CRACK IDENTIFICATION IN COMPOSITE BEAM USING CAUSAL B-SPLINE WAVELETS OF FRACTIONAL ORDER

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Summary

An application of composite elements in machinery design became very popular due to the unique properties of these materials. Therefore, it is obvious to develop appropriate methods for their diagnostics and monitoring. One the group of methods, intensively developed in the last decades, is the analysis of vibration data of the structures using wavelet transform. The method proposed in this paper is based on the analysis of normal modes of vibration of damaged elements in order to detect and identify singularities in the signal, based on which the conclusion about damage presence and location could be made. The application of fractional B-spline wavelets during wavelet transform of the signal improves the sensitivity of the method and allows for tuning-up the order of an applied wavelet for achieving the most adequate result for the investigated problem.

IDENTYFIKACJA PĘKNIĘCIA W BELCE KOMPOZYTOWEJ Z WYKORZYSTANIEM KAUZALNYCH FALEK B-SPLAJNOWYCH RZĘDU UŁAMKOWEGO

Streszczenie

Streszczenie. Zastosowanie elementów kompozytowych w budowie maszyn stało się bardzo popularne ze względu na wyjątkowe właściwości tych materiałów. Dlatego oczywista jest potrzeba opracowania odpowiednich metod do ich diagnostyki i monitoringu. Jedną z grup metod, intensywnie rozwijanych w ostatnich dekadach, jest analiza danych drganiowych struktur z wykorzystaniem transformacji falkowej. Metoda zaproponowana w tym artykule oparta jest na analizie postaci własnych drgań uszkodzonych elementów w celu detekcji i identyfikacji osobliwości w sygnale, na podstawie której można uzyskać konkluzje dotyczące obecności i położenia uszkodzenia. Zastosowanie falek B-splajnowych rzędu ułamkowego podczas transformacji falkowej sygnału zwiększa czułość metody i pozwala na dopasowanie rzędu zastosowanej falki w celu uzyskania najbardziej adekwatnego wyniku dla rozpatrywanego problemu.

1. INTRODUCTION

The application of element made of polymeric composites in the machinery design has many advantages due to the great properties of such materials, e.g. superior strength-to-mass ratios in comparison with metals and metallic alloys, resistance to the environmental corrosion and some aggressive chemical media. However, such materials characterizes by different mechanisms of damaging and degradation in operation conditions. They have lower resistance to impact loading than metallic materials. Some types of damages, e.g. delamination or debonding, could occur in polymeric composites, but not occurred in metals. Therefore, it is suitable to develop appropriate methods for structural diagnostics and monitoring, which will be sensitive for such types of damages.

From the wide range of methods used for structural diagnostics of composite elements one of the greatest groups constitute the vibration-based methods. These methods based generally on the extracting the information about eventual damage by using advanced signal processing analyzes. In the last decades a great popularity achieved the methods which use the wavelet transform-based algorithms for damage detection, localization and identification. An application of wavelet-based methods allows for detection and identification of even small damages by localizing the singularities in the vibration signals.

The damage identification in beam structures was studies by several researchers. The scientific group from the Aristotle University in Thessaloniki studied the damage identification problem based on the numerical and experimental data [1,2] with use of continuous wavelet transform (CWT) and applied symlet of order 4. The authors of [3] analyzed modal shapes of beams obtained from the numerical analyzes using symlets, Gaussian and Daubechies' wavelets of different order, while the authors of [4] used Gabor wavelets. Rucka and Wilde [5] presented the comparative analysis of different wavelets in order to select appropriate ones for damage identification problem in beams. They conduct, that the Gabor wavelets provide the best approximation basing on the set of wavelet chosen for comparative analysis. All of the presented studies used CWT-based algorithms.

In the previous works of the author [6,7] the new algorithm was proposed, which is based on discrete wavelet transform (DWT). The CWT-based algorithms are redundant and therefore needs more computation time for the analysis; comparing CWT- and DWT-based algorithms the latter is about four times faster [8]. The DWT has a limitation in applicable wavelets, e.g. they should be orthogonal (biorthogonal, semiortogonal) and compactly supported. The Gabor wavelets applied by the authors of [4,5] could not be applied in DWT-based methods, however, by applying the B-spline wavelets the problem could be eliminated considering a fact, that the B-spline wavelets converge asymptotically to the Gabor function during increase of their order [9]. This statement was confirmed in the comparative studies presented in [7,8].

