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THE EQUATION OF TEMPERATURE FIELD FOR THE BEGINNING PHASE OF THE CONTINUOUS INGOT FORMING AND ITS APPLYING IN PRACTICE

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The work concerns one of the fundamental problems associated with the process of the continuous casting, which are the heat occurences taking place in the beginning phase of the ingot forming. It is derived the equations, which allow to determine in what distance of surface of the liquid metal mirror the process of cuticle's growing begins. These equations includes all the essential parameters of the process and can be used in the analysis of the heat effects taking place in the ingot, in the initial phase of its forming.

Introduction

Among many factors determining the moment of the beginning of the cuticle forming process, the following parameters have the essential influence:

- pouring temperature,
- fullering velocity,
- kind of the processing material,
- ingot's dimensions,
- parameters determining the process of heat exchange between the ingot and surrounding its medium.

At the same time, it is appeared the question what is the dependence between these parameters and what is their influence over the process.

The derived in the paper equations includes all the essential parameters of the process and can be used in the analysis of the effects taking place in the ingot in the beginning phase of its forming, among other, these equations allow to determinate, in what distance of surface of the liquid metal mirror, the process of cuticle's growing begins (distance ξ on Fig. 1).

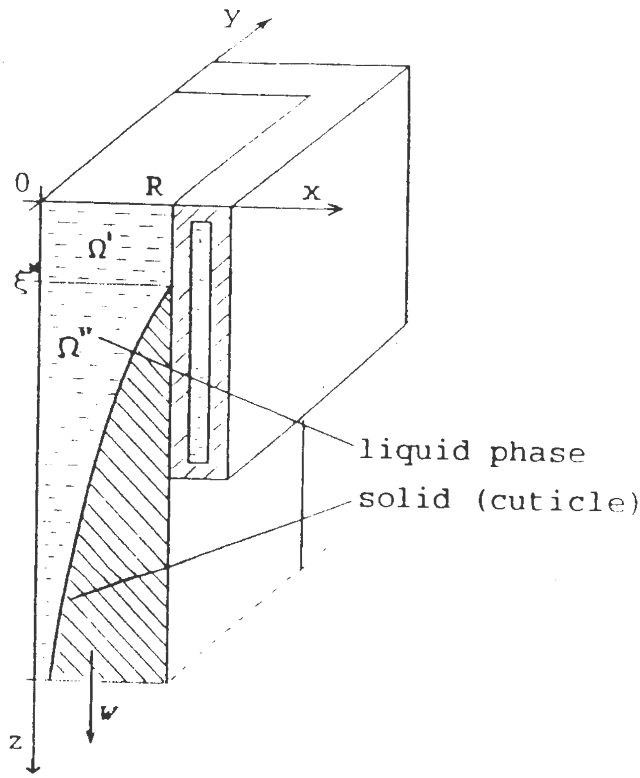

Fig. 1. The modelled area

The Mathematical Model

It is considered flat ingots of 2R thickness produced on the vertical continuous casting machine. With assumed geometry of the areas, the small conductivity in the direction of the ingot fullering and more distant side walls, the failure–free work of the machine causes the generation of the pseudo–stacionary temperature field in the ingot–crystallizer system, which, in the longitudinal section of the liquid phase, above the cuticle (area Ω' on Fig. 1), in immovable coordinate system, in the connection with solidified ingot, is described by the equation

$$w \frac{\partial T}{\partial z} = a_1 \frac{\partial^2 T}{\partial x^2} \qquad 0 < x < R \qquad 0 \le z < \xi^* \tag{1a}$$

instead in the area, in which the cuticle appears (area Ω") by the equations

$$w \frac{\partial T_i}{\partial z} = a_i \frac{\partial^2 T_i}{\partial x^2} \qquad 0 < x < R \qquad z > \xi^* \qquad i = 1, 2 \tag{1b}$$

where x, z are space coordinates, T_i (x, z) – temperature, respectively of liquid phase (i = 1) and solid phase (i = 2), w – fullering velocity and a_i – thermal diffusivity coefficients. The equations (1a) and (1b) are parabolic equations, in which the coordinate z serves as the time and the whole of the problem is completed by the boundary conditions:
– in the symmetry axis of the ingot

