$\alpha = \frac{R}{2L} \;;\; \beta = \sqrt{\frac{1}{LC}-\alpha^2} \;;\; \operatorname{tg}\gamma = -\frac{\beta}{\alpha}$$

$$\operatorname{tg}\varepsilon = \frac{\frac{1}{\omega\sqrt{LC}}\cos(\psi-\varphi)\sin\gamma}{\sin(\psi-\varphi) - \frac{1}{\omega\sqrt{LC}}\cos(\psi-\varphi)\cos\gamma} = \frac{\frac{1}{\omega}\beta\cos(\psi-\varphi)}{\sin(\psi-\varphi) - \frac{\alpha}{\omega}\cos(\psi-\varphi)} \;\Big|:\omega\sin(\psi-\varphi)$$

$$\underline{\operatorname{tg}\varepsilon} = \frac{\operatorname{ctg}(\psi-\varphi)\beta}{\omega-\alpha\operatorname{ctg}(\psi-\varphi)} = \underline{\frac{\beta}{\operatorname{ctg}(\psi-\varphi)-\alpha}}$$

$$i_p = \frac{U_m}{Z\beta\sqrt{LC}}e^{-\alpha t}\left[\sin(\psi-\varphi)\sin\beta t\cos\gamma - \sin(\psi-\varphi)\cos\beta t\sin\gamma - \frac{1}{\omega\sqrt{LC}}\cos(\psi-\varphi)\sin\beta t\right]$$

$$i_p = \frac{U_m}{Z\beta\sqrt{LC}}e^{-\alpha t}\left\{\underbrace{\left[\sin(\psi-\varphi)\cos\gamma - \frac{1}{\omega\sqrt{LC}}\cos(\psi-\varphi)\right]}_{m'}\sin\beta t - \underbrace{\left[\sin(\psi-\varphi)\sin\gamma\right]}_{n'}\cos\beta t\right\}$$

$$i_p = \frac{U_m}{Z\beta\sqrt{LC}}e^{-\alpha t}N'\sin(\beta t-\varepsilon') \;;\; N' = \sqrt{m'^2+n'^2} \;;\; \operatorname{tg}\varepsilon' = \frac{n'}{m'}$$

$$(m'^2+n'^2) = \sin^2(\psi-\varphi)\cos^2\gamma - \frac{2}{\omega\sqrt{LC}}\sin(\psi-\varphi)\cos(\psi-\varphi)\cos\gamma + \frac{1}{\omega^2LC}\cos^2(\psi-\varphi) + \sin^2(\psi-\varphi)\sin^2\gamma$$

$$(m'^2+n'^2) = \sin^2(\psi-\varphi) - \frac{\alpha}{\omega}\sin 2(\psi-\varphi) + \frac{1}{\omega^2LC}\cos^2(\psi-\varphi) + \cos^2(\psi-\varphi) - \cos^2(\psi-\varphi)$$

$$m'^2+n'^2 = 1-\cos^2(\psi-\varphi)\left(1-\frac{1}{\omega^2LC}\right) - \frac{R}{2\omega L}\sin 2(\psi-\varphi) \qquad \boxed{N=N'}$$

$$N' = \sqrt{m^2+n^2} = \sqrt{1-\cos^2(\psi-\varphi)\left(1-\frac{1}{\omega^2LC} - \frac{R}{2\omega L}\sin 2(\psi-\varphi)\right.}$$

$$\operatorname{tg}\varepsilon' = \frac{n'}{m'} = \frac{\sin(\psi-\varphi)\sin\gamma}{\sin(\psi-\varphi)\cos\gamma - \frac{1}{\omega\sqrt{LC}}\cos(\psi-\varphi)} \;\Big|:\cos(\psi-\varphi) = \frac{\operatorname{tg}(\psi-\varphi)\beta\sqrt{LC}}{\operatorname{tg}(\psi-\varphi)\alpha\sqrt{LC}-\frac{1}{\omega\sqrt{LC}}} = \frac{\beta\operatorname{tg}(\psi-\varphi)}{\alpha\operatorname{tg}(\psi-\varphi)\frac{1}{\omega LC}}$$

$$i_p = \frac{U_m}{Z\beta\sqrt{LC}}e^{-\alpha t}N\sin(\beta t-\varepsilon')$$

$$\boxed{u'' = -\frac{U_m}{Z\omega C}\cos(\omega t+\psi-\varphi) - \frac{U_m}{Z\beta C}e^{-\alpha t}N\sin(\beta t-\varepsilon)}$$

$$\boxed{i = \frac{U_m}{Z}\sin(\omega t+\psi-\varphi) + \frac{U_m}{Z\beta\sqrt{LC}}e^{-\alpha t}N\sin(\beta t-\varepsilon')}$$

$u'' = f(t,\psi)$ $\quad \frac{\partial u''}{\partial t} = 0 \quad \frac{\partial^2 u''}{\partial t^2} = A \; ; \; \frac{\partial^2 u''}{\partial \psi^2} = B \; ; \; \frac{\partial^2 u''}{\partial t \partial \psi} = C$

$i = f_1(t,\psi)$ $\quad \frac{\partial u''}{\partial \psi} = 0$

$\left.\begin{matrix} A<0 \\ B<0 \end{matrix}\right\}$ maximum $\qquad \left.\begin{matrix} A>0 \\ B>0 \end{matrix}\right\}$ minimum

$\frac{u''_m}{i_m} = \frac{U_m\beta\sqrt{LC}}{Z\beta C \cdot U_m} = \frac{\sqrt{LC}}{C} = \sqrt{\frac{L}{C}} = Z_w$ oporność pozorna drgań własnych

$\frac{u''^2_m}{i_m^2} = \frac{L}{C} \qquad u''^2_m C = i_m^2 L \qquad \underbrace{\frac{u''^2_m C}{2}}_{\text{energia pola elektr.}} = \underbrace{\frac{i_m^2 L}{2}}_{\text{energia pola magnet.}}$

gdyby nie było tłumienia czyli $R\approx 0$, wtedy energi oscylowałaby bez końca

$N = f(\psi)$ gdy $N = \sqrt{M}$; $M = f(\psi)$ szukamy maximum N

$M = 1 - \cos(\psi-\varphi)\left(1-\frac{1}{\omega^2 LC}\right) - \frac{R}{2\omega L}\sin 2(\psi-\varphi) = f(\psi)$

$\frac{dM}{d\psi} = \sin 2(\psi-\varphi)\left(1-\frac{1}{\omega^2 LC}\right) - \frac{R}{\omega L}\cos 2(\psi-\varphi)$

$\frac{d\cos^2\alpha}{d\alpha} = -\sin 2\alpha \; ; \; \frac{dM}{d\psi} = 0$ przyrównujemy do 0

$\sin 2(\psi-\varphi)\left(1-\frac{1}{\omega^2 LC}\right) = \frac{R}{\omega L}\cos 2(\psi-\varphi) \; ; \; \left(1-\frac{1}{\omega^2 LC}\right)\operatorname{tg} 2(\psi-\varphi) = \frac{R}{\omega L}$

