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NUMERICAL MODELING OF CONCRETE STRUCTURES – RESEARCH ACHIEVEMENTS OF PROFESSOR STANISŁAW MAJEWSKI

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Abstract

Research and professional interests of Professor Stanislaw Majewski throughout the period of his work at the Technical University of Silesia were associated with the widely understood design of building structures. In his early career he worked on the sizing optimization of prestressed beams, then turned his focus to lightweight sandwich structures, but his greatest achievements are associated with the introduction of numerical methods for engineering design and research, mainly of concrete, but also masonry and subsoil. Developed by Professor Majewski MAPRET program was in the early 90s of the last century, the Polish synonym of computerization in design offices. The experience gained in programming Professor used in his greatest project, built since 1995 package MAFEM, allowing FEM analysis of planar and spatial problems. It is based on author material model of concrete, masonry and soil. The first version of this program was the basis of Professor Majewski habilitation dissertation and later versions helped to promote five PhDs. Despite a well-deserved of Professor's retirement, a program MAFEM is still used and developed by his successors and students.

Streszczenie

Zainteresowania naukowe i zawodowe Profesora Stanisława Majewskiego w całym okresie jego pracy na Politechnice Śląskiej były związane z szeroko rozumianym projektowaniem konstrukcji budowlanych. W początkowym okresie swej kariery zajmował się optymalizacją wymiarowania belek sprężonych, później skierował swoje zainteresowania na lekkie konstrukcje warstwowe. Jego największe dokonania związane są jednak z wprowadzaniem metod numerycznych do projektowania inżynierskiego i badań naukowych, głównie konstrukcji betonowych, ale również murowych oraz podłoża gruntowego. Opracowany przez Profesora Majewskiego program MAPRET był na początku lat 90-tych ubiegłego wieku synonimem komputeryzacji polskich biur projektowych. Zdobyte doświadczenie Profesor wykorzystał w budowanym od 1995 roku pakiecie programów MAFEM, służącym analizie MES płaskich i przestrzennych zagadnień. Bazuje on na autorskim modelu materiałowym betonu, muru oraz gruntu. W pierwszej wersji program posłużył powstaniu pracy habilitacyjnej Profesora, a jego późniejsze wersje pozwoliły na wypromowanie pięciu doktorów. Pomimo przejścia Profesora na zasłużoną emeryturę program MAFEM jest nadal wykorzystywany i rozbudowywany przez jego wychowanków.

Keywords: **FEM; Numerical modelling; Elasto-plastic material model.**

1. INTRODUCTION

Modeling may be defined as the process of solving physical problems by appropriate simplification of reality. Problem recognition is usually initiated by laboratory and in situ model tests. They are used by scientists to obtain data to develop empirical or semi-empirical algorithms. The construction of a mathematical model with appropriate assumptions is the first step of theoretical modeling. Such a model usually takes the form of algebraic or differential equations. Unfortunately in most engineering problems, these mathematical problems cannot be simply solved analytically. A numerical solution is required preceded by development of an appropriate numerical model. This model should usually be carefully calibrated and validated against earlier collected data and analytical results. Verified model may be implemented for suitable solutions. Finally interpretation of the numerical results in the conventional form of graphics (eg. maps of stresses), charts, tables may be successfully used in engineering design.

There is a long history of empirical modeling in civil engineering. Approximate numerical solutions of mathematical problems are already found in antiquity, and were very popular in ancient India and China. Bases of numerical methods are given in studies of such great mathematicians as Newton, Euler, Lagrange, Gauss, Jacobi, Fourier, Chebyshev. In the late 19th and early 20th century numerical analysis was not recognized as a mathematical discipline and developed to solve the problems arising in geodesy, astronomy, physics and engineering. Visible expansion of modern numerical analysis appeared in the 1940s, as a result of the development of first digital electronic computers. It should be mentioned that before that invention, computations were done by hand, by slide ruler, or other analogical devices. The important person in the development of FEM is Richard Courant, who was the director of the famous Mathematical Institute in Gottingen. In a paper "Variational methods for the solution of problems of equilibrium and vibrations" [1] he demonstrated ideas (mesh discretization of a continuous domain into a set of discrete sub-domains) later defined as finite element method. The final development of FEM began around 1960 in the field of aerospace and civil engineering. The first who recognized the general potential for using the finite element method to resolve problems in civil engineering was professor C.A. Zienkiewicz, who developed tools based on computational mechanics. For his work he received honorary degrees from dozens of countries, among them was degree granted by Silesian University of Technology. Procedure of honoured professor Zienkiewicz was promoted by polish pioneer of finite elements method in concrete mechanics, professor Stanisław Majewski.