The previously presented DWT-based algorithm could be improved by applying for the analysis fractional B-spline wavelets developed by the Swiss scientific group [10]. The authors proposed a generalization of B-spline wavelets to the non-integer (fractional) order. Such a generalization makes possible to tune-up the order of the wavelet to the value, which will be the most suitable for the investigated problem.

In this paper the application of fractional B-spline wavelets for the detection and identification of a small crack in composite was presented and discussed. The analysis was performed based on numerical data obtained from the finite element (FE) model of cracked composite beam. The comparative analysis of fractional B-spline wavelets of different order was performed in order to select the best one for the damage identification problems.

2. FRACTIONAL B-SPLINE SCALING FUNCTIONS AND WAVELETS

The fractional B-spline wavelets behave as fractional differentiators [11], which could be a useful property for the investigated problem of damage identification. The fractional B-spline scaling function could be obtained by applying the fractional difference of the single-sided power function:

$$\beta^{\alpha}(x) = \frac{1}{\Gamma(\alpha+1)} \sum_{k=0}^{\infty} (-1)^{k} \binom{\alpha+1}{k} (x-k)^{\alpha} , \quad (1)$$

where α is the fractional order of the scaling function, for $\alpha > -1/2$ the scaling function is square-integrable, i.e. $\beta \in L^2(\mathbf{R})$ and $\Gamma(\alpha+1)$ is the Euler's gamma function, which defined as

$$\Gamma(\alpha+1) = \int_{0}^{\infty} x^{u} e^{-x} \, dx . \quad (2)$$

The fractional scaling functions are not compactly supported and not symmetric with exceptions, when α is an integer, which cause that they could not be used in DWT. However, the algorithm of fractional wavelet transform (FrWT) could be constructed using Fourier series [10].

The fractional B-spline scaling functions satisfies the two-scale relation

$$\beta^{\alpha}\left(\frac{x}{2}\right) = \sum_{k \in Z} 2^{-\alpha} \binom{\alpha+1}{k} \beta^{\alpha}(x-k), \quad (3)$$

which, together with other satisfied properties (for details see [11]), make it able to use for Mallat's multiresolution analysis.

The fractional wavelet function of order α could be given in explicit form as [10]:

$$\psi^{\alpha}\left(\frac{x}{2}\right)=\sum_{k\in Z}(-1)^{k}2^{-\alpha}\sum_{l\in Z}\binom{\alpha+1}{l}\beta_{*}^{2\alpha+1}\cdot\cdot(l+k-1)\beta^{\alpha}(x-k), \qquad (4)$$

where $\beta_{*}^{2\alpha+1}(\ )$ is the symmetric version of fractional B-spline scaling function of degree α obtained by convolution. The fractional B-spline wavelets satisfy the semi-orthogonality property, i.e. $\langle\beta^{\alpha}(\cdot),\psi^{\alpha}(\cdot-k)\rangle=0$, as their integer analogues. They have $\lceil\alpha\rceil+1$ vanishing moments:

$$\int_{-\infty}^{\infty}x^{n}\psi^{\alpha}(x)dx=0. \qquad (5)$$

The fractional B-spline wavelet functions of order $0\le\alpha<6$ with the step of α of 0.2 were presented in Fig.1. Note that the same color in Fig.1 was applied for every $\lfloor\alpha\rfloor$.

