

## RESEARCH OF SENSITIVITY TEMPERATURE FIELD IN THE COOLING CASTING

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### SUMMARY

In the paper a problem of sensitivity analysis of the casting mould is presented. The parameter in relation to which the sensitivity is analysed is the thermal diffusivity of the mould material. In the final part of the paper the example of computations is shown.

### 1. MATHEMATICAL DESCRIPTION OF THE PROBLEM

When designing a mould for a casting considered a question might arise: How will a change of thermophysical parameters of the mould material effect temperature field in the casting and in the mould itself? Where will the effect be the strongest and what will its character be? Sensitivity analysis concerning this type of problems can answer these questions.

In the presented work we will consider the effect of the parameter  $a = \lambda/(c\rho)$  of the mould sand upon the temperature field such an approach is induced by the fact that while changing composition of the sand, at the same time we change its density and the thermal capacity as well as the thermal conductivity. We will consider a certain domain  $D$  which is a sum of sub-areas  $D_1$  and  $D_2$ :

$$D = D_1 + D_2$$

In order to simplify the consideration let us assume, that the area  $D$  is 2D one. Let us assume that the area  $D_1$  corresponds to the casting, while  $D_2$  is filled with the moulding sand.

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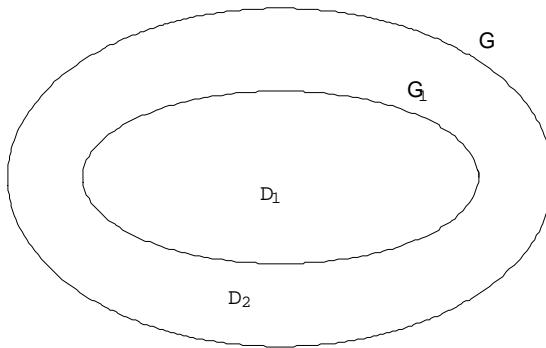


Fig. 1. Domain considered.  
Rys. 1. Rozważany obszar.

The temperature field in the domain  $D$  is described by equations:

$$\frac{\partial T_i}{\partial t} = a_i \left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right), \quad i = 1, 2$$

At the moment  $t = t_0$  the temperature is the known function  $T_{0i}$ :

$$T_i(P, t_0) = T_{0i}, \quad P \in D, \quad i = 1, 2.$$

Furthermore, boundary conditions are given:

- on the boundary  $\Gamma_l$

$$\begin{aligned} -\lambda_1 \frac{\partial T_1}{\partial \bar{n}} &= -\lambda_2 \frac{\partial T_2}{\partial \bar{n}}, \\ T_1|_{\Gamma_l} &= T_2|_{\Gamma_l} \end{aligned}$$

whereas on the boundary  $\Gamma$  I, II or III type of the conditions are given.

$$\begin{aligned} (\alpha) \quad & T(\Gamma) = \varphi(\Gamma), \\ (\beta) \quad & -\lambda_2 \frac{\partial T_2}{\partial \bar{n}} = \psi(\Gamma), \\ (\gamma) \quad & -\lambda_2 \frac{\partial T_2}{\partial \bar{n}} = \alpha(T_2 - T_\infty). \end{aligned}$$

In the above equations we denote respectively:

$\lambda_i$  – thermal conductivity,  $c_i$  – thermal capacity,  $\rho_i$  – mass density,  $\alpha$  – heat transfer coefficient,  $\varphi, \psi$  – known functions.

To estimate the effect of parameter  $a$  upon so determined temperature field we must obtain equation defining derivative of the function  $T_i$  in relation to the parameter  $a$ :

$$T^a = \frac{\partial T}{\partial a}$$

in the considered domain  $D$ . We will obtain the equations defining the requested field  $T^a$  by differentiation with respect to  $a$  the mathematical description of temperature field. As the result we will obtain the following dependencies allowing to calculate  $T^a$ :

$$\begin{aligned} \frac{\partial T_1^a}{\partial t} &= a_1 \left( \frac{\partial^2 T_1^a}{\partial x^2} + \frac{\partial^2 T_1^a}{\partial y^2} \right), \\ \frac{\partial T_2^a}{\partial t} &= \frac{\partial^2 T_2^a}{\partial x^2} + \frac{\partial^2 T_2^a}{\partial y^2} + a_2 \left( \frac{\partial^2 T_2^a}{\partial x^2} + \frac{\partial^2 T_2^a}{\partial y^2} \right), \\ T_i(P, t_0) &= 0, \quad P \in D, \quad i = 1, 2. \end{aligned}$$

On the boundary  $\Gamma_l$  function  $T_i^a$  must comply with the requirement:

$$\begin{aligned} T_1^a \Big|_{\Gamma_l} &= T_2^a \Big|_{\Gamma_l}, \\ -\lambda_1 \frac{\partial T_1^a}{\partial \bar{n}} &= -c_2 \rho_2 \frac{\partial T_2}{\partial \bar{n}} - \lambda_2 \frac{\partial T_2^a}{\partial \bar{n}}. \end{aligned}$$

Moreover, the field  $T_2^a$  must comply with boundary conditions adequate for these given for the temperature field:

$$(\alpha) \quad T^a(\Gamma) = 0,$$

$$(\beta) \quad -c_2\rho_2 \frac{\partial T_2}{\partial \bar{n}} - \lambda_2 \frac{\partial T_2^a}{\partial \bar{n}} = \frac{\partial \psi}{\partial a},$$

$$(\gamma) \quad -c_2\rho_2 \frac{\partial T_2}{\partial \bar{n}} - \lambda_2 \frac{\partial T_2^a}{\partial \bar{n}} = \alpha(T_2^a - T_\infty)$$

(assuming that  $\alpha$  does not depend on the thermophysical parameters of mould sand). As it is evident from the above description, to calculate sensitivity field, the knowledge of temperature field is necessary, not only the knowledge of conditions determining the temperature field. In that case for every moment of time for which we calculate the distribution of sensitivity we must calculate the distribution of temperature in the area  $D$  beforehand.