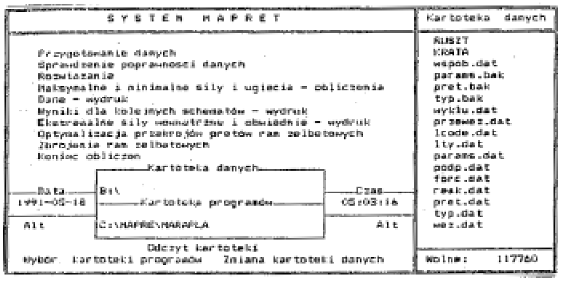
2. JOINING THE STATIC AND STRENGTH ANALYSIS

In 1972, when preparing an opinion about Dr. Majewski, Professor Niewiadomski wrote "Scientific activity of Dr Majewski is characterized by valuable trend of combining academic work with the need of engineering practice". It is an extremely accurate assessment of career development of Professor Majewski, expressed already at the very beginning.

Interests and main directions of activity of Professor Majewski can be divided into three periods. In the first, from the beginning of his work for Silesian University of Technology in 1962 to the mid-seventies after the initial interest in theory of prestressed structures the main direction of his scientific work was focused on structural applications of plastics. Among the achievements of this period can be distinguished a doctoral thesis on the problems of the theory of light structures, several publications, six awarded certificates of copyright for making the invention and four certificates of making utility designs. Since the early seventies, because of a loss of industrial interest in lightweight structures, Professor Majewski moved his main activity into the design, particularly work on a reinforced concrete skeleton for housing. The undoubted advantage of such a focus of interest was in close contact with the design and construction practice executive and the resulting experience useful in teaching. Disadvantages include the need to move away from direct involvement in research and the associated weakening of publishing activities.

Return to scientific activity took place at early eighties. The main direction of interest Professor Majewski sent to a numerical analysis of structures using the finite element method and computer aided design. In a sense, it was still associated with an intense commitment to design and the consequent demand for tools to analyze more complex structural problems. At that time, professor Majewski started his work on set of programs enabling computer-aided design of planar and spatial bar structures, initially written for microcomputer ZX-Spectrum, later rebuild for growing IBM-PC platform. Originally created for his own purposes and completed in cooperation with his son Bohdan and associate Grzegorz Wandzik system called MAPRET was very well received by the industry. At that time it was the only one originally Polish program, which has been linked to the problem of full static analysis and dimensioning. In the early 90s MAPRET was the main structural program used by many design offices in Poland and almost all offices in Upper Silesia. MAPRET [2] enabled considerable, as for possibility of contemporary computers, analyzes of 2D frames (and later also 3D) of any geometrical shape, loaded in any direction at any point, including the temperature and displacement of nodes. Possibilities were limited by number of bars (equal to 850), number of nodes (270, including 200 support nodes), types of loads (100) and possible load schemes (20). Results were presented in a form of:

- tables (displacements of nodes, deflections of bars, internal forces, reactions),
- charts (diagrams of internal forces, deformations, envelopes),
- graphs (sections with specified reinforcement, reinforcement placement at the length of calculated member).



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SYSTEM MAPRET                                              Kartoteka danych

   Przygotowanie danych                                      RUSZT
   Sprawdzenie poprawności danych                            KRATA
   Rozwiazania                                               wspob.dat
   Maksymalne i minimalne sily i ugiecia - obliczenia        params.bak
   Dane - wydruk                                             pret.bak
   Wyniki dla kolejnych schematów - wydruk                   typ.bak
   Ekstremalne sily wewnętrzne i obwiednie - wydruk          wyklu.dat
   Optymalizacja przekrojów pretów ram zelbetowych           przewez.dat
   Zbrojenia ram zelbetowych                                 lcode.dat
   Koniec obliczen                                           lty.dat
                     Kartoteka danych                        params.dat
                                                             podp.dat
   Data          B:\                              Czas       forc.dat
 1991-05-18          Kartoteka programów        05:03:16     reak.dat
                                                             pret.dat
  Alt            C:\MAPRE\MARAPLA                  Alt       typ.dat
                                                             wez.dat
                     Odczyt kartoteki
  Wybor  kartoteki programów   Zmiana kartoteki danych       Wolne:     117760
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Figure 1.
Main menu of the MAPRET system

