NUMERICAL MODELLING OF THE IMPACT OF UNDERGROUND MINING ON PIPELINES. PART I – IMPACT OF CONTINUOUS DEFORMATIONS

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Abstract

Underground infrastructure systems located on the areas of mining impact are exposed to additional impacts related to continuous land deformation resulting from underground exploitation. Such impacts increase the failure rates of the systems. The work discusses the impact of respective deformation indicators on various types of systems, taking into account diverse material solutions for pipelines. The assumptions, scope and results of a 3D numerical analysis of a pipe-soil system model are also presented (Z_Soil acad. ver. 11.03). The analysis is considering a case where a piping is situated parallel to and perpendicular to the front of mining exploitation.

Streszczenie

Sieci infrastruktury podziemnej zlokalizowane w rejonach wpływów górniczych narażone są na dodatkowe oddziaływania, związane z ciągłą deformacją terenu, będącą skutkiem podziemnej eksploatacji. Oddziaływania te wpływają na wzrost awaryjności sieci. W pracy omówiono wpływ poszczególnych wskaźników deformacji na różne rodzaje sieci, z uwzględnieniem zróżnicowanych rozwiązań materiałowych rurociągów. Przedstawiono także założenia, zakres i wyniki numerycznej analizy modelu układu rura-grunt (Z_Soil acad. ver. 11.03). W analizie tej rozpatrzono przypadek usytuowania rurociągu równolegle oraz prostopadle do frontu eksploatacji górniczej.

Keywords: **Continuous mining deformations; Pipelines; FEM.**

1. INTRODUCTION

Extensive water supply systems, sewage collection and treatment systems, gas systems and district heating systems have to be constructed in order for a contemporary urban agglomeration to function. Their efficient and safe operation is conditioning the commonly expected living standards of population as well as the appropriate functioning of industrial facilities [4]. In the case of technical infrastructure located on mining areas (GZW, LGOM), the issue of correct designing of underground systems and their construction as well as adequate operation requires collaboration between the system designers and operators and mining companies. The effects of intensive mining, often occurring multiple times, have a destructive effect on lines, fittings and accompanying structures, especially that such systems' technical conditions as well as the materials and protection systems applied for receiving the effect of underground mining, vary greatly. The scale of this phenomenon is signified by elevated failure rates of the systems [5], [7], [9]. Characteristic damages to the systems have been observed (Fig. 1) such as the cracks of steel pipes at welded points and along the pipe side surface, leakage of expansion joints, collapse of pipes and destruction of their insulation, damages of fittings, damage of slide skids on the supports of heat distribution pipes, or, at last but not least, – in case of sewers – a changed grade line of sewers, bent sewage pipes or subsidence of wells.

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b

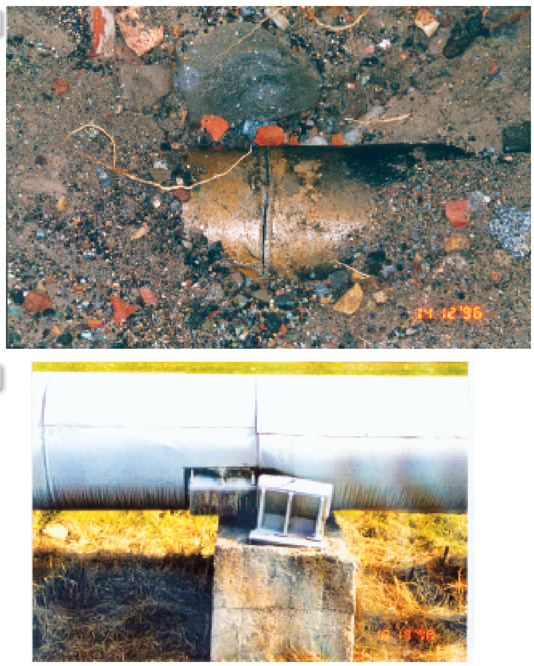

Figure 1.
Examples of pipeline damages: a) cracks of steel pipe at welded points, b) damage of skids and protective jacket of heat distribution pipe (*own archive*)

Underground mining exploitation causes changes in the rock mass resulting in land surface deformations such as extensive subsidence basins (continuous deformations) or local collapse, cones, thresholds or crevices (discontinuous deformations) [15]. They can also disrupt water conditions and cause mining tremors. This paper is analysing the impact of underground continuous exploitation on buried pipelines, and the other part of this article will analyse the functioning of pipelines in the area of discontinuous deformations.

2. DESCRIPTION OF MATERIAL SOLUTIONS FOR BURIED PIPELINES

The primary material solutions used in buried pipelines include metals, concrete, vitrified clay, a wide range of plastics (PVC, PE, PP) and glass-reinforced thermosetting plastics pipes (GRP) [16]. The specific material solution should be so selected as to suit the functional and operational conditions of a pipeline, as well as the conditions of laying a pipeline in the ground, anticipated loads and the method of laying a pipe in the ground (with or without excavation).