$\operatorname{tg} 2(\psi-\varphi) = \frac{R}{\omega L\left(1-\frac{1}{\omega^2 LC}\right)} = \frac{R}{\omega L - \frac{1}{\omega C}} = \operatorname{ctg}\varphi$

$\operatorname{tg} 2(\psi-\varphi) = \operatorname{ctg}\varphi = \operatorname{tg}(90-\varphi) = \operatorname{tg}(270-\varphi)$

1). $\operatorname{tg} 2(\psi_1-\varphi) = \operatorname{tg}(90-\varphi)$

2). $\operatorname{tg} 2(\psi_1-\varphi) = \operatorname{tg}(270-\varphi)$

$\frac{d^2M}{d\psi^2} = \left(1-\frac{1}{\omega^2 LC}\right)2\cdot\cos 2(\psi-\varphi) + \frac{2R}{\omega L}\sin(\psi-\varphi) = 2\cos 2(\psi-\varphi)\left[\left(1-\frac{1}{\omega^2 LC}\right) + \frac{R}{\omega L}\operatorname{tg} 2(\psi-\varphi)\right]$

$= 2\cos 2(\psi-\varphi)\left[\left(1-\frac{1}{\omega^2 LC}\right) + \frac{R}{\omega L}\operatorname{tg}(90-\varphi)\right] = \frac{2R}{\omega L}\cos 2(\psi-\varphi)\left[\left(\frac{\omega L}{R}-\frac{1}{\omega RC}\right) + \underbrace{\operatorname{tg}(90-\varphi)}_{\operatorname{ctg}\varphi}\right]$

$\frac{\omega^2 LC - 1}{\omega RC} = \frac{1}{R}\left(\omega L - \frac{1}{\omega C}\right) = \frac{\omega L - \frac{1}{\omega C}}{R} = \operatorname{tg}\varphi$

$\frac{d^2M}{d\psi^2} = 2\frac{R}{\omega L}\cos 2(\psi_1-\varphi)\left[\underbrace{\operatorname{tg}\varphi + \operatorname{ctg}\varphi}_{>0}\right]$

1). $2\psi_1 - 2\varphi = 90 - \varphi$

$2\psi_1 = 90 + \varphi$

$\psi = \frac{1}{2}(90+\varphi)$

$\cos 2\left[\frac{1}{2}(90+\varphi)-\varphi\right] = \cos(90+\varphi-2\varphi) = \cos(90-\varphi) > 0$

[ ] $\frac{d^2M}{d\psi^2}$ } minimum

2). $2(\psi_1''-\varphi) = 270 - \varphi$

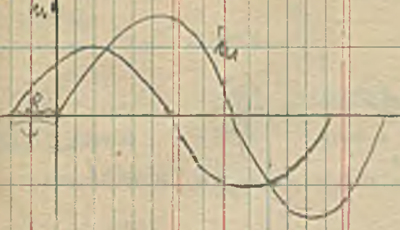
$2\psi_1'' - 2\varphi = 270 - \varphi$

$\psi_1'' = \frac{1}{2}(270+\varphi)$

$\cos 2\left[\frac{1}{2}(270+\varphi)-\varphi\right] = \cos(270+\varphi-2\varphi) = \cos(270-\varphi)$

$\frac{d^2M}{d\psi^2} < 0$ maximum

a). $\psi - \varphi = 0$ $\quad \psi = \varphi$ prąd ustalony przechodzi przez zero.



$$N = \sqrt{1-\left(1-\frac{1}{\omega^2 LC}\right)} = \sqrt{\frac{1}{\omega^2 LC}} = \frac{1}{\omega\sqrt{LC}}$$

$$u_p'' = -\frac{U_m}{Z\beta C} e^{-\alpha t} N \sin(\beta t - \varepsilon)$$

$$u_{pm}'' = -\frac{U_m}{Z\beta C}\frac{1}{\omega\sqrt{LC}} e^{-\alpha t} \quad ; \quad u_u'' = -\frac{U_m}{Z\omega C}$$

$$\frac{u_{pm}''}{u_{um}''} = \frac{Z\omega C}{Z\beta C\omega\sqrt{LC}} e^{-\alpha t} = \frac{1}{\beta\sqrt{LC}} e^{-\alpha t}$$

$\alpha = \frac{R}{2L}$; $R \approx 0$ $\alpha = 0$ to $\beta = \frac{1}{\sqrt{LC}}$

$\frac{u_{pm}''}{u_{um}''} = \frac{1}{\beta\sqrt{LC}} = \frac{\beta}{\beta} = 1$ Przepięcie najwyżej w tym wypadku 2 razy większe od napięcia ustalonego na kondensatorze.

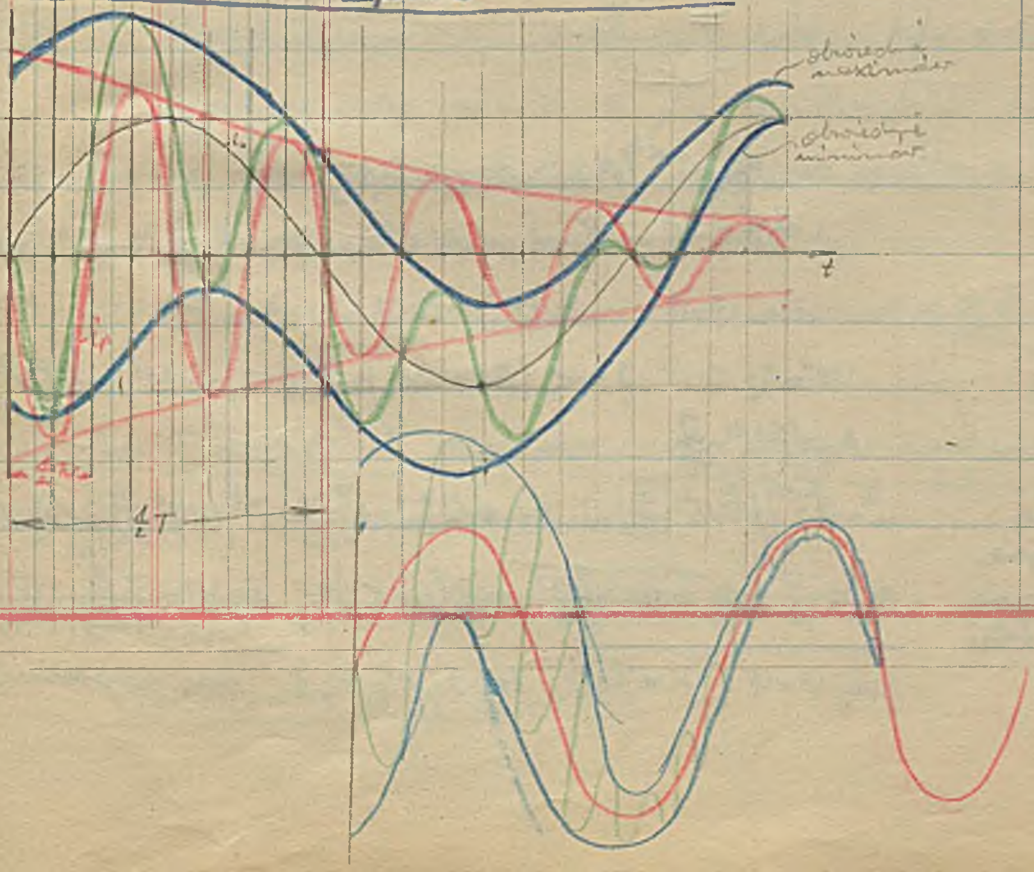
$i_p = -\frac{U_m}{Z\beta\sqrt{LC}} e^{-\alpha t} N \sin(\beta t - \varepsilon')$; $\quad tg\,\varepsilon' = \frac{tg(\psi-\varphi)\beta\omega LC}{tg(\psi-\varphi)\alpha\omega LC - 1}$ sprawdzić uprzednio!

