DAMAGE IDENTIFICATION BASED ON STATIONARY WAVELET TRANSFORM OF MODAL DATA

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Summary

Wavelet-based methods applicable for structural damage assessment have been extensively developed over the last decade. These methods are based on the application of wavelet transform to the modal shapes of vibration of structural elements in order to identify the damages. The most of the developed algorithms were based on continuous and discrete wavelet transform. However, due to the energy leakages during application of the discrete wavelet transform and reduction of spatial dimensions of an original signal the obtained results could be inappropriate, especially in the case of small damages. Following this, a new algorithm for spatial damage detection in composite structures based on stationary wavelet transform of modal shapes of vibration was proposed. The proposed algorithm was tested on data of the numerical model of a square composite plate with multiple damages of various shapes. Several advantages of the proposed approach using stationary wavelet transform with respect to discrete wavelet transform were shown and discussed based on the tested example. The detectability performance of the proposed algorithm was tested on the noised data with various levels. Due to the great effectiveness and low computational complexity the presented approach could be implemented in hardware and used in practical structural damage assessment problems as well.

Keywords: structural damage identification, stationary wavelet transform, discrete wavelet transform, energy leakage, modal analysis, composite structures

IDENTYFIKACJA USZKODZEŃ NA PODSTAWIE STACJONARNEJ TRANSFORMACJI FALKOWEJ DANYCH MODALNYCH

Streszczenie

Metody falkowe stosowane do strukturalnej oceny uszkodzeń są intensywnie rozwijane w ciągu ostatniego dziesięciolecia. Metody te opierają się na zastosowaniu transformacji falkowej do postaci własnych drgań elementów strukturalnych w celu identyfikacji uszkodzeń. Jednak, ze względu na przecieki energii podczas zastosowania dyskretnej transformacji falkowej i redukcji wymiarów sygnału wejściowego otrzymane wyniki mogą być nieodpowiednie, szczególnie w przypadku małych uszkodzeń. W związku z tym zaproponowano nowy algorytm dwuwymiarowej identyfikacji uszkodzeń w strukturach kompozytowych oparty na stacjonarnej transformacji falkowej postaci własnych drgań. Zaproponowany algorytm testowano na danych z modelu numerycznego kwadratowej kompozytowej płyty z wielokrotnymi uszkodzeniami o różnym kształcie. Przedstawiono i omówiono niektóre zalety zaproponowanego podejścia w porównaniu do dyskretnej transformacji falkowej na podstawie testowanych danych. Wykrywalność uszkodzeń przy pomocy zaproponowanego algorytmu przetestowano na danych zaszumionych o różnym poziomie. Ze względu na wysoką efektywność i małą złożoność obliczeniową zaproponowane podejście może być zaimplementowane sprzętowo i stosowane w praktycznych problemach oceny uszkodzeń strukturalnych.

Słowa kluczowe: strukturalna identyfikacja uszkodzeń, stacjonarna transformacja falkowa, dyskretna transformacja falkowa, przeciek energii, analiza modalna, struktury kompozytowe

1. INTRODUCTION

Modern requirements of structural damage assessment (SDA) and structural health monitoring (SHM) of composite structures demand on the development of effective methods, which will be sensitive to the particular types of damages in the early stage of their development, applicable in various measurement conditions and be non-destructive and low-cost. From the great variety of currently applied methods the vibration-based methods seem to be ones of the most appropriate. However, the analysis of natural frequencies and modal shapes do not give complete information about the condition of a structure. In order to identify the damages in a tested structure the additional signal processing techniques should be applied to the modal data.

In the recent years a great interest was paid to the wavelet-based methods, which ensure very high sensitivity to the abrupt changes of the processed signals, and could be applied for SDA and SHM problems as well. The idea of application of wavelet transforms to the modal shapes of vibration is based on detection of local reduction of stiffness caused by the damages. Besides the high sensitivity of wavelet based methods their great advantage is that it is not necessary to use a reference model (e.g. the healthy structure) for evaluation the structural condition.