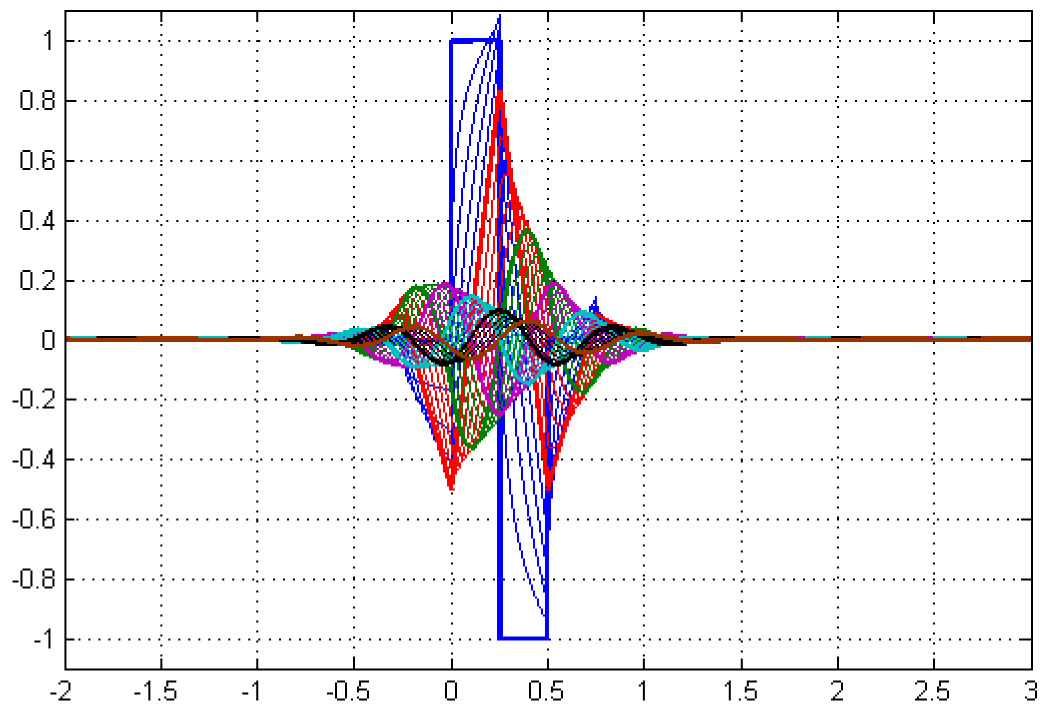

Fig.1. Fractional B-spline wavelet functions of order $0\le\alpha<6$

3. .DATA PREPARATION AND DESCRIPTION OF THE APPLIED METHOD

In order to test the causal fractional B-spline wavelets' effectiveness in the damage identification problem the numerical model of a composite beam was prepared. A small damage was inputted into the model.

3.1. NUMERICAL MODEL

The model was prepared using MSC Marc/Mentat® commercial software. In order to obtain modal shapes of a cantilever beam possibly close to the realistic ones the model was defined as 3-dimensional. The dimensions of a beam were as follows: length of 0.2 m, width of 0.01 m and thickness of 0.0024 m. The material of a beam was defined as orthotropic (epoxy resin reinforced by carbon fiber) with the following properties [12]: Young's moduli – $E_{11}=82$ GPa, $E_{22}=82$ GPa, $E_{33}=8.5$ GPa; shear moduli – $G_{12}=5.2$ GPa, $G_{23}=3.05$ GPa, $G_{31}=3.47$ GPa and Poisson's ratios – $\nu_{12}=0.312$, $\nu_{23}=0.29$, $\nu_{31}=0.27$.

The lay-up of a composite consisted of 12 layers with the orientation given by the structural formula: $[0/60/-60]_{2S}$. The geometrical model was meshed using hexagonal 8-node elements with following quantities: 256 elements on the length – in order to fulfill the criterion of FrWT (see [10] for details), 10 elements on the width and 3 elements on the thickness.

The damage was inputted as trough-the-width crack with distance from a clamped side of 0.15 m, gap size of 0.0005 m and the depth of 0.0004 m (the thickness of two layers). The mesh around the damage was prepared as more dense, however only equidistant nodes were considered during data extraction. The crack was located between the considered nodes in order to avoid the situation when the node position is changes in the thickness direction of a beam.

The numerical analysis was performed in order to evaluate modal shapes of a beam. For the next studies five bending modes were chosen. Then the bending displacements were collected for the nodes along the length of a beam on the half-width position. Obtained data were converted into Matlab® format.

3.2. DAMAGE IDENTIFICATION METHOD

Collected data were firstly preconditioned for reduction of a boundary effect. The whole-point symmetric padding method was chosen for reducing this effect.