$\frac{i_{pm}}{i_{um}} = \frac{1}{\beta\omega LC} = \frac{\beta^2}{\beta\omega} = \frac{\beta}{\omega}$; $\quad tg\,\delta = \frac{\beta}{\omega\, tg(\psi-\varphi) - \alpha} = -\frac{\beta}{\alpha}$; $tg\,\gamma = \frac{\beta}{\alpha}$; $\delta = -\gamma$

$tg\,\varepsilon' = 0 \quad \varepsilon' = 0$

$$i = \frac{U_m}{Z}\sin\omega t - \frac{U_m}{Z\beta\omega LC} e^{-\alpha t}\sin\beta t$$

zakładany $\beta = 4\omega$



b). $\psi-\varphi=90°$; $N=\sqrt{1-\cos^2(\psi-\varphi)(1-\frac{1}{\omega^2LC})-\frac{R}{\omega L}\sin 2(\psi-\varphi)}=1$

$u''_{pm}=-\frac{U_m}{Z\beta C}e^{-\alpha t}$; $u''_{um}=\frac{U_m}{Z\omega C}$

$\frac{u''_{pm}}{u''_{um}}=\frac{Z\omega C}{Z\beta C}e^{-\alpha t}$ gdy $R\approx0$, $\alpha=0$ to $\frac{u''_{pm}}{u''_{um}}=\frac{\omega}{\beta}$

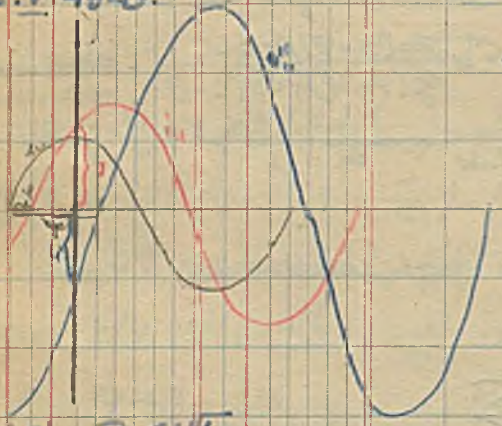
zachodzi gdy okres drgań wymuszonych > od okresu drgań własnych

$\frac{i_{pm}}{i_{um}}=\frac{1}{\beta\sqrt{LC}}e^{-\alpha t}$; gdy $R\approx0$; $\alpha=0$ $\beta=\frac{1}{\sqrt{LC}}$

$\frac{i_{pm}}{i_{um}}=\frac{1}{\beta LC}=\frac{\beta}{\beta}=1$ przetężeń w tym wypadku mieć nie można, gdyż prąd łączny może osiągnąć dwukrotną wartość.

III. R,L,C. 1). $u=U_m\sin(\omega t+\psi)$ 2). zanikanie prądu, wyłączenie źródła

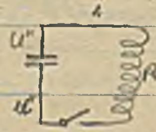
16.V.1949.

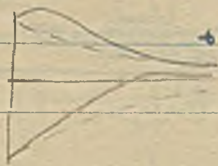


$t=0$ $J=\frac{U_m}{Z}\sin(\psi-\varphi)$

$U''=-\frac{U_m}{Z\omega C}\cos(\psi-\varphi)=\frac{U_m}{Z\omega C}\sin(\psi-\varphi-90°)$

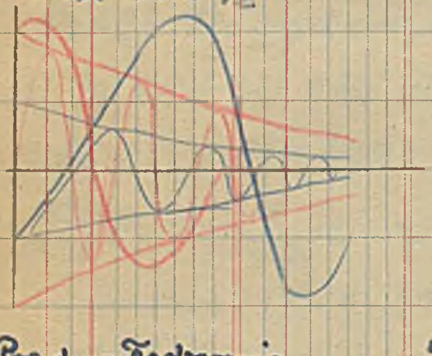
1). $R>2\sqrt{\frac{L}{C}}$ 2). $R=2\sqrt{\frac{L}{C}}$ 3). $R<2\sqrt{\frac{L}{C}}$





$t=0$, $u''=U''$; $i=J$

ad 3). $R<2\sqrt{\frac{L}{C}}$



$u''=e^{-\alpha t}(A\sin\beta t+A\cos\beta t)$

$\frac{du''}{dt}=-Ce^{-\alpha t}(-\alpha A_1\sin\beta t+\beta A_1\cos\beta t-\alpha A_2\cos\beta t-\beta A_2\sin\beta t)$

dla $t=0$ $u''=U''$ $i=J$

$U''=A_2$ $J=-C(\beta A_1-\alpha A_2)=-C(\beta A_1-\alpha U'')=\alpha CU''-C\beta A$

$A_1=\frac{\alpha}{\beta}U''-\frac{J}{C\beta}$

$u''=e^{-\alpha t}[(\frac{\alpha}{\beta}U''-\frac{J}{C\beta})\sin\beta t+U''\cos\beta t]=\frac{U''}{\beta}e^{-\alpha t}[(\alpha U''-\frac{J}{C})\sin\beta t+U''\beta\cos\beta t$

$u''=\frac{U''}{\beta}e^{-\alpha t}\sqrt{\alpha^2+\beta^2}\sin(\beta t+\gamma')$; $tg\gamma'=\frac{\beta}{\alpha+tg(\psi-\varphi)}=\frac{\beta}{\alpha}$

Prąd wyładowania przebiega oscylacyjnie, z tym, że zaczyna się już od pewnej wartości dla chwili $t=0$

gdy $\psi=\varphi$; $\psi=\varphi=0$ to $tg\gamma'=tg\gamma$ jak poprzednio.

Stany nieustalone linii długich

$-\frac{\partial u}{\partial x} = Ri + L\frac{\partial i}{\partial t}$

$-\frac{\partial i}{\partial x} = Gu + C\frac{\partial u}{\partial t}$

Wielkości chwilowe sinusoidalne możemy zastąpić wartościami symbolicznymi. Ponieważ nie znamy przebiegu zmienności naszych funkcji w czasie nie możemy stosować rachunku symbolicznego.

