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RECONSTRUCTION OF BOUNDARY CONDITION ON THE INTERNAL SURFACE OF CONTINUOUS CASTING MOULD

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SUMMARY

In the paper the method of identification of heat flux on the contact surface between the cast slab and continuous casting mould is presented. The algorithm is constructed using the boundary element method supplemented by the least squares criterion. In the final part the example of computations is shown.

1. INTRODUCTION

The identification of boundary conditions on the contact surface between continuous casting and mould belongs to the group of the inverse problems. In such case the equation describing the course of thermal processes in domain considered and also the part of boundary conditions is given, while the remaining conditions are not determined. In order to solve this type of the problem, the additional information concerning the course of the process (e.g. temperature values at the set of internal points from continuous casting mould region) must be known. The inverse problem considered in this paper is solved using the boundary element method. This method seems to be very effective in the case of the boundary values identification. The basic BEM algorithm is coupled with the numerical procedure resulting from the least squares criterion.

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2. GOVERNING EQUATIONS

The symmetrical fragment of continuous casting mould is considered – c.f. Figure 1. The steady state temperature field $T(x)$ in this 2D domain which thermal conductivity λ is a constant value describes the following energy equation

$$\boldsymbol{x} \in \Omega: \quad \frac{\partial^2 \boldsymbol{T}(\boldsymbol{x})}{\partial \boldsymbol{x}_1^2} + \frac{\partial^2 \boldsymbol{T}(\boldsymbol{x})}{\partial \boldsymbol{x}_2^2} = 0 \tag{1}$$

This equation is supplemented by the boundary conditions, namely

$$\begin{aligned} \boldsymbol{x} \in \Gamma_2: &\quad \boldsymbol{q}(\boldsymbol{x}) = 0 \\ \boldsymbol{x} \in \Gamma_3: &\quad \boldsymbol{q}(\boldsymbol{x}) = \alpha_0 [\boldsymbol{T}(\boldsymbol{x}) - \boldsymbol{T}_0] \\ \boldsymbol{x} \in \Gamma_4: &\quad \boldsymbol{q}(\boldsymbol{x}) = \alpha_w [\boldsymbol{T}(\boldsymbol{x}) - \boldsymbol{T}_w] \end{aligned} \tag{2}$$

while the value of heat flux between the cast slab and continuous casting mould is unknown (boundary Γ_1). In equations (2) $q(x) = -\lambda\, \partial T / \partial n$ denotes the heat flux at the boundary point x, $x=(x_1, x_2)$, α_0 [W/m^{2}K] is the heat transfer coefficient between CCM and environment, T_0 is ambient temperature, α_w is the heat transfer coefficient between CCM and cooling water, T_w is the mean temperature of cooling water.

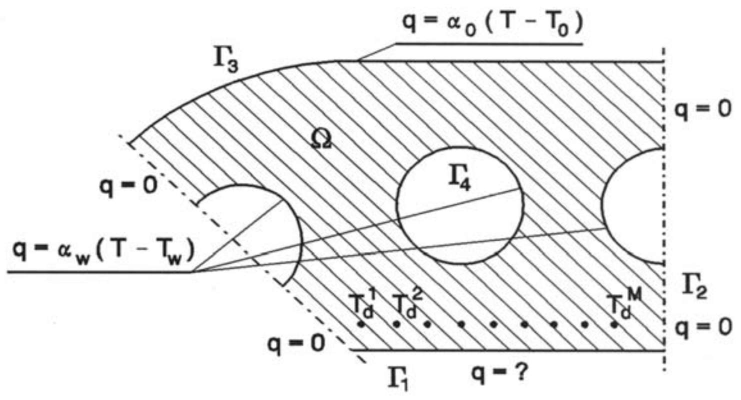

Fig. 1. Domain considered
Rys. 1. Rozważany obszar

Additionaly, the temperature values T_d^1, T_d^2, ..., T_d^M at the set of internal points are known (e.g. on the basis of measurements).