The application of wavelet-based methods for the structural damage assessment was introduced by Surace and Ruotolo [1], who applied continuous wavelet transform (CWT) for detection of a crack in a beam-like structure, while the authors of [2] used discrete wavelet transform (DWT) for identification of impact damages in composite structures. However, both approaches presented in the above-cited works do not allow for localization of damages. Further studies in this area resulted in development of CWT-based methods, which allow for detection and localization of damages. Descriptions of several approaches applied to the SDA problem can be found in [3-7]. The authors detected and localized the cracks in the beam-like structures using various types of wavelets. The more computationally efficient algorithms, which based on DWT and allow for the damage localization, were developed by the author [8,9]. Alternatively, Zhong and Oyadiji developed three methods of SDA for beam-like structures [10-12] based on the stationary wavelet transform (SWT), which allows for increasing the localization ability with respect to DWT due to the time invariance and of SWT and the lack of the energy leakage presented during application of DWT [13].

Considering the practical necessities of SDA it was obvious to extend the developed wavelet-based methods to the spatial geometric domain. 2D SDA problems were studied by several authors [14-16], which used 2D CWT for the damage detection in rectangular plates. Another approach based on 2D DWT was introduced by the author in [17] and developed in further studies [18,19].

Due to the great results obtained by Zhong and Oyadiji it was decided to extend the SWT-based method to the spatial geometric domain and analyze the results of such an approach. For this purpose the numerical model of a composite plate with modeled multiple damages was used for the analysis. The SWT-based algorithm was applied to the numerical data and the sensitivity analysis was performed on the numerical data noised on the various levels. Obtained results show that 2D SWT-based approach provides better localization capability and computational effectiveness than the DWT-based one.

2. PRELIMINARIES AND METHODOLOGY

2.1 1D AND 2D STATIONARY WAVELET TRANSFORM

The stationary wavelet transform is an undecimated modification of a classical DWT based on Mallat's multiresolution algorithm. The lack of decimation in SWT is its main advantage which ensures the translation invariance with respect to DWT obtained by omitting the downsamplers during the analysis. In DWT algorithm the decomposition procedure causes reduction of length of the signal realization twice in each direction and for each decomposition level, whereas in SWT the length of signal realization remains the same as for the original signal regardless of the level of decomposition. This property also implies the disadvantage of SWT, i.e. it is redundant with respect to DWT.

The SWT decomposition uses two functions: the scaling function (x) and the wavelet function (x), which are associated with low-pass and high-pass filters, respectively. These functions satisfy the two-scale relation [20]:

$$2^{-\frac{1}{2}}\varphi\left(\frac{x}{2}-k\right)=\sum_{n=-\infty}^{\infty}h(n-2k)\varphi(x-n), \quad (1)$$

$$2^{-\frac{1}{2}}\psi\left(\frac{x}{2}-k\right)=\sum_{n=-\infty}^{\infty}g(n-2k)\varphi(x-n), \quad (2)$$

where $\{h_n\}$ and $\{g_n\}$ are the impulse responses of low-pass and high-pass quadrature mirror filters, respectively. The decomposition procedure in SWT results in

approximation $c_{j,k}$ and detail $d_{j,k}$ sets of coefficients at resolution 2^{-j}, where j is the level of decomposition. Considering (1) and (2) they could be presented in the form [20]:

$$c_{j+1,k} = \sum_{n=-\infty}^{\infty} h(n-2k) c_{j,n} , \tag{3}$$

$$d_{j+1,k} = \sum_{n=-\infty}^{\infty} g(n-2k) c_{j,n} \tag{4}$$

or in the discrete form:

$$\tilde{c}_{j+1,k} = \sum_{l=-\infty}^{\infty} h(l) \tilde{c}_{j,k+2^j l} , \tag{5}$$

$$\tilde{d}_{j+1,k} = \sum_{l=-\infty}^{\infty} h(l) \tilde{d}_{j,k+2^j l} . \tag{6}$$