$-\frac{\partial^2 u}{\partial^2 x} = R\frac{\partial i}{\partial x} + L\frac{\partial^2 i}{\partial t \partial x}$

$-\frac{\partial^2 u}{\partial^2 x} = -R\left(Gu + C\frac{\partial u}{\partial t}\right) - L\left(G\frac{\partial u}{\partial t} + C\frac{\partial^2 u}{\partial t^2}\right)$

$\frac{\partial^2 u}{\partial x^2} = RGu + RC\frac{\partial u}{\partial t} + LG\frac{\partial u}{\partial t} + LC\frac{\partial^2 u}{\partial^2 t} = RGu + (RC+LG)\frac{\partial u}{\partial t} + LC\frac{\partial^2 u}{\partial^2 t}$

$u = m \cdot n \qquad m = f_1(t) \qquad \frac{\partial u}{\partial x} = m\frac{dn}{dx}\ ;\ \frac{\partial^2 u}{\partial x^2} = m\frac{d^2 n}{dx^2}$

$n = f_2(x) \cdot \qquad \frac{\partial u}{\partial t} = n\frac{dm}{dt}\ ;\ \frac{\partial^2 u}{\partial t^2} = n\frac{d^2 m}{dt^2}$

$m\frac{d^2 n}{dx^2} = RGmn + (RC+LG)n\frac{dm}{dt} + LCn\frac{d^2 m}{d^2 t} \quad /:mn$

$\underbrace{\frac{1}{n}\frac{d^2 n}{dx^2}}_{f_1(x)} = \underbrace{RG + \frac{1}{m}(RC+LG)\frac{dm}{dt} + \frac{1}{m}LC\frac{d^2 m}{d^2 t}}_{f_2(t)} = const$

dla każdego czasu t funkcja f ma być konstans, możliwe gdy cała funkcja równa jest constans. lewa strona dla jakiegokolwiek miejsca spełniać się musi.

$f(x) = g(y)$ gdy $x = x_1$ to $f(x_1) = a = g(y)$

$f_1(x) = const,\ f_2(t) = const;\ f_1(x) = f_2(t) = const = \pm b^2;\ \frac{1}{n}\frac{d^2 n}{dx^2} = \pm b^2$

1). $\frac{d^2 n}{dx^2} = nb^2\ ;\ \frac{d^2 n}{dx^2} - nb = 0\ ;\ y'' - a^2 y = 0$

$\underline{n = A_1 e^{bx} + A_2 e^{-bx}}$ wypadek: $x = \infty$; $n = \infty$ nie istnieje bo wszystko miałoby się równać ∞.

2). $\frac{1}{n}\frac{d^2 n}{dx^2} = -b^2 \qquad \frac{d^2 n}{dx^2} + b^2 n = 0\ ;\ y'' + a^2 y = 0$

$\frac{d^2 m}{d^2 t} + \frac{RC+LG}{LC}\frac{dm}{dt} + \left(\frac{RG}{LC} + \frac{b^2}{LC}\right)m = 0$

$\underline{n = A_1 \sin bx + A_2 \cos bx}$

$$\frac{d^2 m}{dt^2} + \underbrace{\left(\frac{R}{L}+\frac{G}{C}\right)}_{p}\frac{dm}{dt} + \underbrace{\left(\frac{RG+b^2}{LC}\right)}_{q} m = 0 \qquad y''+py'+qy=0$$

1). $\left.\begin{array}{l} K_1=-\frac{p}{2}+\sqrt{\frac{p^2}{4}-q} \\ K_2=-\frac{p}{2}-\sqrt{\frac{p^2}{4}-q}\end{array}\right\}$

$\left.\begin{array}{l} a_1=-K_1 \\ a_2=-K_2.\end{array}\right\}$

$a_2>a_1$

$m=A_1e^{-a_1t}+A_2e^{-a_2t}$

2). $K_1=K_2=K=-\frac{p}{2}$

$\sqrt{\frac{p^2}{4}-q}=0$

$\frac{p^2}{4}=q$

$m=e^{-at}(A_1+A_2t)$

$-\alpha=a$

3). $\left.\begin{array}{l} K_1=-\alpha+j\beta \\ K_2=-\alpha-j\beta\end{array}\right\}$

$\alpha=\frac{p}{2}$; $\beta=\sqrt{q-\frac{p^2}{4}}$

$\frac{p^2}{4}<q$; $\sqrt{-\left(q-\frac{p^2}{4}\right)}=j\beta$

$m=e^{-\alpha t}(B_1\sin\beta t+B_2\cos\beta t)$

napięcie jest aperiodyczną lub periodyczną f. czasu.

$\frac{p^2}{4}<q$; $\frac{1}{4}\left(\frac{R^2}{L^2}+\frac{2RG}{LC}+\frac{G^2}{C^2}\right)<\frac{RG}{LC}+\frac{b^2}{LC}$; $\frac{1}{4}\frac{R^2}{L^2}-\frac{1}{2}\frac{RG}{LC}+\frac{1}{4}\frac{G^2}{C^2}<\frac{b^2}{LC}$

$\boxed{\left(\frac{1}{2}\frac{R}{L}-\frac{1}{2}\frac{G}{C}\right)^2<\frac{b^2}{LC}}$ warunek periodyczności funkcji

17. $\overline{\text{V}}$. 1949 r.

$\alpha=\frac{p}{2}=\frac{R}{2L}+\frac{G}{2C}$; $\beta=\sqrt{q-\alpha^2}$

$\frac{RG+b^2}{LC}-\frac{R^2}{4L^2}-\frac{RG}{2LC}-\frac{G^2}{4C^2}=\frac{RG}{LC}+\frac{b^2}{LC}-\frac{R^2}{4L^2}-\frac{RG}{2LC}-\frac{G^2}{4C^2}=\underbrace{\left(\frac{R^2}{4L^2}-\frac{RG}{2LC}+\frac{G^2}{4C^2}\right)}_{\left(\frac{R}{2L}-\frac{G}{2C}\right)^2}$

$\beta=\sqrt{\frac{b^2}{LC}-\left(\frac{R}{2L}-\frac{G}{2C}\right)^2}=f(b)$; $u=m\cdot n$

$u=e^{-\alpha t}(A_1\sin bx+A_2\cos bx)(B_1\sin\beta t+B_2\cos\beta t)$

$u=e^{-\alpha t}(\underbrace{A_1B_1}_{A_1'}\sin bx\sin bt+\underbrace{A_2B_1}_{B_1'}\cos bx\sin\beta t+\underbrace{A_1B_2}_{A_2'}\sin bx\cos\beta t+$

$+\underbrace{A_2B_2}_{B_2'}\cos bx\cos\beta t)$

$u=e^{-\alpha t}[(A_1'\sin\beta t+A_2'\cos\beta t)\sin bx+(B_1'\sin\beta t+B_2'\cos bt)\cos bx$

$u=e^{-\alpha t}\left[\underbrace{\sqrt{A_1'^2+A_2'^2}}_{A}\sin(\beta t+\gamma)\sin bx+\underbrace{\sqrt{B_1'^2+B_2'^2}}_{B}\sin(\beta t+\partial)\cos bx\right]$

$\operatorname{tg}\gamma=\frac{A_2'}{A_1'}$ $\qquad$ $\operatorname{tg}\partial=\frac{B_2'}{B_1'}$

$u=e^{-\alpha t}[A\sin(\beta t+\gamma)\sin bx+B\sin(\beta t+\partial)\cos bx]$

cztery stałe całkowania A, γ, B, ∂.

