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FRACTAL CHARACTERISTIC OF HARDNESS IN ALLOY WITH A STRUCTURAL GRADIENT

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SUMMARY

Fractal analysis of structural gradient taking into account the extent of its occurrence is presented in the paper. The power relations obtained allow to provide a fractal description of the changes of hardness in alloy with a structural gradient of two matrix components.

1. INTRODUCTION

Properties of cast iron in casts are determined by a structure the type of which, being formed in solidification and self-cooling processes, is a composite function of many factors. In order to ensure specified functional properties, casts are often produced characterised by a specified size and range of the structural gradient. An example are cast iron rolls (of rolling mills) whose measure of quality is a specified hardness profile along the cross-section.

The purpose of the paper is to present a fractal characteristic of the size and range of a structural gradient conditioning the course of hardness changes of a permanent mould cast of a roll.

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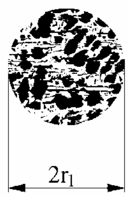
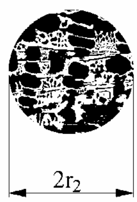
2. FRACTAL DEPICTION OF DIFFERENT ASPECTS OF STRUCTURAL GRADIENT IN A DESCRIPTION OF HARDNESS CHANGES OF A PERMANENT MOULD CAST OF A ROLL

A change of the intensity of solidification of a permanent mould cast brings about formation of a structure with a complex morphology which changes along the roll's radius. Microscopic observations confirm the occurrence of a changeable geometric form and a gradient fraction of graphite, pearlite and cementite (fig. 1), which conditions hardness distribution corresponding to this gradient. Since the hardness of an alloy is a function of the hardness of a matrix [1][2], pearlite and cementite are subjected to a fractal analysis. Results of scaling of the volume fraction of the above-mentioned matrix components are discussed in detail in paper [2].

The solution obtained in that paper depicted the hardness along the cross section of a roll as a function of the volume fraction of matrix components and fractal dimensions characterizing morphology.

The present paper expands the solutions obtained so far and covers with a fractal analysis not only the size of the structural gradient occurring but also the range of its occurrence along the radius of a roll.

Distance from the roll's surface R [mm]			
0	5	10	15
*Size of radius **r** of indentation in hardness testing*			
			

Distance from the roll's surface R [mm]			
20	25	30	162,5
*Size of radius **r** of indentation in hardness testing*			

[1] *Cz.Podrzucki: Żeliwo Wyd.ZG STOP, Kraków 1991;*
[2] *J.Chmiela, J.Cybo, D.Słota: Fraktalny opis zmian twardości żeliwa wykazującego na przekroju odlewu gradient udziału składników, Solidification of Metals and Alloys. No.36, 1998, 63-69*

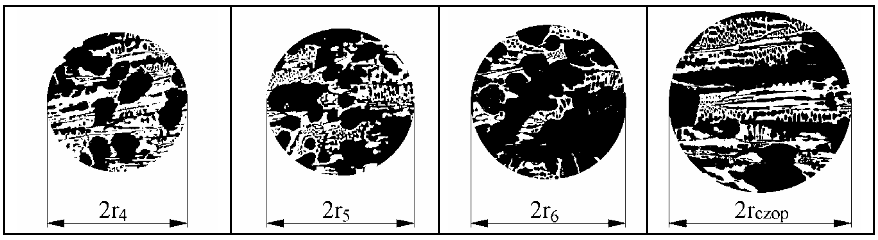


Fig. 1. Morphology of structure components along the radius of a roll's cross-section
Rys1. Morfologia składników struktury wzdłuż promienia przekroju poprzecznego walca

2.1. Fractal description of hardness changes for a structural gradient occurring throughout the section

In case when a permanent mould cast of a roll is characterized by the occurrence of a structural gradient of both matrix components along the whole cross-section, the volume fraction scaling is applied in a two-stage way. This means adequate relations not only in the variability range of the "r" radius of hardness indentation but also within the whole range of the "R" radius of the roll. Results of scaling of the position of volume fraction of matrix components are shown in fig. 2.