Considering the separability property of scaling and wavelet functions the SWT could be extended to the 2D domain. The scaling and wavelet functions could be obtained as a tensor product of combinations of 1D scaling and wavelet functions:

$$\begin{aligned} \Phi(x,y) &= \varphi(x)\varphi(y), \\ \Psi_1(x,y) &= \varphi(x)\psi(y), \\ \Psi_2(x,y) &= \psi(x)\varphi(y), \\ \Psi_3(x,y) &= \psi(x)\psi(y), \end{aligned} . \tag{7}$$

where Φ is a 2D scaling function and Ψ_i (i = 1,2,3) are the directional-oriented (horizontal, vertical and diagonal) 2D wavelet functions. The decomposition of a 2D signal could be associated with a two-stage filtering using the same set of low-pass and high-pass filters along the specific directions. This results in one 2D set of approximation coefficients and three 2D sets of detail coefficients, which could be presented in the form:

$$\tilde{c}_{j+1,k,m} = \sum_{l=-\infty}^{\infty} h(l) h(l) \tilde{c}_{j,k+2^j l, m+2^j p} , \tag{8}$$

$$\tilde{d}_{1,j+1,k,m} = \sum_{l=-\infty}^{\infty} h(l) h(p) \tilde{d}_{1,j,k+2^j l, m+2^j p} . \tag{9}$$

$$\tilde{d}_{2,j+1,k,m} = \sum_{l=-\infty}^{\infty} h(p) h(l) \tilde{d}_{2,j,k+2^j l, m+2^j p} , \tag{10}$$

$$\tilde{d}_{3,j+1,k,m} = \sum_{l=-\infty}^{\infty} h(p) h(p) \tilde{d}_{3,j,k+2^j l, m+2^j p} . \tag{11}$$

2.2 NUMERICAL MODEL

The numerical data for the SDA problem were obtained from the finite element method analysis using MSC Marc/Mentat® commercial software. A model of a square composite plate with a side length of 300 mm and a thickness of 2.5 mm was meshed using hexagonal 8-node elements. The plate was clamped along the edges. The material properties of a plate were defined as follows. The stiffness matrix and specific mass of a single layer of transversally isotropic glass-epoxy laminate was defined, $C_{11} = C_{22} = 48.01$ GPa, $C_{12} = C_{21} = 19.98$ GPa, $C_{13} = C_{23} = C_{31} = C_{32} = 6.592$ GPa, $C_{33} = 11.42$ GPa, $C_{44} = C_{55} = 3.533$ GPa, $C_{66} = 14.01$ GPa, = 1794 kg/m^3, and then the 12-layered laminate was defined through modeller with the following lay-up: $[0/60/-60]_{2S}$.

The damages were modeled by excluding the elements in particular regions. In the investigated case three damages were modeled: two square exclusions with various dimensions and a crack with a width of 1 mm. All of the damages have the same depth of 0.5 mm (20% of total thickness). The scheme of a plate with additional dimensions was presented in Fig.1. The surfaces of the modeled plates contained 64×64 equidistant nodes. The normal modes analysis was performed for the prepared numerical model. First five resulted bending modal shapes were considered in further analysis. For this purpose the nodal values of displacements of the modal shapes on the surface of a plate were collected.

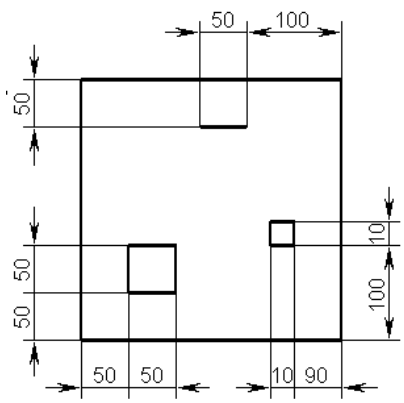

Fig. 1. A scheme of the modeled plate

2.3 METHODOLOGY

Collected data in the form of square matrices of nodal displacements for particular modal shapes were subjected to analysis using SWT-based algorithm. After the decomposition only the sets of detail coefficients were considered since they contain the diagnostic information about the structural condition of a plate. Due to the strong dependence between the magnitudes of displacements and the magnitudes of resulted detail coefficients it is necessary to consider more than one modal shape assuming that the damages presence and positions are

unknown. The direction-oriented detail coefficients were added up in order to obtain the result of damage identification in all available directions. For the purpose of enhancing the detectability of damages the absolute values of resulted sets were added up. The determination of absolute values was resulted by a fact that the coefficients have positive and negative values, which could superpose during adding of the sets.