Szukamy równania dla prądu.

$$-\frac{\partial i}{\partial x}=Gu+C\frac{\partial u}{\partial t}$$

$$-\frac{\partial i}{\partial x}=e^{-\alpha t}\left[GA\sin(\beta t+\gamma)\sin bx+GB\sin(\beta t+\delta)\cos bx-C\alpha A\sin(bt+\gamma)\sin bx+\right.$$
$$\left.+\beta CA\cos(\beta t+\gamma)\sin bx-C\alpha B\sin(\beta t+\delta)\cos bx+C\beta B\cos(\beta t+\delta)\cos bx\right]$$

$$\frac{\partial i}{\partial x}=e^{-\alpha t}\left\{\left[-GA\sin(\beta t+\gamma)+C\alpha A\sin(\beta t+\gamma)-C\beta A\cos(\beta t+\gamma)\right]\sin bx+\left[-GB\sin(\beta t+\delta)+\right.\right.$$
$$\left.\left.+C\alpha B\sin(\beta t+\delta)-C\beta B\cos(\beta t+\delta)\right]\cos bx\right\}$$

$$\frac{\partial i}{\partial x}=e^{-\alpha t}\left\{A\left[(C\alpha-G)\sin(\beta t+\delta)-C\beta\cos(\beta t+\delta)\right]\sin bx+B\left[(C\alpha-G)\sin(\beta t+\delta)-C\beta\cos(\beta t+\delta)\right]\overset{\cos bx}{}\right.$$

$$\frac{\partial i}{\partial x}=e^{-\alpha t}\sqrt{(C\alpha-G)^2+(\beta C)^2}\left[A\sin(\beta t+\gamma-\varphi)\sin bx+B\sin(\beta t+\delta-\varphi)\cos bx\right]$$

$\sqrt{(C\alpha-G)^2+(C\beta)^2}\sin(\beta t+\delta-\varphi)$ obliczyć z — po lewej stronie.

$$\left[(C\alpha-G)^2+C^2\beta^2\right]=\left[C\left(\frac{R}{2L}+\frac{G}{2C}\right)-G\right]^2+C^2\left[\frac{b^2}{LC}-\left(\frac{R}{2L}-\frac{G}{2C}\right)^2\right]=$$

$$=C^2\left(\frac{R^2}{4L^2}+\frac{RG}{2LC}+\frac{G^2}{4C^2}\right)-2GC\left(\frac{R}{2L}+\frac{G}{2C}\right)+G^2+C^2\frac{b^2}{LC}-C^2\frac{R^2}{4L^2}+\frac{C^2RG}{2LC}-C^2\frac{G^2}{4C^2}=$$

$$=\cancel{\frac{C^2R^2}{4L^2}}+\cancel{C\frac{RG}{2L}}+\cancel{\frac{G^2}{4}}-\cancel{\frac{CGR}{L}}-\cancel{G^2}+\cancel{G^2}+C^2\frac{b^2}{L\cancel{C}}-\cancel{\frac{C^2R^2}{4C^2}}+\cancel{C\frac{RG}{2L}}-\cancel{\frac{G^2}{4}}=\frac{C^2b^2}{LC}=b^2\frac{C}{L}$$

$$\frac{\partial i}{\partial x}=e^{-\alpha t}b\sqrt{\frac{C}{L}}\left[A\sin(\beta t+\gamma-\varphi)\sin bx+B\sin(\beta t+\delta-\varphi)\cos bx\right]$$

$$i=b\sqrt{\frac{C}{L}}\int e^{-\alpha t}\left[A\sin(\beta t+\gamma-\varphi)\sin bx+B\sin\beta t+B\sin(\beta t+\delta-\varphi)\cos bx\right]dx$$

$$i=b\sqrt{\frac{C}{L}}\,e^{-\alpha t}\frac{1}{b}\left[-A\sin(\beta t+\gamma-\varphi)\cos bx+B\sin(\beta t+\delta-\varphi)\sin bx\right]+D$$

$$tg\,\varphi=\frac{C\beta}{\alpha C-G}=\frac{\beta}{\alpha-\frac{G}{C}}=-\frac{\beta}{\frac{G}{C}-\alpha}=-\frac{\beta}{\frac{G}{2C}-\frac{R}{2L}}=\frac{\beta}{\frac{R}{2L}-\frac{G}{2C}}\quad;\quad\alpha=\frac{R}{2L}+\frac{G}{2C}$$

Prąd przesunięty wzgl. napięcia o ∢φ w czasie.

$\cos\alpha=\sin(\alpha+90)$; $-\sin(\alpha-90)=\cos\alpha$; $-\cos\alpha=\sin(\alpha-90)$

$\cos(90\pm\alpha)=\mp\sin\alpha$; $\sin\alpha=\cos(\alpha-90)$

Prąd na drodze (w przestrzeni) przesunięty o 90°.

— · —

Rozpatrywaliśmy wszystko dla wartości chwilowych.

Rozpatrywać będziemy linię przy przyłożonym napięciu stałym.

27.V.1949r.

$$-\frac{\partial i}{\partial x}=e^{-\alpha t}\cdot b\sqrt{\frac{C}{L}}\left[A\sin(\beta t+\gamma+\varphi)\sin bx+B\sin(\beta t+\delta+\varphi)\cos bx\right]$$

$$\mathrm{tg}\,\varphi=\frac{C\beta}{G-\alpha C}=\frac{\beta}{\frac{G}{C}-\alpha}=\frac{\beta}{\frac{G}{C}-\frac{R}{2L}-\frac{G}{2C}}=-\frac{\beta}{\frac{G}{2C}-\frac{R}{2L}}$$

$$i=-b\sqrt{\frac{C}{L}}\int e^{-\alpha t}\left[A\sin(\beta t+\gamma-\varphi)\sin bx+B\sin(\beta t+\delta+\varphi)\cos bx\right]dx+D$$

$$i=-e^{-\alpha t}\cancel{b}\sqrt{\frac{C}{L}}\frac{1}{\cancel{b}}\left[-A\sin(\beta t+\gamma+\varphi)\cos bx+B\sin(\beta t+\delta+\varphi)\sin bx\right]+D$$

dla stanu przejściowego $D=0$, dla ustalonego również równe 0.

$$\boxed{u=e^{-\alpha t}\left[A\sin(\beta t+\gamma)\sin bx+B\sin(\beta t+\delta)\cos bx\right]}$$

$$\boxed{i=e^{-\alpha t}\sqrt{\frac{C}{L}}\left[A\sin(\beta t+\gamma+\varphi)\cos bx-B\sin(\beta t+\delta+\varphi)\cos bx\right]}$$

Sinusoidy fazy po drodze nie zmienia, a więc fale stojące.

Fale te na drodze przesunięte wzgl. siebie o 90°

$\cos bx=\sin(bx+90°)$; $-\sin bx=\cos(bx+90°)$; $\frac{U_m}{i_m}=-\sqrt{\frac{L}{C}}$ oporność porom drgań własny

$\mathrm{tg}\,\varphi<0$ $\frac{R}{2L}>\frac{G}{2C}$; $\frac{R}{L}>\frac{G}{C}$ prąd przesunięty w tył.