The procedure of an application of FrWT could be considered as filtering of the input signal by the set of filters. For fulfilling the two-scale relation (3) it is necessary to orthogonalize the filters. The orthogonalized high pass $H_{\perp}^{\alpha}(z)$ and low pass $G_{\perp}^{\alpha}(z)$ filters following [11] could be presented as:

$$H_{\perp}^{\alpha}\left(e^{j\omega}\right) = H^{\alpha}\left(e^{j\omega}\right)\sqrt{\frac{A^{2\alpha+1}\left(e^{j\omega}\right)}{A^{2\alpha+1}\left(e^{2j\omega}\right)}}, \quad (6)$$

$$G_{\perp}^{\alpha}\left(e^{j\omega}\right) = e^{-j\omega} H_{\perp}^{\alpha}\left(-e^{-j\omega}\right), \quad (7)$$

where

$$H^{\alpha}\left(e^{j\omega}\right) = \sqrt{2}\left(\frac{1+e^{-j\omega}}{2}\right)^{\alpha+1} \quad (8)$$

is the non-orthogonal scaling filter for causal fractional B-spline wavelet and

$$A^{\alpha}\left(e^{j\omega}\right) = \sum_{n\in Z} e^{-jn\omega} \int \beta^{\alpha}(x)\beta^{\alpha}(x+n)dx \quad (9)$$

is the autocorrelation filter of a B-spline.

Using the filters defined in such a way, the decomposition of a signal could be performed. After filtering and downsampling of an input signal we obtain the approximation and detail coefficients.

Basing on previous studies, it should be noted that the coefficients a strongly dependent on the magnitudes of displacements, i.e. the greater magnitude of displacements the better identification of a singularity. Therefore, it is necessary to consider more then one modal shape assuming that the position of a damage is not a priori known. Adding up the absolute values of sets of detail coefficients could be an effective tool of magnifying the magnitudes of detail coefficients in the damage location. The schematic representation of the applied algorithm was shown in Fig.2. Parameter M denotes the number of considered modes.

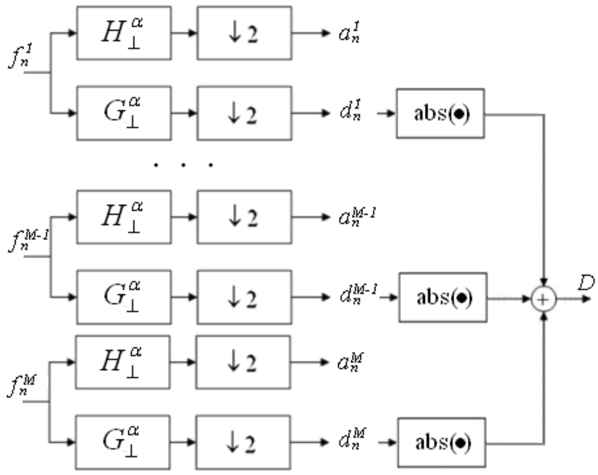

Fig.2. Scheme of the algorithm of a damage identification based on single-level FrWT with adding up of sets of detail coefficients

4. RESULTS AND DISCUSSION

The analysis of five chosen modal shapes of a cantilever beam was carried out using causal fractional B-spline wavelets. Results of the analysis show, that in the case of low-order of the applied wavelets the filtering of signals was not sufficient, i.e. the sets of detail coefficients consist a trend of the given modal shape. The reason of such a phenomenon is small number of vanishing moments of these wavelets. In order to apply the algorithm presented above the details coefficients need to be de-trended. The de-trending procedure was carried out as follows: an original set of detail coefficients was approximated by the spline function with low smoothing parameter in order to avoid the approximation of coefficients, which contain singularities. Then, the set of approximated coefficients was subtracted from the original set. An example of de-trending procedure was presented in Fig.3. The analysis was carried out for the third bending mode with applied wavelet of order of $\alpha = 1.4$.

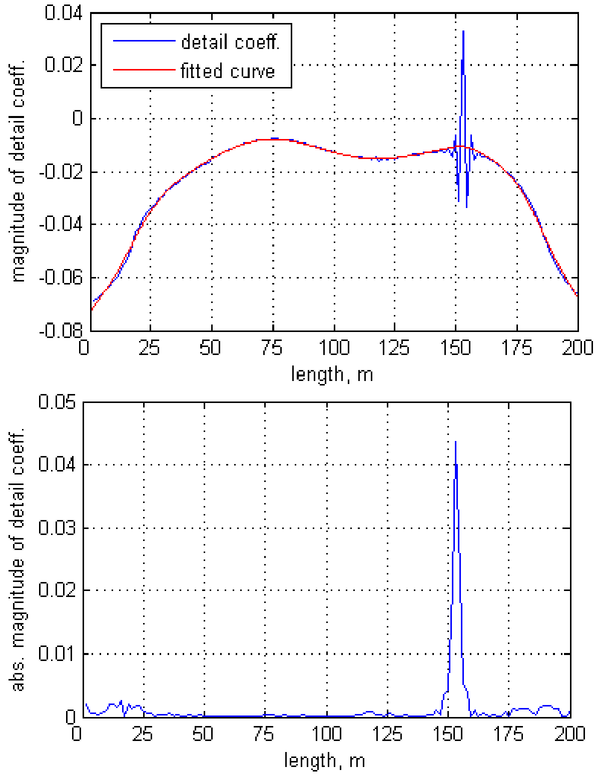

Fig.3. Approximation of the detail coefficients and absolute values of the de-trended set of detail coefficients