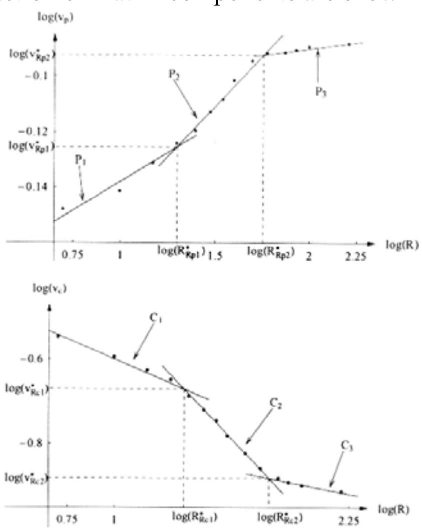


Fig. 2 Fractal diagrams of scaling of the position of volume fraction (of pearlite υ_p and cementite υ_c; lines "P_1", "P_2" and "C_1", "C_2" – roll face; lines "P_3" and "C_3" - roll neck)

Rys.2. Wykresy fraktalne skalowania położenia udziału objętościowego (perlitu υ_p i cementytu υ_c; proste „P_1", „P_2" i „C_1", „C_2" – beczka walca; proste „P_3" i „C_3" - czop walca)

By transforming the logarithmic relations presented in a graphic form in fig. 2, power relations 1 and 2 are obtained which describe the size of fraction volume of pearlite and cementite respectively, as a function of the position along the roll's section radius "R":

$$\upsilon_p(R) = \upsilon^*_{Rpq} \cdot \left(\frac{R}{R^*_{Rpq}} \right)^{D_k} , \tag{1}$$

where $k = 5$, $q = 1$ for $R \in (0, R^*_{Rp1}]$, $k = 6$, $q = 1$ for $R \in (R^*_{Rp1}, R_{Rp2}]$ or $k = 7$, $q = 2$ for $R \in (R^*_{Rp2}, R_{\max})$ and $(R^*_{Rp1}, \upsilon^*_{Rp1})$ and $(R^*_{Rp2}, \upsilon^*_{Rp2})$ $(R^*_{Rc2}, \upsilon^*_{Rc2})$ are points of the scaling law change (intersections of lines P_1 and P_2 as well as P_2 and P_3). Whereas D_5, D_6 and D_7 denote the slopes of the lines P_1, P_2 and P_3.

$$\upsilon_c(R) = \upsilon^*_{Rcs} \cdot \left(\frac{R}{R^*_{Rcs}} \right)^{-D_l} , \tag{2}$$

where $l = 8$, $s = 1$ for $R \in (0, R^*_{Rc1}]$, $l = 9$, $s = 1$ for $R \in (R^*_{Rc1}, R_{Rc2}]$ or $l = 10$, $s = 2$ for $R \in (R^*_{Rc2}, R_{\max})$ and $(R^*_{Rc1}, \upsilon^*_{Rc1})$ and $(R^*_{Rc2}, \upsilon^*_{Rc2})$ are points of the scaling law change (intersections of lines C_1 and C_2 as well as C_2 and C_3). Whereas D_8, D_9 i D_{10} denote the slopes of the lines C_1, C_2 and C_3. The slopes of the straight lines shown in the fractal diagram are in a known way connected with the fractal dimensions determined from scaling laws. Knowing equations (1) and (2) as well as from hardness measurements of the indentations sizes which are directly connected with the volume fraction amounts υ_p(R) i υ_c(R) adequate to their positions, relation (3) is obtained:

$$r(\upsilon_p(R), \upsilon_c(R)) = \sqrt{r^*_p r^*_c} \left(\frac{\upsilon^*_{Rpq}}{\upsilon^*_p} \right)^{\frac{D_i}{2}} \left(\frac{\upsilon^*_{Rcs}}{\upsilon^*_c} \right)^{-\frac{D_j}{2}} \frac{\sqrt{R^{(D_k D_i + D_l D_j)}}}{\sqrt{\left(R^*_{Rpq}\right)^{D_k D_i} \left(R^*_{Rcs}\right)^{D_l D_j}}} \tag{3}$$

where: $i = 1$ for $R \in (0, R^*_p]$, $i = 2$ for $R \in (R^*_p, R_{\max})$, $j = 3$ for $R \in (0, R^*_c]$, $j = 4$ for $R \in (R^*_c, R_{\max})$, $k = 5$, $q = 1$ for $R \in (0, R^*_{Rp1}]$, $k = 6$, $q = 1$ for $R \in (R^*_{Rp1}, R_{Rp2}]$ or $k = 7$, $q = 2$ for $R \in (R^*_{Rp2}, R_{\max})$, $l = 8$, $s = 1$ for $R \in (0, R^*_{Rc1}]$, $l = 9$, $s = 1$ for

$R \in (R^*_{Rc1}, R_{Rc2}]$ or $l = 10$, $s = 2$ for $R \in (R^*_{Rc2}, R_{\max})$. Parameters R^*_p and R^*_c are selected so that $\upsilon_p(R^*_p) = \upsilon^*_p$ and $\upsilon_c(R^*_c) = \upsilon^*_c$.