$\mathrm{tg}\,\varphi>0$ $\frac{R}{L}<\frac{G}{C}$ -"- -"- w przód.

$\mathrm{tg}\,\varphi=\infty$ $\frac{R}{L}=\frac{G}{C}$ warunek dla linii nieodkształcającej.
$\varphi=90°$

1). Linia przy biegu Luzem. (otwarta na końcu)

długość l, przyłożone napięcie $U=\mathrm{const}$.

a). gdy $x=0$ na początku linii; $u=U$, $u_u=U$; $u_p=0$

b). -"- $x=l$ - $i=0$; $i_u=0$; $i_p=0$

$$(u_p)_{x=0}=e^{-\alpha t}\left[A\underbrace{\sin(\beta t+\gamma)\sin bx}_{=0}+B\sin(\beta t+\delta)\underbrace{\cos bx}_{=1}\right]=0$$

$0=e^{-\alpha t}B\sin(\beta t+\delta)$; $B=0$

$$\boxed{u_p=Ae^{-\alpha t}\sin(\beta t+\gamma)\sin bx}$$

$$\boxed{i_p=e^{-\alpha t}\sqrt{\frac{C}{L}}A\sin(\beta t+\gamma+\varphi)\cos bx}$$

, $(i_p)_{x=l}=0=A\sqrt{\frac{C}{L}}e^{-\alpha t}\sin(\beta t+\gamma+\varphi)\cos bl$

czyli $\cos bl$ musi być równy 0; to

$bl=\frac{\pi}{2}$; $\frac{3}{2}\pi$; $\frac{5}{2}\pi$;;

$b_1=\frac{1}{2}\frac{\pi}{l}$; $b_3=\frac{3}{2}\frac{\pi}{l}$

$b_5=\frac{5}{2}\frac{\pi}{l}$

$$\boxed{u_p=e^{-\alpha t}\sum_{k=1}^{k=\infty}A_{2k-1}\sin(\beta_{2k-1}t+\gamma_{2k-1})\sin b_{2k-1}x}$$

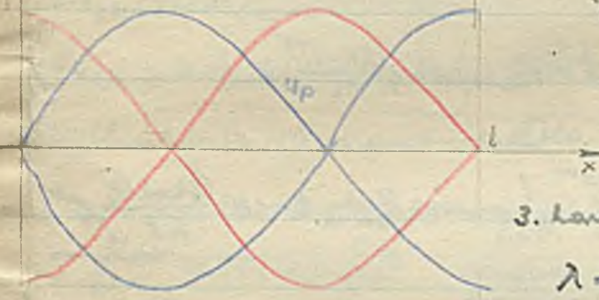
$$\boxed{i_p=e^{-\alpha t}\sqrt{\frac{C}{L}}\sum_{k=1}^{k=\infty}A_{2k-1}\sin(\beta_{2k-1}t+\gamma_{2k-1}+\varphi_{2k-1})\cos b_{2k-1}x}$$

1. harmoniczna.

Przejściowe wartości będą krzywymi odkszt., malejącymi w czasie, da się rozłożyć na harm.

Krzywe odkszt. symetr. wzgl. osi x i wzgl. osi y

Nie wszystkie harmoniczne mogą istnieć.



Biorę chwilę, w której $\sin(\beta_{2k-1}t+\gamma_{2k-1})=1$

$\sin\frac{1}{2}\frac{\pi}{l}l=\sin\frac{\pi}{2}=1$ otrz. ćwiartkę sinusoidy

Fala stojąca zmieniająca się w czasie.

3. harm. $b_3=\frac{3}{2}\frac{\pi}{l}$ $(u)_{x=l}$ $\sin b_3 l=\sin\frac{3}{2}\frac{\pi}{l}l=\sin\frac{3}{2}\pi=1$

$\lambda=\frac{4}{3}l$ $l=\frac{3}{4}\lambda$

5. harmoniczna

$(u_p)_{x=l}$ $\sin\frac{5}{2}\frac{\pi}{l}\cdot l=\sin\frac{5}{2}\pi=\sin\frac{\pi}{2}=1$

$\cos\frac{\pi}{2}=0$ $l=\frac{5}{4}\lambda$ $\lambda=\frac{4}{3}l$

28.V.1949 r.

$$u_p=e^{-\alpha t}\sum_{k=1}^{k=\infty}A_{2k-1}\sin(\beta_{2k-1}t+\gamma_{2k-1})\sin b_{2k-1}x$$

$$\operatorname{tg}\varphi_{2k-1}=-\frac{\beta_{2k-1}}{\frac{R}{2L}-\frac{G}{2C}}$$

$$i_p=\sqrt{\frac{C}{L}}e^{-\alpha t}\sum_{k=1}^{k=\infty}A_{2k-1}\sin(\beta_{2k-1}t+\gamma_{2k-1}+\varphi_{2k-1})\cos b_{2k-1}x$$

$$b_{2k-1}=\frac{(2k-1)\pi}{2l}\qquad \beta_{2k-1}=\sqrt{\frac{b^2}{LC}-\left(\frac{R}{2L}-\frac{G}{2C}\right)^2}=\sqrt{\frac{(2k-1)^2\pi^2}{4l^2LC}-\left(\frac{R}{2L}-\frac{G}{2C}\right)^2}$$

$t=0$, $u=0$. $u=u_u+u_p$

$u_u=U$ $u_p=-U$ na całej linii dla czasu $t=0$

$i=0$, $i_u=0$, $i_p=0$; Gdy podstawimy te wartości brzegowe do równań wyjściowych.

$$(u_p)_{t=0}=\sum_{k=1}^{k=\infty}A_{2k-1}\sin\gamma_{2k-1}\sin\frac{(2k-1)\pi}{2l}x=-U$$

$-U=\sum_{k=1}^{k=\infty}\underbrace{A_{2k-1}\sin\gamma_{2k-1}}_{D_{2k-1}}\sin(2k-1)m$ jest wyrażony szereg Fouriera

$\frac{\pi}{2l}x = m$; $-U = \sum_{k=1}^{k=\infty} D_{2k-1} \sin(2k-1)m$

$$\sin\gamma_{2k-1} = \frac{\beta_{2k-1}}{\sqrt{\left(\frac{R}{2L}-\frac{G}{2C}\right)^2 + \beta_{2k-1}^2}} = \frac{\beta_{2k-1}}{\sqrt{\left(\frac{R}{2L}-\frac{G}{2C}\right)^2 + \frac{b_{2k-1}^2}{LC} - \left(\frac{R}{2L}-\frac{G}{2C}\right)^2}} = \frac{\beta_{2k-1}}{b_{2k-1}}\sqrt{LC}$$