3. ANALYZES AND RESULTS

3.1 DAMAGE IDENTIFICATION

One of the most important factors, which have an influence on the detectability and accuracy of localization of damages during the analysis, is the type of applied wavelet and its order [17]. In the previous studies the authors applied various wavelets for the investigated SDA problems: Chang and Chen [3,4] used Gabor wavelets, the authors of [5,6,10] used symlet of order 4 selected by trials and errors method, Gentile and Messina used Gabor wavelets, while the author of [8,9,17] used B-spline wavelets. Considering a fact that there are no strict rules for selection of wavelets for the wavelet analysis the comparative study based on commonly used wavelets was performed on the numerical data. The analysis was carried out using above-presented wavelet-based algorithm. The results of comparative study were presented in Fig.2. For clarity of presentation here and further the following symbols of wavelets were used: *db* for Daubechies, *bsp* for B-spline, *sym* for symlets, *coif* for coiflets and *bior* and *rbio* for biorthogonal and reversed biorthogonal wavelets respectively.

It could be noticed that the application of Haar and *bior*1.5 wavelets ensure the most accurate localization of damages, however the resulted patterns presented in Fig.2 are biased by the noise caused by an inappropriate filtering of the modal shapes. In the case of small damages and/or additional noise from the measurements these wavelets may not properly detect and localize the damages. The application of Daubechies wavelets as well as coiflets ensures better filtering capabilities, i.e. the damage locations are well separated from the noise, however the shape of damages was identified inaccurately. The inaccuracy degree increases with the increase of an order of the wavelet (cf. Fig.2b-d and Fig.2i-j). The same inaccuracies and blurring the boundaries of damages were observed for symlets and high-order biorthogonal and reversed biorthogonal wavelets (see Fig.2g, 2h, 2m, 2p). The most accurate localization with the low level of noise could be observed for the following wavelets: *bsp*3 (Fig.2e), *bior*3.3 (Fig.2l), *rbio*1.5 and *rbio*3.3 (Fig.2n and 2o). This means that the best results could be obtained using the low-order wavelets during the analysis. This implies a compromise between the wavelet stability and accuracy of localization, i.e. for the low-order wavelets the localization accuracy is high, but the influence of noise is also high, while during the increase of the order of the wavelet the influence of noise became much lower, but the power leakage is observable due to the high number of vanishing moments of these wavelets.

It should be noticed that during the analysis only single-level decomposition was performed due to increasing distortion of patterns when the higher-level decompositions are applied.

3.2 COMPARISON OF SWT- AND DWT-BASED ALGORITHMS

In order to emphasize advantages of the proposed approach the obtained results were compared with a popular DWT-based approach. As it was mentioned before, the decomposition based on DWT is characterized by the reducing the dimensions of pattern in each direction with respect to the original pattern and thus, the localization accuracy decreases. The exemplary results of such decomposition for selected wavelets were presented in Fig.3. Following this, in order to obtain the patterns with the same dimensions as the original pattern the DWT-based reconstruction procedure should be applied. Note that the reconstruction algorithm should be applied only for detail coefficient sets, which could be realized by setting the approximation coefficients values to zero. The results of DWT-based decomposition/reconstruction were presented in Fig.4.