3. METHOD OF SOLUTION

The boundary integral equation for the problem considered is the form [1]

$$\boldsymbol{B}(\xi)\boldsymbol{T}(\xi)+\int_{\Gamma}\boldsymbol{T}^{*}(\xi,\ \boldsymbol{x})\boldsymbol{q}(\boldsymbol{x})\ \mathrm{d}\Gamma=\int_{\Gamma}\boldsymbol{q}^{*}(\xi,\ \boldsymbol{x})\boldsymbol{T}(\boldsymbol{x})\ \mathrm{d}\Gamma \tag{3}$$

where $\xi \in \Gamma$ is the observation point, $B(\xi)\in (0, 1)$, $T^{*}(\xi, x)$ is the fundamental solution and $q^{*}(\xi, x) = -\lambda\partial T^{*}(\xi, x) / \partial n$.

In numerical realization, the boundary Γ is divided into N constant boundary elements Γ_j. Additionally we assume that N_1 nodes belong to the boundary Γ_1, the nodes $N_1+1, ..., N_2$ belong to Γ_2, the nodes $N_2+1, ..., N_3$ belong to Γ_3, while nodes $N_3+1, ..., N$ – to the boundary Γ_4. The integrals in equation (3) are substituted by sum of integrals, namely

$$\xi_i \in \Gamma: \quad \frac{1}{2}\boldsymbol{T}(\xi_i)+\sum_{j=1}^{N}\boldsymbol{G}_{ij}\,\boldsymbol{q}(\boldsymbol{x}_j)=\sum_{j=1}^{N}\hat{\boldsymbol{H}}_{ij}\,\boldsymbol{T}(\boldsymbol{x}_j) \tag{4}$$

where

$$\boldsymbol{G}_{ij}=\int_{\Gamma_j}\boldsymbol{T}^{*}(\xi_i,\ \boldsymbol{x})\,\mathrm{d}\Gamma_{\mathrm{j}}, \qquad \hat{\boldsymbol{H}}_{ij}=\int_{\Gamma_j}\boldsymbol{q}^{*}(\xi_i,\ \boldsymbol{x})\,\mathrm{d}\Gamma_{\mathrm{j}} \tag{5}$$

It should be pointed out that the value of coefficient $B(\xi_i) = 1/2$ and the form of equation (4) result from the assumption that along the boundary element Γ_j the temperature and heat flux are constant (so-called constant boundary elements are considered). Equation (4) can be written as follows

$$\xi_i \in \Gamma: \quad \sum_{j=1}^{N}\boldsymbol{G}_{ij}\,\boldsymbol{q}_j=\sum_{j=1}^{N}\hat{\boldsymbol{H}}_{ij}\,\boldsymbol{T}_j \tag{6}$$

where $H_{ij}=\hat{H}_{ij}$ for $i \neq j$, while $H_{ij} = -1/2$ for $i = j$.
Taking into account the boundary conditions (2), the equation (6) takes a form

$$\sum_{j=1}^{N_2}\boldsymbol{G}_{ij}\,\boldsymbol{q}_j+\sum_{j=N_2+1}^{N_3}\alpha_0\,\boldsymbol{G}_{ij}\left(\boldsymbol{T}_j-\boldsymbol{T}_0\right)+\sum_{j=N_3+1}^{N}\alpha_w\,\boldsymbol{G}_{ij}\left(\boldsymbol{T}_j-\boldsymbol{T}_w\right)=\sum_{j=1}^{N}\boldsymbol{H}_{ij}\,\boldsymbol{T}_j \tag{7}$$

or

$$-\sum_{j=1}^{N_2}\boldsymbol{H}_{ij}\,\boldsymbol{T}_j+\sum_{j=N_2+1}^{N}\left(\alpha\boldsymbol{G}_{ij}-\boldsymbol{H}_{ij}\right)\boldsymbol{T}_j=-\sum_{j=1}^{N}\boldsymbol{G}_{ij}\,\boldsymbol{q}_j+\sum_{j=N_2+1}^{N}\alpha\,\boldsymbol{G}_{ij}\,\boldsymbol{T}^{\infty} \tag{8}$$

where $\alpha = \alpha_0$, $T^{\infty} = T_0$ for $j=N_2+1, ..., N_3$ and $\alpha = \alpha_w$, $T^{\infty} = T_w$ for $j=N_3+1, ..., N$.
The formulae (9) create the linear system of equations