As it could be noticed, the damage location could be clearly identified. The de-trended detail coefficients will be more effective during addition of such sets for particular modal shapes, i.e. the de-rending improve the emphasizing the location of a crack.

which could be motivated by a fact that the have only one vanishing moment (see (5)). In order to determine the best α for the damage identification problem the Matlab routine was developed. The effectiveness of a given wavelet defined by the order α was evaluated as follows. The order α was stepped by 0.01 from 0 to 6. For each case the resulted set D obtained by addition of sets of detail coefficients for particular modal shapes (see Fig.2) the effectiveness parameter was evaluated following the formula:

$$e^{\alpha} = \frac{\max\left(D^{\alpha}\right)}{\mathrm{Me}\left(D^{\alpha}\right)}, \tag{10}$$

where $\mathrm{Me}\left(D^{\alpha}\right)$ is the median of a given set.

Obtained results for all considered wavelets were presented in Fig.4. Results show that in the cases when $\alpha < 1$ the crack does not recognized clearly, which was caused by low number of vanishing moments. The best results were obtained for $\alpha = 1.41$, for which $e^{\alpha} = 9.272$. Another peak could be noticed for $\alpha = 3.36$, for which $e^{\alpha} = 9.168$. The higher values of e^{α} were obtained for fractional α, which confirms the rightness of application of B-spline wavelets of fractional order for the damage identification problems. In comparison with classical B-spline

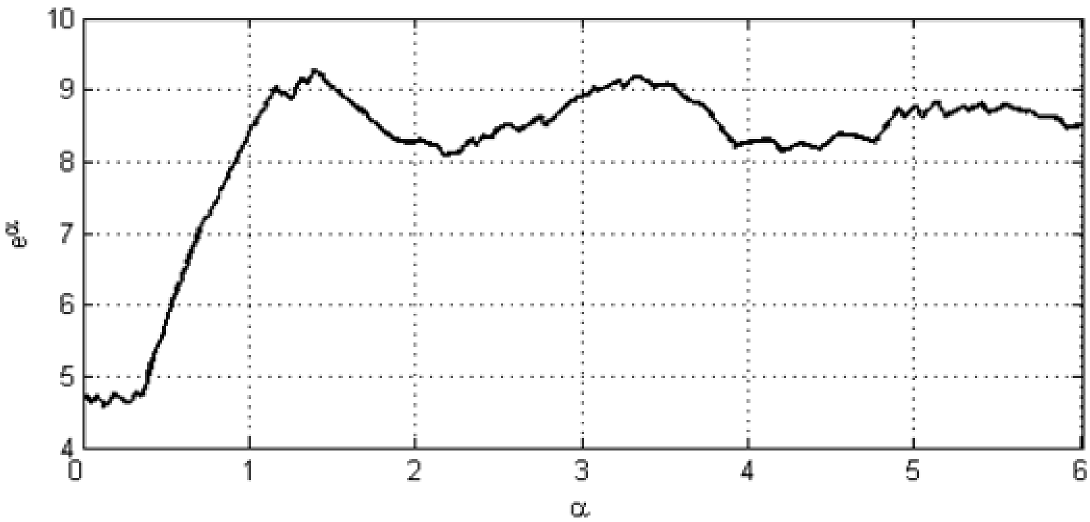

Fig.4. The influence of the order of causal fractional B-spline wavelets on effectiveness of crack identification

The application of low-order causal fractional B-spline wavelets reveal great results, however, it should be considered that due to the increase of α the values of detail coefficients became more distorted due to an increase of the number of vanishing moments. The analysis show that the wavelets with an order of $-0.5 < \alpha < 0$ reveal poor detectibility of a crack,

wavelets (when α is integer) the fractional ones improve the sensitivity of damage detection. Some exemplary results for various α were presented in Fig.5.