The experience gained in developing the program for the analysis of bar structures Professor Majewski used in his work on the FEM system for the analysis of solid elements, called MAFEM. At the beginning program was able to analyze 2D problems, while later it was extended on 3D tasks. Program was designed for calculations of macro scale problems considering 3D stress and strain states. As earlier, Professor Majewski did not try to adapt any of the already operating on the market computing processors (such as Ansys), but created an entire program from the scratch. This approach has had some drawbacks, like the possibility to use only rectangular finite elements, but allowed for better integration of material model with the processing module. As a result, it was possible to improve the incremental-iterative procedures that must be used in the analysis of nonlinear problems. Implemented into MAFEM relatively simple elasto-plastic material model of concrete at the beginning was used to solve problems focused on concrete structures, later also on masonry including problems of interaction between structure and subsoil. Developed software Professor Majewski used in his work on habilitation dissertation and later in more than thirty publications and five completed doctoral dissertations on the issue of the computer simulation of the structure.

3. ELASTO-PLASTIC MATERIAL MODEL FOR CONCRETE

In the introduction to the description of the MWW3 elasto-plastic model for concrete, published in the Polish Archives of Civil Engineering [3], Profesor Majewski wrote: "… why another elasto-plastic model while a number of more advanced models had been developed in the last decade? Accepting the last part of this opinion in general let me emphasize that elasto-plastic modeling still can be useful in structural analysis not necessarily as the top achievement in this area, but instead of the linearly elastic approach, which is still very popular there. Dozens of real structural problems, including 3D tasks can be realistically analyzed in terms of elasto-plasticity. The simple definition of material parameters on the basis of laboratory tests is a strong advantage of this approach". Is not this thesis still valid? Most of today, more and more advanced material models of concrete is applicable only in solving problems on a micro-scale. The largest applications commonly used in the structural design are still based on the linear-elastic solution and nothing indicates that this situation was subject to change in the near future.

Implemented in MAFEM system material model was designed mainly to analyse 3D problems in a complex stress state. The crucial issue in the development of the model was the failure criterion. Professor Majewski based his consideration on the three-parameter Willam-Warnke model [4], which uses second order polynomials to describe the shape of boundary surface meridians. Each of the meridian comprised of rectilinear part and parabola tangentially connected with them. Defined in this way tensile meridian of the boundary surface intersects the points corresponding to the concrete uniaxial tensile strength f_t, and uniform, biaxial compression f_{cc}, and the meridian through the point corresponding to the uniaxial compression strength of concrete f_c. In the zone of uniform tensile stress all meridians intersect the axis of the mean stress at the point of triaxial tensile strength f_{ttt}.

In the already cited paper [3] professor Majewski wrote: "The behavior of concrete in a stress state is so complicated that the failure criterion cannot be defined just on the basis of a simple theoretical assumption (e.g. Rankine's assumption about the decisive role of extreme principal stress). A realistic approach must take into account of a wide range of laboratory tests…". Basis for modification of the Willam-Warnke model were his own and taken from the literature results of laboratory tests of concrete under triaxial stress state. Looking for their best straight line approximation with additional requirement, that this line must pass the point corresponding with the biaxial compressive strength of the concrete following formula for tensile merdian was derived:

$$t_o^t = \frac{\tau_{okt}^t}{f_c} = -0{,}470153\frac{\sigma_m}{f_c} + 0{,}184824$$

Analogically, looking for the best straight-line approximation of the results located along the compressive meridian, with the requirement that this line should cross the mean stress axis in this same point sm as the tensile meridian does, leads to formula:

$$t_o^c = \frac{\tau_{okt}^c}{f_c} = -0{,}713363\frac{\sigma_m}{f_c} + 0{,}280434$$

The approximation of concrete strength for both meridians is acceptable for $\sigma_m \leq -2f_{cc}/3$. To improve the accuracy of the model, for bigger mean stresses, closing cap was used, to obtain smooth failure surface, tangent to rectilinear parts of meridians. Location and shape of meridians and their cap are shown in the Figure 2.