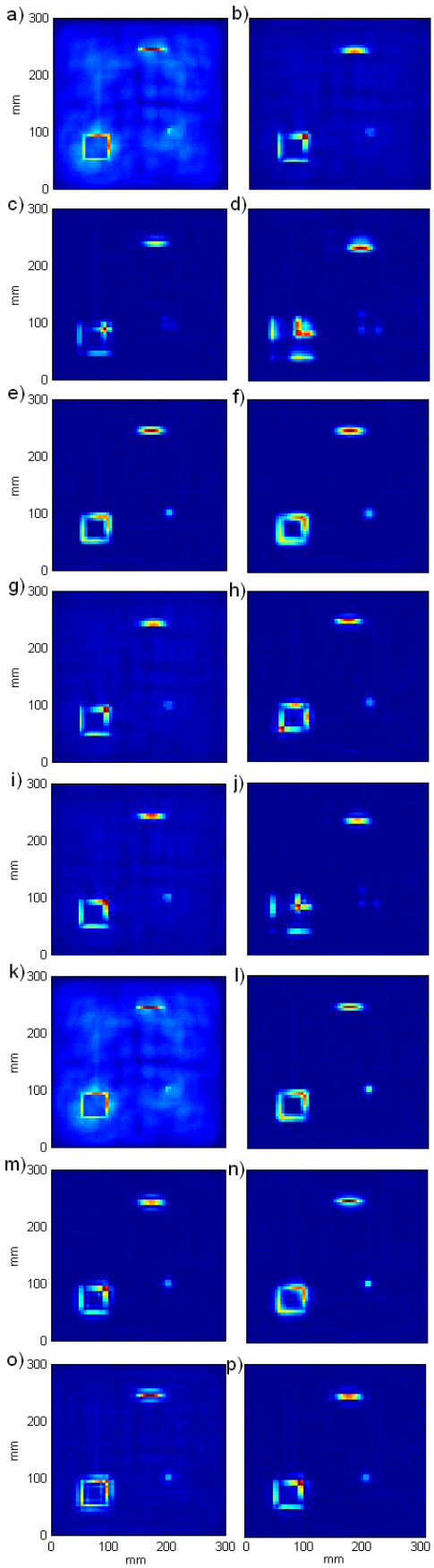

Fig. 2. Results of SWT-based decomposition using a) Haar, b) *db*2, c) *db*3, d) *db*5, e) *bsp*3, f) *bsp*5, g) *sym*2, h) *sym*4, i) *coif*1, j) *coif*3, k) *bior*1.5, l) *bior*3.3, m) *bior*5.5, n) *rbio*1.5, o) *rbio*3.3, p) *rbio*5.5 wavelets.

It can be noticed that during the DWT-based decomposition only *sym*4 wavelet detects and localizes properly the positions and shapes of damages. The Haar wavelet does not detected the boundaries of a bigger square-shaped damage, while in the other cases the distortions caused by downsampling operation are observable. The decomposition/reconstruction algorithm based on DWT shows the worst results, i.e. in all cases the great distortions around the locations of the damages (caused by down- and upsampling) are observable.

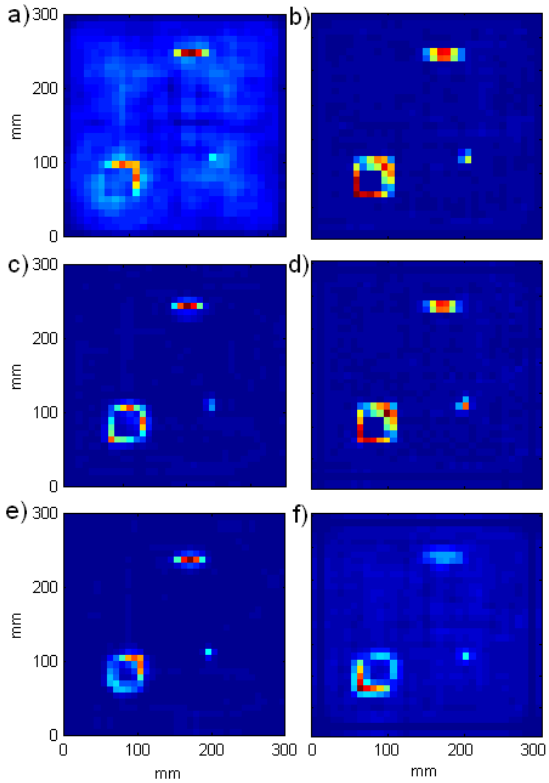

Fig. 3. Results of DWT-based decomposition using a) Haar, b) *bsp*3, c) *sym*4, d) *bior*3.3, e) *rbio*1.5, f) *rbio*3.3 wavelets.

3.3 INFLUENCE OF NOISE

In order to evaluate the detectability and accuracy of localization of damages in the real measurement conditions the analysis of influence of the noise was performed. The modal shapes obtained from the numerical analysis were noised by the noise with normal distribution on various levels. The noising procedure was performed using the signal-to-noise ratio in decibel scale (SNR_{dB}) determined for a given modal shape according to the following expression:

$$SNR_{dB} = 10\log_{10}\left(\frac{f(x,y)^2}{n(x,y)^2}\right), \qquad (12)$$

where $f(x,y)$ is the original signal (displacements of a modal shape) and $n(x,y)$ is the corresponded noise matrix.