$$A_{2k-1}\cdot\sin\gamma_{2k-1} = -U\frac{4}{(2k-1)\pi}$$

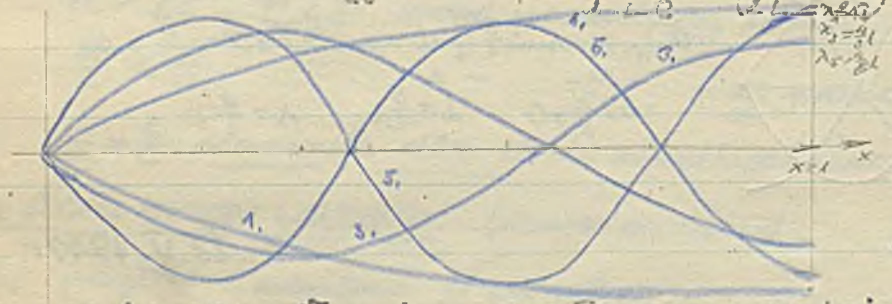
$$\boxed{A_{2k-1} = -U\frac{4}{\pi(2k-1)}\frac{b_{2k-1}}{\beta_{2k-1}\sqrt{LC}} = -U\frac{4}{(2k-1)\pi}\frac{(2k-1)\pi}{\beta\sqrt{LC}\,2l} = -U\frac{2}{l\cdot\beta_{2k-1}\sqrt{LC}}}$$

$$u_p = -U\frac{2}{l\sqrt{LC}}e^{-\alpha t}\sum_{k=1}^{k=\infty}\frac{1}{\beta_{2k-1}}\sin(\beta_{2k-1}t - \varphi_{2k-1})\sin\frac{(2k-1)\pi}{2l}x$$

$$i_p = -U\frac{2}{l\sqrt{LC}}\sqrt{\frac{C}{L}} \quad ; \quad i_p = -U\frac{2}{l.L}e^{-\alpha t}\sum_{k=1}^{k=\infty}\frac{1}{\beta_{2k-1}}\sin\beta_{2k-1}t\cdot\cos\frac{(2k-1)\pi}{2l}x$$

$$u_{p1} = U\frac{2}{l\sqrt{LC}}e^{-\alpha t}\frac{1}{\beta_1}\sin(\beta_1 t - \varphi_1)\sin\frac{\pi}{2l}x$$

1. harmoniczna $b_1 = \frac{\pi}{2l}$ $\beta_1 = \sqrt{\frac{b_1^2}{LC} - \left(\frac{R}{2L} - \frac{G}{2C}\right)^2}$



Fale te są zanikającymi w czasie drganiami.

linia otwarta, dług. l, przyłożonym napięciu $u = U_m\sin(\omega t + \psi)$

Rozumowanie to samo dotąd, dopóki nie bierzemy zależności od czasu

$x = 0$, $x = l$; Przeprowadzamy poprzednie rozumowanie i otrz.
$u_p = 0$, $i_p = 0$ | te same równania:

$$u_p = e^{-\alpha t}\sum_{k=1}^{k=\infty} A_{2k-1}\sin(\beta_{2k-1}t + \gamma_{2k-1})\sin b_{2k-1}x$$

$$i_p = \sqrt{\frac{C}{L}}e^{-\alpha t}\sum_{k=1}^{k=\infty} A_{2k-1}\sin(\beta_{2k-1}t + \gamma_{2k-1} + \varphi_{2k-1})\cos b_{2k-1}x$$

Warunki dla czasu $t=0$ dla całej linii

$u = 0$; $u = u_u + u_p$; $(u_p)_{t=0} = -u_u$ $u_u = f(x$

$i = 0$; $i = i_u + i_p$; $(i_p)_{t=0} = -i_u$ $i_u = f_1(x)$

$t=0$:

$$-u_n=\sum_{k=1}^{k=\infty}\underbrace{A_{2k-1}\sin\gamma_{2k-1}}_{M_{2k-1}}\sin\frac{(2k-1)\pi}{2l}X=\sum_{k=1}^{k=\infty}M_{2k-1}\sin(2k-1)m\;;\;m=\frac{\pi x}{2l}$$

$$M_{2k-1}=-\frac{4}{\pi}\int_0^{\frac{\pi}{2}}u_n\sin(2k-1)m\,dm$$ u_n - jest funkcją miejsca na linii

30. $\underline{\overline{\text{V}}}$. 1949 r.

$$\left.\begin{aligned}M_{2k-1}&=-\frac{4}{\pi}\int_0^{\frac{\pi}{2}}u_n\sin(2k-1)m\,dm\\N_{2k-1}&=-\frac{4}{\pi}\int_0^{\frac{\pi}{2}}i_n\cos(2k-1)m\,dm\end{aligned}\right\}\quad m=\frac{\pi}{2l}x$$ $u_n=f(x)$

$i_n=f_2(x)$

$M_{2k-1}=A_{2k-1}\sin\gamma_{2k-1}$ φ_{2k-1} znane

$N_{2k-1}=A_{2k-1}\sin(\gamma_{2k-1}+\varphi_{2k-1})$ znajdujemy A_{2k-1}, γ_{2k-1}

ogólnikowe rozwiązanie. Dla jakiegokolwiek stanu <u>linii nieodkszt.</u>

Linie takie spotykamy w praktyce.

$$-\frac{\partial u}{\partial x}=Ri+L\frac{\partial i}{\partial t}$$ równania zawsze ważne.

$$-\frac{\partial i}{\partial x}=Gu+C\frac{\partial u}{\partial t}$$

$$-\frac{\partial^2u}{\partial x^2}=R\frac{\partial i}{\partial x}+L\frac{\partial^2 i}{\partial t\partial x}$$

$$\mp\frac{\partial^2u}{\partial x^2}=\mp RGu\mp RC\frac{\partial u}{\partial t}\mp GL\frac{\partial u}{\partial t}\mp CL\frac{\partial^2u}{\partial t^2}$$

$$\frac{\partial^2u}{\partial x^2}=RGu+(RC+GL)\frac{\partial u}{\partial t}+CL\frac{\partial^2u}{\partial t^2}$$

o rozwiązanie ~~[illegible]~~:

$$u=e^{-\alpha t}[\underbrace{A\sin(\beta t+\gamma)\sin bx+B\sin(\beta t+\delta)\cos bx}_{m}]$$

tałe całkowania $m=f(x,t)$ $u=e^{-\alpha t}\cdot m$ $\alpha=\frac{R}{2L}+\frac{G}{2C}$

emy linię nieodkształcającą: $\frac{R}{L}=\frac{G}{C}$ $\alpha=\frac{R}{L}=\frac{G}{C}$

$$\frac{\partial u}{\partial x}=e^{-\alpha t}\cdot\frac{\partial m}{\partial x}\;;\;\frac{\partial^2u}{\partial x^2}=e^{-\alpha t}\cdot\frac{\partial^2m}{\partial x^2}$$

$$\frac{\partial u}{\partial t}=e^{-\alpha t}\left(-\alpha m+\frac{\partial m}{\partial t}\right)\;;\;\frac{\partial^2u}{\partial t^2}=e^{-\alpha t}\left(+\alpha^2m+\frac{\partial^2m}{\partial t^2}-2\alpha\frac{\partial m}{\partial t}\right)$$

$$e^{-\alpha t}\frac{\partial^2m}{\partial x^2}=e^{-\alpha t}\left\{RGm+(RC+GL)\left(-\alpha m+\frac{\partial m}{\partial t}\right)+CL\left(\alpha^2m-2\alpha\frac{\partial m}{\partial t}+\frac{\partial^2m}{\partial t^2}\right)\right\}$$

$$\frac{\partial^2m}{\partial x^2}=RGm-\alpha m(RC+GL)+(RC+GL)\frac{\partial m}{\partial t}+\alpha^2m\,CL-2CL\alpha\frac{\partial m}{\partial t}+CL\frac{\partial^2m}{\partial t^2}$$

$$\frac{\partial^2m}{\partial x^2}=m(\underbrace{RG-\alpha RC-\alpha GL+\alpha^2CL}_{=0})+\frac{\partial m}{\partial t}(\underbrace{RC+GL-2CL\alpha}_{=0})+CL\frac{\partial^2m}{\partial t^2}$$