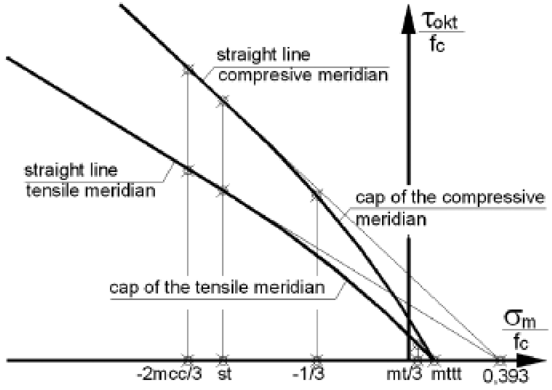

Figure 2.
Meridians of the cap

Location of any meridian between the tensile and compressive one could be calculated according to the assumed shape of the deviatoric section of the failure surface. Two possible shapes were tested in the model MWW: elliptical Willam Warnke (Fig. 3a) and hexagonal Coulomb-Mohr (Fig. 3b). Unquestionable advantage of the first is its smoothness, but in some investigations the second one gives better approximation.

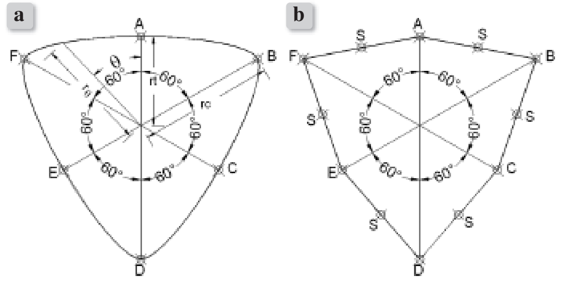

Figure 3.
Deviatoric sections of the failure surface: a) elliptical Willam-Warnke b) hexagonal Coulomb-Mohr approximation

For the initial stresses $\sigma_m \leq e_{lim} f_c$ model adopts a linear stress-strain relation. Over this limit only plastic deformation occurs. Zones of plastic and elastic deformations are separated by the yield surface. Its initial shape is determined by the parameter e_{lim} treated as the scale factor. Elasticity limit is expressed in terms of the uniaxial compressive strength f_c:

$$e_{lim} = 1 - \exp(-f_c/80)$$

Evolution of the yield surface hardening or softening is described by non-dimensional yield functions y_v, y_i where function y_v depends on the sum of increments of the plastic part of volumetric and deviatoric strain, while function y_i depends only on the plastic part of the deviatoric strain.

To enable irreversible strains on stress paths running below the critical state line (hydrostatic pressure), a closing cap was proposed to close the yield surface on the compressive side. To achieve the smoothness of the surface circular meridians of the closing cap are tangent to the appropriate rectilinear meridians and they are passing coordinate m_{ccc}. That point indicates the non-dimensional hydrostatic pressure limit of elastic volumetric strain and when hydrostatic pressure exceeds this value, plastic volumetric strain appears.

Considering the isotropic hardening rule the equation of the yield surface is derived from the equation of the yield surface. Formulas describing the yield surface in deviatoric cross section are functions of Lode's angle.

4. APPLICATIONS OF THE MAFEM

4.1. Thermal shrinkage stresses in early age massive concrete – Prof. Barbara Klemczak

The elasto-plastic material model developed for mature concrete has been adapted for the early age

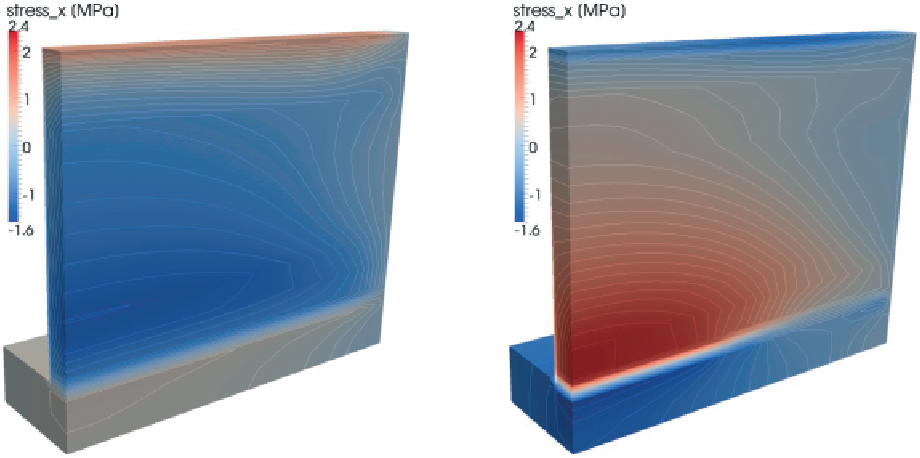

Figure 4.
The map of stress distribution in t = 52 h (left) and in t = 340 h (right) in exemplary ¼ of bridge abutment [10]