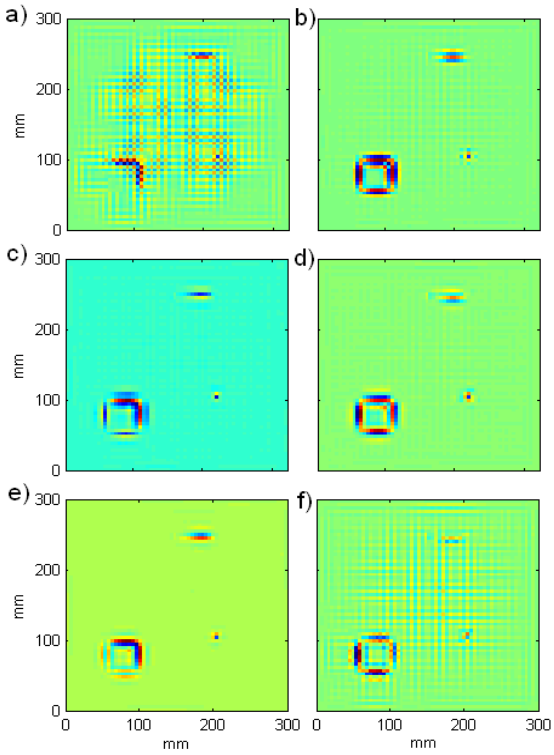

Fig. 4. Results of DWT-based decomposition/reconstruction using a) Haar, b) *bsp*3, c) *sym*4, d) *bior*3.3, e) *rbio*1.5, f) *rbio*3.3 wavelets.

The next operations of SWT-based decomposition were similar as those described in the section 2.3. It was assumed that the given case noised on the certain level is robust to noise if all three damages were clearly detectable. The analyzes were performed for the several wavelets in order to analyze their effectiveness in identification of damages in the case of noisy data. For each of the selected wavelets (based on the results presented in section 3.1) the critical values of SNR_{dB} for various wavelets were determined. The exemplary results for these critical values were presented in Fig.5.

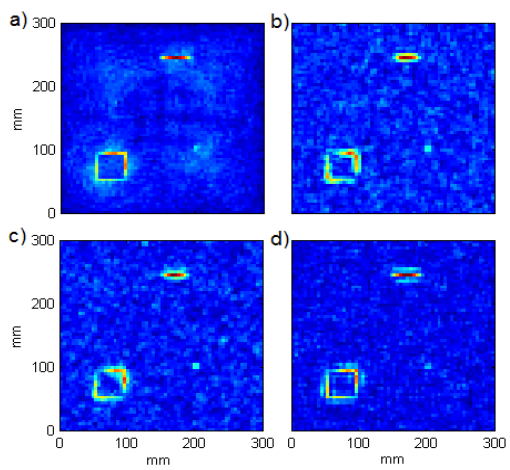

Fig. 5. Results of SWT-based decomposition using a) Haar wavelet for $SNR_{dB} = 22$ dB, b) *bior*3.3 wavelet for $SNR_{dB} = 20$ dB, c) *rbio*1.5 wavelet for for $SNR_{dB} = 18$ dB, f) *rbio*3.3 wavelet for $SNR_{dB} = 19$ dB.

It was observed that the proposed algorithm is characterized by the great robustness to the noise. This allows for using even standard measurement equipment (e.g. accelerometers) for carrying out the experimental studies. Additionally, it could be observed that the SNR_{dB} value and corresponding detectability of damages is depended on the order of the applied wavelet. This phenomenon is caused by a fact that the wavelets with small number of vanishing moments and shorter effective support detect the sudden changes in the signal better and do not cause the energy leakages in the damaged areas.

4. CONCLUSIONS

The presented approach of structural damage identification in composites was based on the stationary wavelet transform of modal shapes of vibration. Obtained results show that the proposed SWT-based approach, regardless the redundancy with respect to DWT-based one, ensures better identification ability and high robustness to the presence of noise. The presented comparative studies confirm the advantages of SWT-based approach with respect to DWT-based one. The analysis of sensitivity to different types of damages of various wavelets show that the low-order B-spline based wavelets ensure the best localization ability and the highest robustness to the noise. Due to the great effectiveness and low computational complexity the algorithm can be implemented on hardware as well. In order to verify the proposed approach the additional experimental studies are planned in further works.

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