$$\frac{\partial T}{\partial x} = 0 \qquad x = 0 \tag{2}$$

– on the surface of the ingot, according to the possessed informations

$$-\lambda_i \frac{\partial T_i}{\partial x} = \alpha (T_i - T^\infty) \qquad x = R \tag{3a}$$

$$\lambda_i \frac{\partial T_i}{\partial x} = q \qquad x = R \tag{3b}$$

whereas i = 1 when $0 < z < \xi^*$ and i = 2 when $z \ge \xi$

– on the surface of phases division

$$T_1 = T_2 \qquad x = \varphi(z) \qquad z \ge \xi^* \tag{4a}$$

and the initial condition:

$$-\lambda_1 \frac{\partial T_1}{\partial x_1} + \lambda_2 \frac{\partial T_2}{\partial x} = \gamma_2\, \kappa \varphi'(z) \qquad x = \varphi(z) \qquad z \ge \xi^* \tag{4b}$$

$$T_1 = T^0 \qquad z = 0, \qquad 0 \le x \le R \tag{5}$$

where T^0 is the pouring temperature, λ – the thermal conductivity coefficient, α is the coefficient of heat penetration, q – is the heat stream, T^∞ – the temperature of the environment, and the function $x = \varphi(z)$, $z \ge \xi^*$ describes the position of the phases division's limit, where $\varphi(\xi^*) = B$.

The Solution of the Problem for the Area above the Cuticle

Let assume, that on the level $z = \xi < \xi^*$, the temperature field $T^1(x, z)$ in the ingot section can be described by

$$T_1(x, \xi) = \phi(x) + T^0 \qquad 0 < x < R \tag{6}$$

where $\phi(x)$ is unknown function of suitable class. After the difference approximation of the left side of the equation (1) and including the condition (5) we obtain

$$w\frac{T_1(x, \xi) - T^0}{\xi} = a_1 \ddot{T}_1(x, \xi) \qquad 0 < x < R \tag{7}$$

From the last equation after including the formula (6) and after the easy transformations we obtain

$$\ddot{\phi}(x) - w(a_1\xi)^{-1}\phi(x) = 0 \qquad 0 < x < R \tag{8}$$

The general integral of the equation (8) has the form

$$\phi(x) = A exp\left(\sqrt{\frac{w}{a_1\xi}}\,x\right) + B exp\left(-\sqrt{\frac{w}{a_1\xi}}\,x\right) \qquad 0 < x < R \tag{9}$$

It means, that the temperature field in the ingot section, on the level $z = \xi$ is described by the function

$$T_1(x, \xi) = A exp\left(\sqrt{\frac{w}{a_1\xi}}\,x\right) + B exp\left(-\sqrt{\frac{w}{a_1\xi}}\,x\right) + T^0 \qquad 0 < x < R \tag{10}$$

which after including of the condition (2) has the form

$$T_1(x, \xi) = D ch\left(\sqrt{\frac{w}{a_1\xi}}\,x\right) + T^0 \qquad 0 < x < R \tag{11}$$

To calculate the constant D we use the conditions (3), which lead to equation

$$-\lambda_1 D\sqrt{\frac{w}{a_1\xi}}\, sh\left(\sqrt{\frac{w}{a_1\xi}}\,R\right) = \alpha\left(D\, ch\left(\sqrt{\frac{w}{a_1\xi}}\,R\right) + T^0 - T^\infty\right) \tag{12}$$

when the coefficient of heat penetration α, between the ingot and the crystallizer is known (condition (3a)), or to the equation

$$-\lambda_1 D'' \sqrt{\frac{w}{a_1 \xi}}\, sh\left(\sqrt{\frac{w}{a_1 \xi}} R\right) = q \tag{13}$$

when this heat exchange is characterized by the stream q (condition (3b)).
From the last equation we obtain

$$D' = \frac{\alpha\,(T^{\infty} - T^0)}{\sqrt{\frac{w}{a_1\xi}}\,\lambda sh\left(\sqrt{\frac{w}{a_1\xi}} R\right) + \alpha ch\left(\sqrt{\frac{w}{a_1\xi}} R\right)} \tag{14a}$$