$$\mathbf{B}_1 \cdot \mathbf{T} = \mathbf{B}_2 \cdot \mathbf{P} \tag{9}$$

where

$$\mathbf{B}_1 = \begin{bmatrix} -\boldsymbol{H}_{11} & ... & -\boldsymbol{H}_{1N_2} & \left(\alpha \boldsymbol{G}_{1N_2+1} - \boldsymbol{H}_{1N_2+1}\right) & ... & \left(\alpha \boldsymbol{G}_{1N} - \boldsymbol{H}_{1N}\right) \\ ... & ... & ... & ... & ... & ... \\ -\boldsymbol{H}_{N1} & ... & -\boldsymbol{H}_{NN_2} & \left(\alpha \boldsymbol{G}_{NN_2+1} - \boldsymbol{H}_{NN_2+1}\right) & ... & \left(\alpha \boldsymbol{G}_{NN} - \boldsymbol{H}_{NN}\right) \end{bmatrix} \tag{10}$$

and

$$\mathbf{B}_2 = \begin{bmatrix} -\boldsymbol{G}_{11} & ... & -\boldsymbol{G}_{1N_2} & \alpha \boldsymbol{G}_{1N_2+1} & ... & \alpha \boldsymbol{G}_{1N} \\ ... & ... & ... & ... & ... & ... \\ -\boldsymbol{G}_{N1} & ... & -\boldsymbol{G}_{NN_2} & \alpha \boldsymbol{G}_{NN_2+1} & ... & \alpha \boldsymbol{G}_{NN} \end{bmatrix} \tag{11}$$

at the same time

$$\mathrm{P} = \begin{bmatrix} \boldsymbol{q}_1 & ... & \boldsymbol{q}_{N_1} & 0 & ... & 0 & \boldsymbol{T}^{\infty} & ... & \boldsymbol{T}^{\infty} \end{bmatrix}^{\mathrm{T}} \tag{12}$$

It should be pointed out that the vector **P** contains the unknown heat fluxes $q_1, ..., q_{N1}$.
From equation (9) results that

$$\mathbf{T} = \mathbf{B}_1^{-1} \cdot \mathbf{B}_2 \cdot \mathbf{P} = \mathbf{U} \cdot \mathbf{P} \tag{13}$$

this means

$$\boldsymbol{T}_j = \sum_{k=1}^{N_1} \boldsymbol{U}_{jk}\, \boldsymbol{q}_k + \sum_{k=N_1+1}^{N} \boldsymbol{U}_{jk}\, \boldsymbol{P}_k\, , \qquad \boldsymbol{j} = 1,\ 2,\ ...,\ \boldsymbol{N} \tag{14}$$

and

$$\boldsymbol{q}_j = \alpha \left(\boldsymbol{T}_j - \boldsymbol{T}^{\infty}\right) = \alpha \left(\boldsymbol{T}_j - \boldsymbol{P}_j\right), \quad \boldsymbol{j} = \boldsymbol{N}_2 + 1,\ ...,\ \boldsymbol{N} \tag{15}$$

The temperatures at internal nodes $\xi_1 \in \Omega$, $i = 1, 2, ..., M$. are calculated on the basis of formula [1]

$$\boldsymbol{T}\left(\xi_i\right) = \sum_{j=1}^{N} \boldsymbol{H}_{ij}^{w}\, \boldsymbol{T}_j - \sum_{j=1}^{N} \boldsymbol{G}_{ij}^{w}\, \boldsymbol{q}_j \tag{16}$$

Putting the dependencies (14) and (15) into equation (16) finally one obtains

$$\boldsymbol{T}\left(\xi_i\right)=\sum_{j=1}^{N_1} \boldsymbol{W}_{ij}\, \boldsymbol{q}_j+\boldsymbol{Z}_i \tag{17}$$

where

$$\boldsymbol{W}_{ij} =-\boldsymbol{G}_{ij}^{w} + \boldsymbol{D}_{ij}^{w} \tag{18}$$

$$\boldsymbol{D}_{ij}^{w}=\sum_{k=1}^{N_2} \boldsymbol{H}_{ik}^{w}\, \boldsymbol{U}_{kj}+\sum_{k=N_2+1}^{N}\left(\boldsymbol{H}_{ik}^{w}-\alpha\, \boldsymbol{G}_{ik}^{w}\right)\boldsymbol{U}_{kj} \tag{19}$$

at the same time

$$\boldsymbol{Z}_i=\sum_{j=N_2+1}^{N} \alpha\, \boldsymbol{G}_{ij}^{w}\, \boldsymbol{P}_j+\sum_{j=N_1+1}^{N} \boldsymbol{D}_{ij}^{w}\, \boldsymbol{P}_j \tag{20}$$