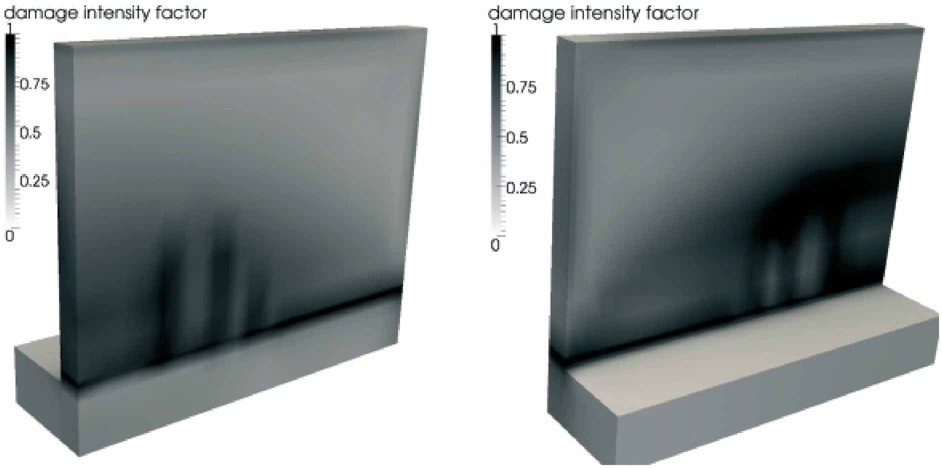

Figure 5.
Damage intensity maps after 20 days in the interior (left) and on the surface (right) of an exemplary ¼ of bridge abutment [10]

concrete. The original model was enriched with elements characteristic for an early age concrete, as well as for the thermal phenomena observed in massive concrete structures. The aging of concrete was considered and additionally the viscous effects were introduced to the model. The applied 3D model allows to recognize stress state and possible damage of the structure during the whole time of concrete curing (Fig. 4, Fig. 5). The model was successfully applied in the analysis of early age thermal and shrinkage stresses formed in massive foundation blocks and slabs as well in reinforced concrete walls cast against an old set foundation [5, 6, 7, 8, 9].

4.2. Punching of RC slab – Dr. Grzegorz Wandzik

Application of the MAFEM into the analyses of punching in a RC slab supported on a column [11]. Elasto-plastic model was used for 3D simulation of some typical tests performed in laboratory conditions. Rather dense FE mesh made possible accurate analysis of stress state in the connection. This type of joint is characterized by complex stress state, where high 3D compression/tension is combined with shear. Combination of high multiaxial compression and shear stresses compared with failure criteria enabled achievement of satisfactory results in prediction of load capacity and deformation of slab adjacent to the column. Obtained results proved the accurateness of elasto-plastic material model for estimation of RC

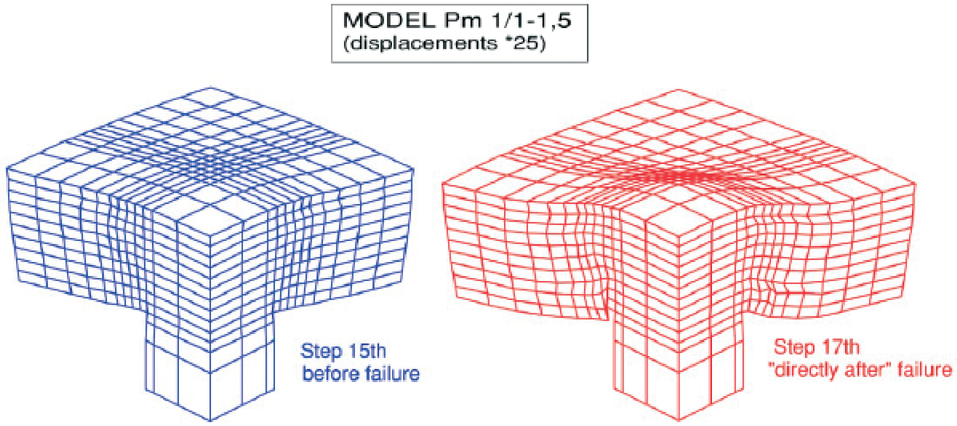

Figure 6.
Deformation of RC slab-column connection in 3D FEM analysis [11]