Basing on results presented in Fig.5 it could be noticed that the magnitude of detail coefficients became higher due to the increase of α. However,

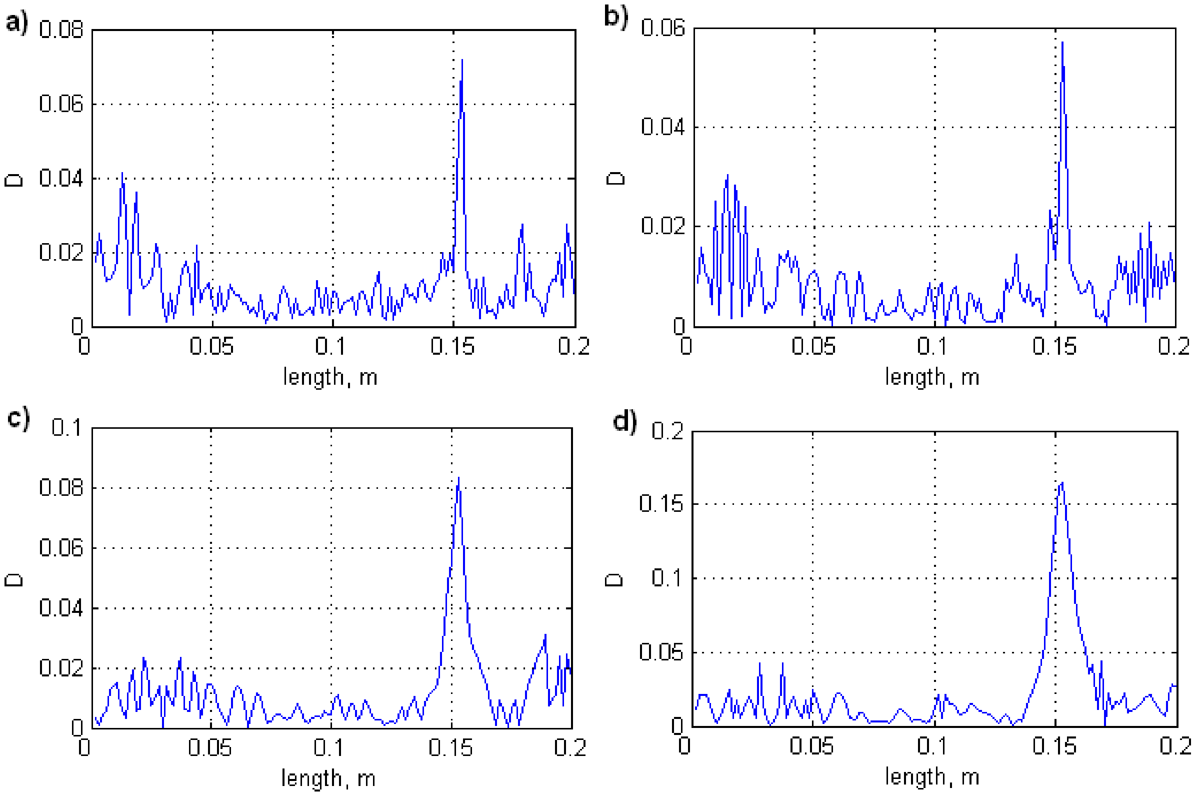

Fig.5. Result of crack identification for $\alpha =$ a) 0.37, b) 1.41, c) 3.36, d) 5.15

the distortion of the coefficients in the damage location also grows with an increase of α, which may produce some difficulties in interpretation, especially in the cases when more cracks will be located near each other. Following this, it is suitable to choose the wavelets of order $1.17 < \alpha < 1.55$ (see Fig.4) in order to evaluate the damage location precisely.

5. CONCLUSIONS

In the following study the evaluation of effectiveness of application of causal fractional B-spline wavelet to the crack identification problem in composite beam was carried out. During the analysis an evaluation of sensitivity to the damage presence with respect to the fractional order of applied wavelet was performed. It was confirmed that the application of fractional B-spline wavelet allow for increasing the accuracy of damage identification in comparison with classical B-spline wavelets. Results of the analysis allow to evaluate the best values of order of wavelets applied to the problem. The study was performed on numerical data and the obtained results need to be validated in the laboratory experiments, which will be carried out in further studies.

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