The least squares criterion leads to the formula (c.f. equation (17))

$$\sum_{i=1}^{M}\left[\boldsymbol{T}\left(\xi_i\right)-\boldsymbol{T}_d^{i}\right]^2=\sum_{i=1}^{M}\left[\sum_{j=1}^{N_1} \boldsymbol{W}_{ij}\, \boldsymbol{q}_j+\boldsymbol{Z}_i-\boldsymbol{T}_d^{i}\right]^2 \rightarrow \mathrm{MIN} \tag{21}$$

where $T(\xi_i)$ are the calculated temperatures at control points shown in Figure 1, while T_d^i are the measured temperatures at the same set of points [1].

On the basis of the necessary condition of several variables function minimum one can find the unknown boundary heat fluxes on the contact surface between cast slab and continuous casting mould.

4. RESULTS OF COMPUTATIONS

In numerical computations the following input data have been accepted: thickness of CCM: 0.05[m], diameter of cooling pipes: 0.02[m], thermal conductivity λ=330[W/mK], heat transfer coefficients α_0=15 [W/m^{2}K], α_w=3675 [W/m^{2}K], ambient temperature T_0=30^{0}C, temperature of cooling water T_w=50^{0}C. At internal points (c.f. Figure 1) one assumes the temperatures

1 – 199, 2 – 207, 3 – 214, 4 – 219, 5 – 223,
6 – 226, 7 – 228, 8 – 230, 9 – 231, 10 – 232,
11–186, 12 – 200, 13 – 209, 14 – 214, 15 – 217

The reconstructed heat fluxes for successive boundary elements are equal

$$q_1 = -932082, \quad q_2 = -1015816, \quad q_3 = -1164980, \quad q_4 = -863793,$$
$$q_5 = -926146, \quad q_6 = -1225011, \quad q_7 = -728346, \quad q_8 = -1383596,$$
$$q_9 = -651530, \quad q_{10} = -1104035,$$

So, the mean value of heat flux on the contact surface between cast slab and CCM is equal q_m= - 999533 [W/m^2]. The results of the inverse problem solution are shown in Figure 2.

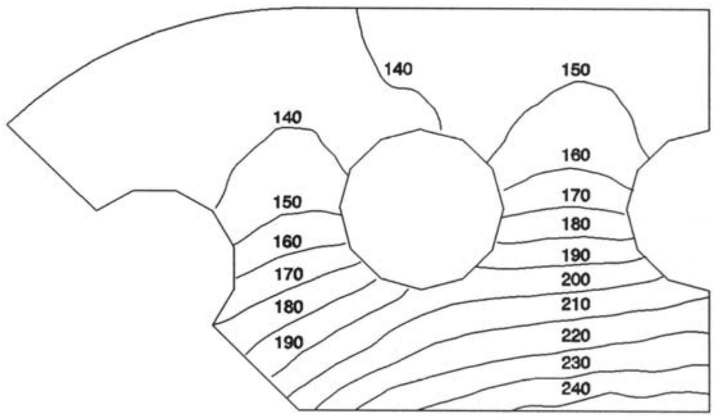

Fig.2. The solution of inverse problem
Rys. 2. Rozwiązanie zadania odwrotnego

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ODTWORZENIE WARUNKU BRZEGOWEGO MIĘDZY WLEWKIEM A KRYSTALIZATOREM URZĄDZENIA COS

STRESZCZENIE

W pracy przedstawiono metodę identyfikacji strumienia ciepła wymienianego między wlewkiem ciągłym a krystalizatorem urządzenia COS. Algorytm zbudowano wykorzystując metodę elementów brzegowych oraz kryterium najmniejszych kwadratów.

Reviewed by prof. Stanisław Jura