## 2. EXAMPLE OF COMPUTATIONS

To illustrate the described algorithm we will calculate the field of thermal sensitivity for the design parameter  $a$  in the casting of square cross-section. The casting is produced from the steel and enclosed with moulding sand. Considering thermal symmetry of the object, the computations have been performed for one quadrant only.

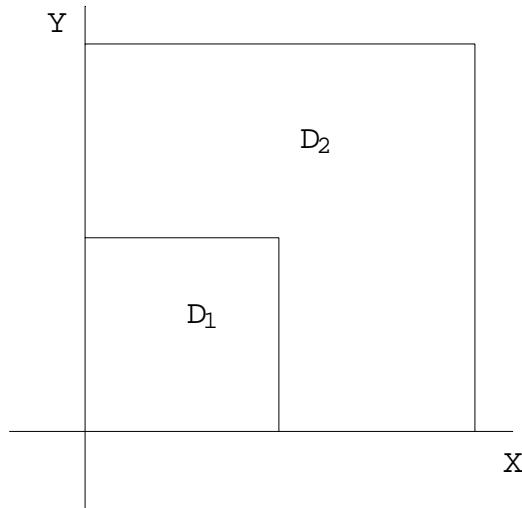


Fig. 2. The casting considered.

Rys. 2. Modelowany odlew.

Below, the results of computations in the system cast – mould for chosen moment of time are presented.

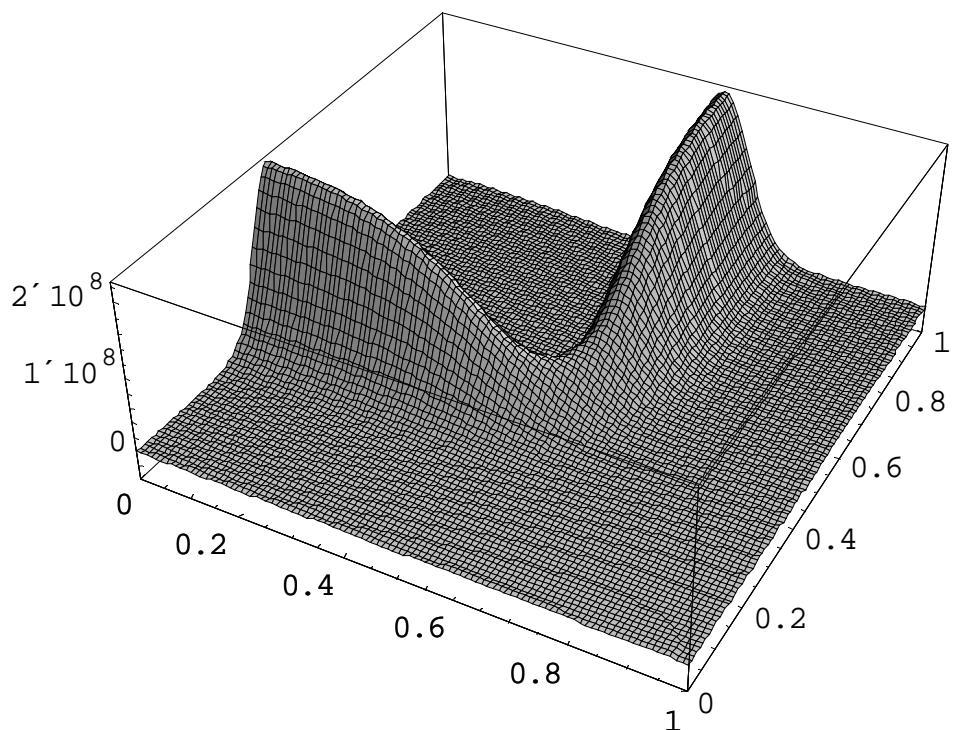


Fig. 3. Sensitivity field calculated for the time  $t_0 + 10$  s.  
Rys. 3. Pole wrażliwości wyznaczone dla chwili  $t_0 + 10$  s.

Obtained distribution of sensitivity field in cross-section allows to estimate the effect of changes of parameter  $a$  upon the distribution of temperature in the volume of the casting and mould.

## REFERENCES

- [1] E. J. Haugh, J. Cea, *Optimization of distributed parameters structures*, Sijhoff & Noordhoff, 2, 1981
- [2] K. Dems, *Sensitivity analysis in thermal problems (I)*, J. Therm. Stresses, 9, 1986
- [3] M. Kleiber (red.), *Komputerowe metody mechaniki ciał stałych*, PWN, Warszawa 1995
- [4] J. Legras, *Praktyczne metody analizy numerycznej*, WNT, Warszawa 1974
- [5] J. Stoer, R. Bulirsch, *Wstęp do metod numerycznych*, t. II, PWN, Warszawa 1980

## BADANIE WRAŻLIWOŚCI POLA TEMPERATURY STYGNĄCEGO ODLEWU

### STRESZCZENIE

W przedstawianej pracy podjęty został problem wyznaczania wrażliwości pola temperatury odlewu stygnącego w formie odlewniczej. Wybranym parametrem projektowym formy (tj. parametrem na którego zmiany badana jest wrażliwość) został współczynnik dyfuzyjności cieplnej  $a$  materiału formierskiego. Wyprowadzone zależności posłużyły do przykładowego wyznaczenia pola wrażliwości w poprzecznym przekroju odlewu.

Reviewed by prof. Zbigniew Górný