It means, that the function describing the temperature field

$$D'' = -\frac{q}{\lambda\sqrt{\frac{w}{a_1\xi}}\, sh\left(\sqrt{\frac{w}{a_1\xi}} R\right)} \tag{14b}$$

in the ingot section, on the level z = ξ have respectively, the form

$$T_1(x, \xi) = \frac{\alpha\,(T^{\infty} - T^0)\, ch\left(\sqrt{\frac{w}{a_1\xi}}\, x\right)}{\sqrt{\frac{w}{a_1\xi}}\,\lambda_1 sh\left(\sqrt{\frac{w}{a_1\xi}} R\right) + \alpha ch\left(\sqrt{\frac{w}{a_1\xi}} R\right)} + T^0 \qquad 0 < x < R \tag{15a}$$

$$T_1(x, \xi) = T^0 \frac{q ch\left(\sqrt{\frac{w}{a_1\xi}}\, x\right)}{\lambda_1 \sqrt{\frac{w}{a_1\xi}}\, sh\left(\sqrt{\frac{w}{a_1\xi}} R\right)} \qquad 0 < x < R \tag{15b}$$

according to this, what information about the heat exchange between the ingot and the crystallizer we use.

The Relationships between the Parameters of the Process and the Beginning Position of the Cuticle

The process of cuticle's forming begins in the moment when the temperature of ingot's surface reaches the crystallization temperature T^*. Let assume, that on the level z = ξ^* the temperature of ingot's surface has reached the temperature T^*. This assuming leads to equations:

$$T^* = \frac{\alpha (T^\infty - T^0)\, ch\left(\sqrt{\frac{w}{a_1 \xi^*}}\, R\right)}{\sqrt{\frac{w}{a_1 \xi^*}}\, \lambda_1 sh\left(\sqrt{\frac{w}{a_1}}\, \xi^* R\right) + \alpha ch\left(\sqrt{\frac{w}{a_1 \xi^*}}\, R\right)} + T^0 \tag{16a}$$

$$T^* = T^0 - \frac{q ch\left(\sqrt{\frac{w}{a_1 \xi^*}}\, R\right)}{\lambda \sqrt{\frac{w}{a_1 \xi^*}}\, sh\left(\sqrt{\frac{w}{a_1 \xi^*}}\, R\right)} \tag{16b}$$

after easy transformations we receive

$$\sqrt{\frac{w}{a_1 \xi^*}}\, th\left(\sqrt{\frac{w}{a_1 \xi^*}}\, R\right) = \frac{\alpha (T^* - T^\infty)}{\lambda_1 (T^0 - T^*)} \tag{17a}$$

and

$$\sqrt{\frac{w}{a_1 \xi^*}}\, th\left(\sqrt{\frac{w}{a_1 \xi^*}}\, R\right) = \frac{q}{\lambda_1 (T^0 - T^*)} \tag{17b}$$

The last equations include all the essential parameters of the process, determining the heat exchange in the beginning phase of continuous casting forming, and can be used in the practice, among others, these equations allow to determine in what distance of surface of the liquid metal mirror, the process of cuticle's growing begins.

The Example of the Calculations

It was considered the cast steel ingot of plate's shape and thermophysical parameters: $\lambda = 30$ Wm/K, $a = 0.0000056\ m^2/s$. Among others, it was analysed the influence of ingot's thickness R and the initial pouring temperature T^0 for the initial cuticle's position.

In the calculations, it was assumed that $w = 0.02$ m/s, $T = 30^0C$, $\alpha = 1200$ W/mK, $q = 1\,800\,000\ W/m^2$.

The effects of calculations were graphic illustrated on Figures 2 and 3.