$RG-RG$ $\alpha GL=\frac{G^2L}{C}$ $\alpha^2CL=\frac{G^2}{C^2}CL=\frac{G^2L}{C}$

$$\left.\begin{aligned}2CL\alpha&=2RC\\RC&=GL\end{aligned}\right\}+$$

$$\frac{\partial^2 m}{\partial x^2} = LC\frac{\partial^2 m}{\partial t^2} \; ; \; \frac{\partial^2 m}{\partial t^2} = \frac{1}{LC}\frac{\partial^2 m}{\partial x^2} ; \; \frac{\partial^2 m}{\partial t^2} = v^2\frac{\partial^2 m}{\partial x^2} \qquad v = \frac{1}{\sqrt{LC}}$$

$m = f_1(x-vt) + f_2(x+vt)$ równanie d'Alemberta

$$\frac{\partial m}{\partial t} = \frac{\partial f_1(x-vt)}{\partial(x-vt)}\cdot(-v) + \frac{\partial f_2(x+vt)}{\partial(x+vt)}v$$

$$\underline{\frac{\partial^2 m}{\partial t^2}} = \frac{\partial^2 f_1(x-vt)}{\partial(x-vt)^2}v^2 + \frac{\partial^2 f_2(x+vt)}{\partial(x+vt)}v^2 = \underline{v^2\left[\frac{\partial^2 f_1(x-vt)}{\partial(x-vt)} + \frac{\partial^2 f_2(x+vt)}{\partial(x+vt)}\right]}$$

$$\frac{\partial m}{\partial t^2} = \frac{\partial f_1(x-vt)}{\partial(x-vt)} + \frac{\partial f_2(x+vt)}{\partial(x+vt)} \; ; \; \underline{\frac{\partial^2 m}{\partial x^2} = \frac{\partial^2 f_1(x-vt)}{\partial(x-vt)^2} + \frac{\partial^2 f_2(x+vt)}{\partial(x+vt)^2}}$$

Funkcję $\frac{\partial^2 m}{\partial t^2} = v^2\frac{\partial^2 m}{\partial x^2}$ można przedstawić w formie równania d'Alemberta

$$\underline{u_p = e^{-\alpha t}[f_1(x-vt) + f_2(x+vt)]}$$

Rozwiązanie ogólne funkcji f_1 i f_2 nie znamy, ale $x-vt$ i $x+vt$ muszą być tak zgrupowane.

Fizykalnie: w danym miejscu dla pewnego czasu jest pewna wartość tej funkcji. W innym miejscu i w innym czasie nie może być takiej samej wartości:

Fala biegnąca.

Szybkość fali biegnącej:

$$f_1(x-vt) = f_1[x+dx - v(t+dt)]$$

$$x - vt = x + dx - vt - v\,dt$$

$v = \frac{dx}{dt}$ prędkość ; $v = \frac{1}{LC}$ prędkość na linii nieodkształcającej.

Będziemy mieć falę biegnącą naprzód ; dla drugiej funkcji $v = -\frac{dx}{dt}$ falę biegnącą wstec

Stąd równa się dwom falom ~~gasnącym~~ : 1 biegnąca od początku do końca, druga odbita biegnie od końca do początku ; obie są gasnące.

$$-\frac{\partial i}{\partial x} = Gu + C\frac{\partial u}{\partial t}$$

$$-\frac{\partial i}{\partial x} = e^{-\alpha t}\left\{Gf_1 + Gf_2 + C\left(-\alpha f_1 - \frac{\partial f_1}{\partial(x-vt)}v - \alpha f_2 + \frac{\partial f_2}{\partial(x+vt)}v\right)\right.$$

$$-\frac{\partial i}{\partial x} = e^{-\alpha t}\left\{(\underbrace{G-\alpha C}_{=0})(f_1+f_2) - vC\left[\frac{\partial f_1}{\partial(x-vt)} - \frac{\partial f_2}{\partial(x+vt)}\right]\right\}$$

$$G - \alpha C = G - \frac{G}{C}C = 0 \qquad Cv = C\frac{1}{\sqrt{LC}} = \sqrt{\frac{C}{L}}$$

$$\frac{\partial i}{\partial x} = e^{-\alpha t}\sqrt{\frac{C}{L}}\left(\frac{\partial f_1}{\partial(x-vt)} - \frac{\partial f_2}{\partial(x+vt)}\right)$$

$$i_p = e^{-\alpha t}\sqrt{\frac{C}{L}}\int\left[\frac{\partial f_1}{\partial(x-vt)} - \frac{\partial f_2}{\partial(x+vt)}\right]dx + D$$

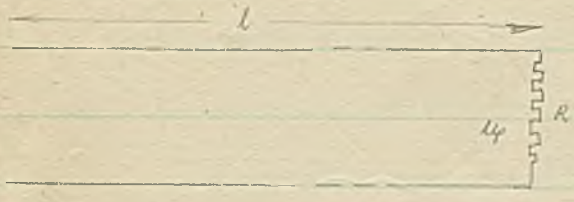
$t = \infty \ ; \quad D = 0$

$\sqrt{\frac{C}{L}} = \frac{1}{\sqrt{\frac{L}{C}}} = \frac{1}{Z}$ oporność falowa linii nieodkształcającej.

$$\underline{i_p = \frac{1}{Z}e^{-\alpha t}[f_1(x-vt) - f_2(x+vt)]}$$

$$\underline{u_p = e^{-\alpha t}[f_1(x-vt) + f_2(x+vt)]}$$

Ogólne ~~rozwiązania~~ dla napięcia i prądu, są to jednakowe fale.



Linia długa zamknięta przez oporność bezindukcyjną.

$(u_p)_{x=l} = e^{-\alpha t}[f_1(l-vt) + f_2(l+vt)]$ to jest u na końcu

$(i_p)_{x=l} = \frac{e^{-\alpha t}}{Z}[f_1(l-vt) - f_2(t+vt)]$ i na końcu

W każdej chwili musi być $i_p = \frac{u_p}{R}$

$$(u_p)_{x=l} = (i_p)_{x=l}R = e^{-\alpha t}\cdot\frac{R}{Z}(f_1 - f_2) = e^{-\alpha t}(f_1 + f_2)$$

$$\frac{R}{Z}(f_{gr} - f_{odb}) = f_{gr} + f_{odb}$$

$$\frac{R-Z}{Z}f_g = \frac{R+Z}{Z}f_{odb}$$

$$f_{odb} = \frac{R-Z}{R+Z}f_{gr}$$

$\frac{R-Z}{R+Z} = \delta$ · spółczynnik odbicia napięcia