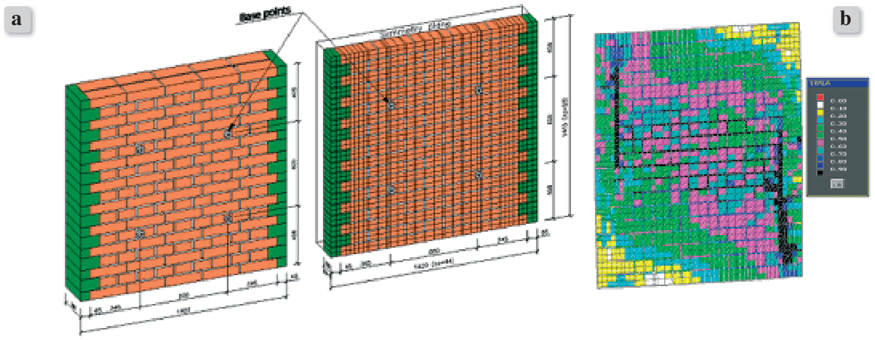

Figure 7.
Model for shear tests: a) lab and FE specimen, b) directions of main tensile stress and effort level in FE for model [13]

elements ultimate load capacity in complex 3D stress state. As a result of numerical analysis, two-stage method was proposed [12].

4.3. Numerical analyses of soil-structure interaction – Prof. Leszek Szojda

A numerical analysis of complex stress state was provided on the basis of compression and shear tests of wall specimens. Necessary parameters of boundary surface have been determined in accompanying laboratory tests. Results of numerical compression and shear analyses were compared with findings of laboratory tests and proved the model accuracy, especially on strength under compression, deformation and cracking. These initially numerical tests were used for analyses of soil-structure interaction of whole dwelling house building under mining subsidence.

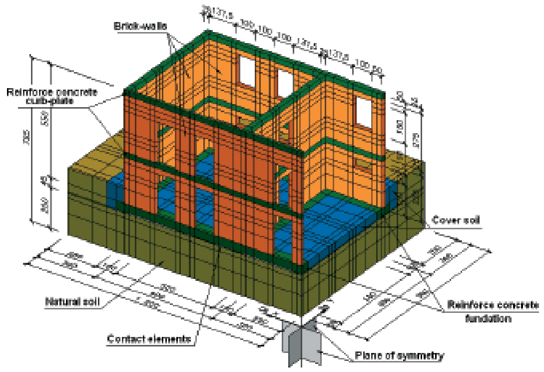

Figure 8.
Numerical model of soil-structure interaction [13]

4.4. Effect of combined 3D stress state on behavior of RC beams – Dr. Rafał Krzywoń

Program MAFEM has been used in the numerical calculations of beam and column elements under combined flexure, compression, shear and torsion [14]. Changes of flexural stiffness for full spectrum of load were analysed. Statistical study allowed to create mathematical procedures, helpful to determine flexural stiffness characteristic.

Some trials to adapt the program in the analysis of foam concrete sandwich foundation slabs were done [15].

4.5. Devlopment, calibration and verification of elastic-plastic model of geological materials – Dr. Małgorzata Orzechowska

The aim of the work [16] was to modify the material model, to allow realistic analysis of building structures, loaded with wide range of strain rates or subjected to high levels of long-term stress, which over time can lead to the destruction of the material. As a basis for modification, visco-elastic visco-plastic model created earlier by Klemczak has been used. The verification has given good results in terms of both quality and quantity, which confirmed the usefulness of the model in the analysis of structures subjected to long-term load or load applied at different strain rates.

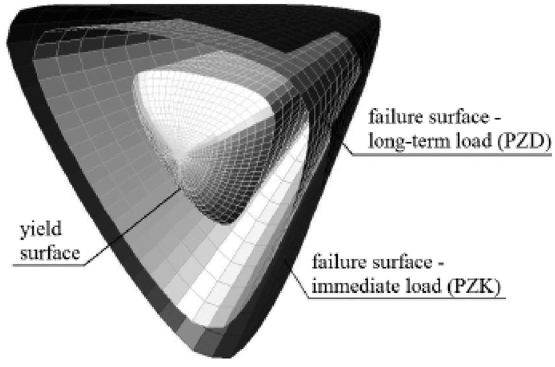

Figure 9.
3D view of boundary surfaces [17]

5. WORD OF GRATITUDE

At this point, we would like to thank Professor Majewski for undertaking the promotion of our dissertations, but mainly for the fact that he showed and still shows what it means to be a good person, with a smile to go through life, bestow other respect.

We are convinced, that opinion about uniqueness of Professor's abilities and his respect to everyone, is shared not only by all his alumni, but by all the people, who had the pleasure to get to know the Professor.

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