The above equations present the relation between the size of the hardness indentation radius and the fractal characteristic of the structural gradient size as a function of position. Brinell hardness is described by the expression (4):

$$HB = \frac{2P}{\pi D\left(D - \sqrt{D^2 - 4r^2}\right)}, \tag{4}$$

where:

P – loading force,
D – globule diameter,
r – indentation radius

Substituting the formula (4) where the denotations: HB=HB(υ_p(R), υ_c(R)) have been applied, for the relation (3), finally the expression (5) is obtained:

$$HB = \frac{2P}{\pi D\left(D - \sqrt{D^2 - 4r^*_p r^*_c \left(\frac{\upsilon^*_{Rpq}}{\upsilon^*_p}\right)^{D_i} \left(\frac{\upsilon^*_{Rcs}}{\upsilon^*_c}\right)^{-D_j} \frac{R^{\left(D_k D_i + D_l D_j\right)}}{\left(R^*_{Rpq}\right)^{D_k D_i} \left(R^*_{Rcs}\right)^{D_l D_j}}}\right)}, \tag{5}$$

where: $i = 1$ for $R \in (0, R^*_p]$, $i = 2$ for $R \in (R^*_p, R_{\max})$, $j = 3$ for $R \in (0, R^*_c]$, $j = 4$ for $R \in (R^*_c, \infty)$, $k = 5$, $q = 1$ for $R \in (0, R^*_{Rp1}]$, $k = 6$, $q = 1$ for $R \in (R^*_{Rp1}, R_{Rp2}]$ or $k = 7$, $q = 2$ for $R \in (R^*_{Rp2}, R_{\max})$, $l = 8$, $s = 1$ for $R \in (0, R^*_{Rc1}]$, $l = 9$, $s = 1$ for $R \in (R^*_{Rc1}, R_{Rc2}]$ or $l = 10$, $s = 2$ for $R \in (R^*_{Rc2}, R_{\max})$. Parameters R^*_p and R^*_c are selected so that $\upsilon_p(R^*_p) = \upsilon^*_p$ and $\upsilon_c(R^*_c) = \upsilon^*_c$.

The relation (5) is a basis for a fractal description of hardness changes in a permanent mould cast of a roll as a function of matrix components of the structural gradient which occurs along the whole section.

Consistency of the results obtained was subjected to confrontation with a functional description of hardness changes (6):

$$HB(R) = \frac{U}{1 + \exp\left[Z(R - W)\right]} + HB_{neck} \tag{6}$$

where: U,W,Z – parameters determined by approximation ;
HB_{neck} – roll neck hardness; R – roll's radius

The results of the calculations obtained from the fractal and functional descriptions are presented in fig. 3.

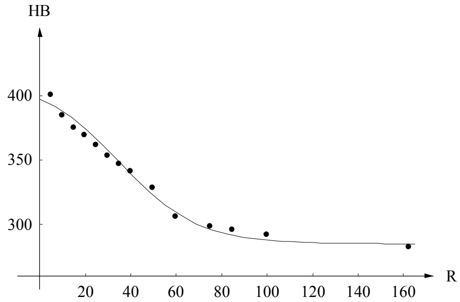


Fig. 3 Profile of hardness changes (• - fractal description, – functional description)
Rys.3 Profil zmian twardości (• - opis fraktalny, – opis funkcyjny)

3. DISCUSSION OF RESULTS

A gradient structure of cast iron in a permanent mould cast of a roll is characterized by the occurrence of opposite gradients of fraction of the components which are distinguished by complex morphology along the roll's radius. In order to make a quantitative description of a structure of such a degree of complexity, a fractal analysis based on simple scaling laws was applied. The existence of power relations between the amount of the volume fraction of matrix components and the place of occurrence along the cross-section "R" and the indentation radius "r" obtained from hardness measurement by Brinell's method, was proved. The fractal characteristic of the gradient structure obtained in this way, taking into account the extent of oppositely directed gradients of components fractions, became a basis for determining the relation between the structure and properties (hardness).
The fractal characteristic of hardness of an alloy derived in this paper takes into account the structural gradient of both components throughout the section. Other cases of structural gradient and its generalization will be presented in the next paper.

FRAKTALNA CHARAKTERYSTYKA TWARDOŚCI STOPU Z GRADIENTEM STRUKTURALNYM

STRESZCZENIE

W pracy przedstawiono fraktalną charakterystykę przeciwnie skierowanych gradientów strukturalnych składników osnowy. Wyprowadzone zależności uwzględniają zmianę prawa skalowania udziału objętościowego składników wzdłuż promienia przekroju poprzecznego, które w powiązaniu z relacjami uzyskanymi z praw skalowania w obszarze promienia odcisku twardości, pozwoliły na uzyskanie fraktalnej charakterystyki twardości stopu.

Reviewed by prof. Józef Gawroński