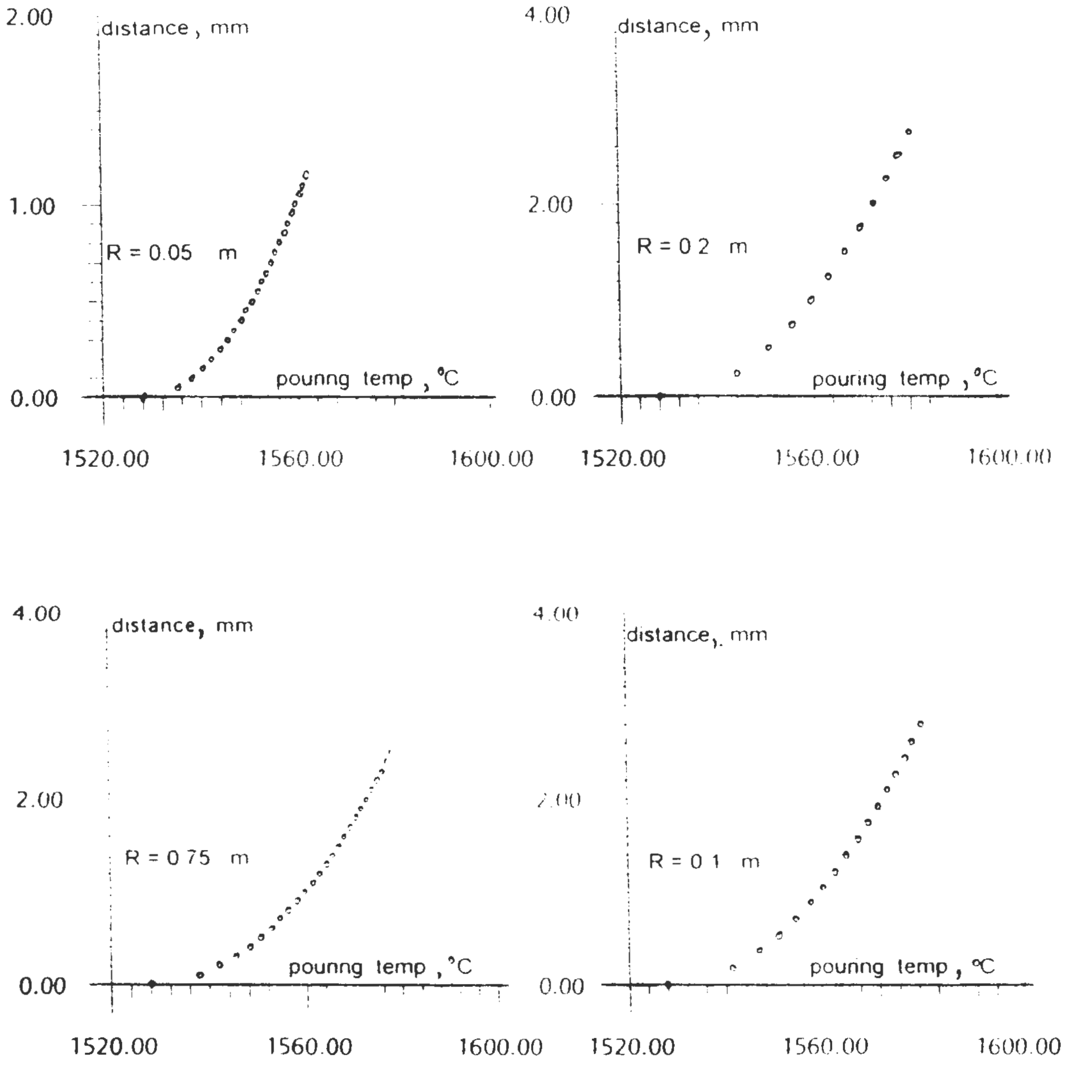

Fig. 2. The effect of calculation for the heat stream q

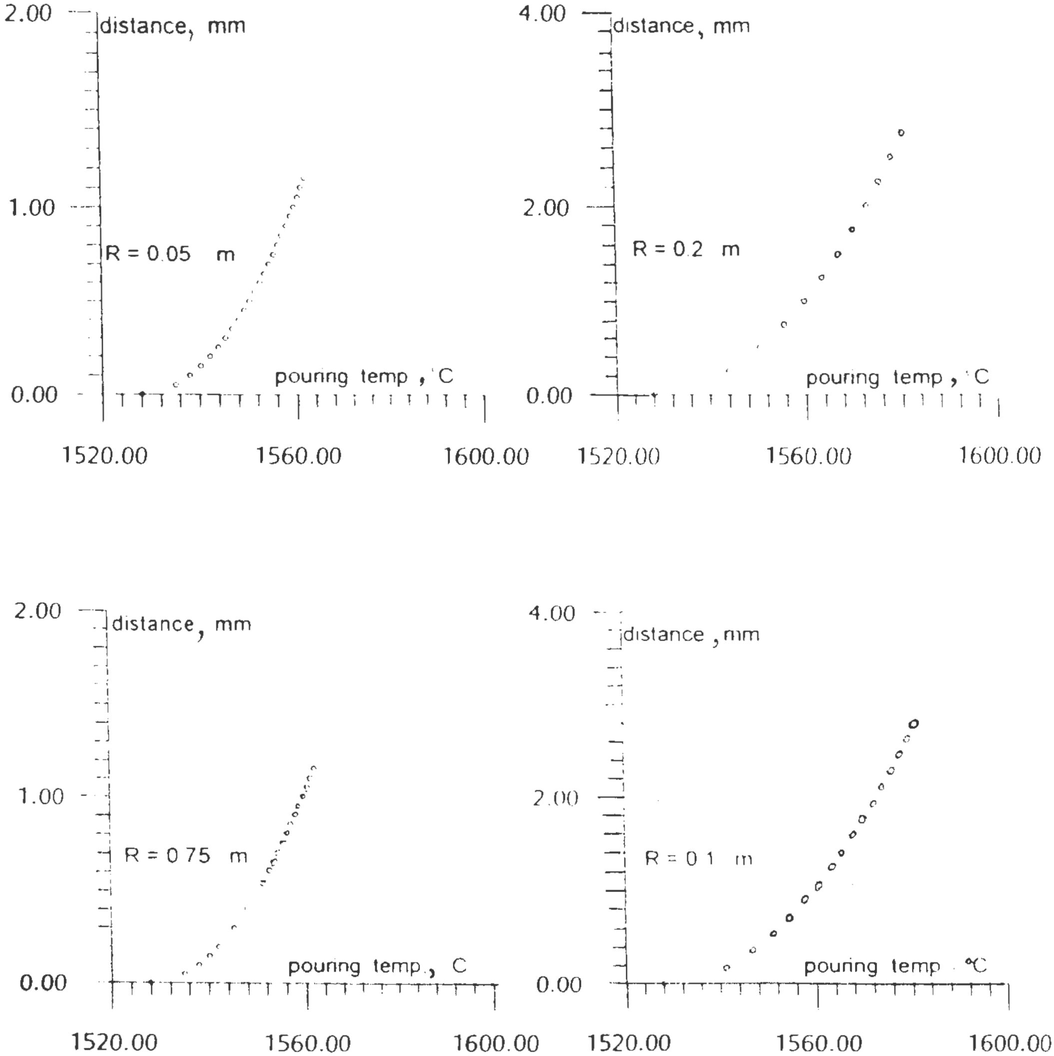

Fig. 3. The effect of calculation for the heat penetration coefficient α

Streszczenie

RÓWNANIE POLA TEMPERATURY DLA POCZĄTKOWEJ FAZY FORMOWANIA SIĘ WLEWKA CIĄGŁEGO I JEGO ZASTOSOWANIE W PRAKTYCE

Skoncentrowano się na jednym z podstawowych problemów, związanych z procesem ciągłego odlewania, jakim są zjawiska cieplne zachodzące w początkowej fazie formowania się wlewka. Wyprowadzono między innymi równania, pozwalające określić w jakiej odległości od powierzchni zwierciadła ciekłego metalu rozpoczyna się proces narastania naskórka. Wyprowadzone równania wiążą ze sobą wszystkie istotne parametry procesu i mogą być wykorzystane do analizy zjawisk cieplnych, zachodzących we wlewku w początkowej fazie jego formowania.