The adopted material solution is of fundamental importance for the interaction between the pipeline and the ground, hence for the method of transmitting loads acting on the buried pipeline [14], [17]. Inflexible pipelines made of traditional materials (concrete, reinforced concrete, vitrified clay), with very thick walls, are transmitting the loads acting onto them as independent static systems without the deformation of the cross section of the pipe. On the other hand, plastic or thin-walled steel pipelines are classified as flexible pipelines and are deforming as a result of the acting loads and represent, together with the surrounding soil, an interworking system. The degree of pipeline ovalisation is dependent on the type of soil surrounding the pipe and its rigidity and compaction ratio. The behaviour of inflexible and flexible pipelines in soil is also associated with their dimensioning criteria. For inflexible pipelines, the criterion includes allowable stresses or rupture force, and for flexible pipelines – relative, permitted bending of the pipe and its buckling.

3. THE IMPACT OF CONTINUOUS LAND DEFORMATIONS ON BURIED PIPELINES

Subsidence basins formed as a result of the conducted mining works are usually described with the Budryk-Knothe theory [3], [13], the main equation of which (in a two-dimensional approach) is the exponential function of the depression basin profile in the form of [13]:

$$w(x) = \frac{w_{max}}{r}\int_{x}^{\infty} \exp\left(-\frac{\pi}{r^2}x^2\right)dx \qquad (1)$$

where:

w_{max} – maximum land depression, $w_{max} = -a\,g$ [15],

a – coefficient of exploitation,

g – thickness of bed,

r – range radius of main impacts, $r = H/tg\beta$,

H – bed deposition depth,

β - angle of range of main impacts.

The other deformation indicators – horizontal displacement $u(x)$, inclination $T(x)$, vertical curvature $K(x)$ and horizontal strain $\varepsilon(x)$ – are the relevant derivatives of the function $w(x)$ or, in the case of horizontal strains, the derivative of the function $u(x)$.

The impact of the relevant mining deformation parameters on the effort state of the pipeline and on changes in the functional conditions of the system is diversified [12], [19]. Land depressions and inclinations are disrupting functional conditions (changes in the slopes of sections of gravity lines, changes in the situation of the lowest and highest points in the system) without an additional effort state of the pipelines. Horizontal deformations and vertical curvatures increase loads acting on the pipeline, and in turn change the stress state of the pipe walls. The influence of a curvature is considered important for pipes with a larger diameter (above 300 mm).

4. THE CONTEMPORARY COMPUTATIONAL METHODS OF PIPELINES IN CONSIDERATION OF THE MINING IMPACT

The Scandinavian method [10] and the method given in German ATV-DVWK-A127 [1] guidelines can be considered the most popular modern computational methods for buried pipelines taking account of pipeline inflexibility and flexibility. These are analytical methods where a pipe ring is considered in the plane state of strain, loaded with the weight of soil and with the surcharge's load. Such methods do not take into account the effects of underground exploitation on pipelines. The manner in which the additional effort state of pipelines is considered on mining areas can be found in specialist literature in the field of construction engineering on mining areas [8], [11], [15], [18] where cases of parallel and perpendicular pipeline situation in relation to the exploitation front are considered. The effect of horizontal deformations ε, of the tensile or compressive nature, on a flat, rigid or deformable pipe ring is considered in the first case. Additional loads related to soil loosening or compaction cause, as appropriate, reduction ($-\Delta p_h$) or growth ($+\Delta p_h$) of horizontal loads p_h related to vertical loads p_v (weight of the ground and surcharge's load). They are described with the following formulae for rigid pipes:

$$+\Delta p_h = 160 \cdot \varepsilon \cdot p_v \tag{2}$$

$$-\Delta p_h = 0.2 \cdot p_v \tag{3}$$

The analogous formulae for flexible pipes are as follows:

$$+\Delta p_h = 120 \cdot \varepsilon \cdot p_v \tag{4}$$

$$-\Delta p_h = 20 \cdot \varepsilon \cdot p_v \tag{5}$$

The other case considered refers to a situation where a pipeline is laid perpendicular to the front of exploitation. The impact of horizontal deformations or/and the impact of a vertical curvature in the longitudinal direction are then considered. The longitudinal force N_ε is created due to soil creeping and the activation of static stresses on the side surface of a straight section of a pipe with the length l, and such force is summing up with longitudinal forces in the pipeline (e.g. coming from internal pressure or thermal effects). The maximum value of this force existing in the centre of the considered pipe segment is determined with the formula:

$$N_\varepsilon = \pi \cdot D_z \cdot \tau \cdot \frac{l}{2} \tag{6}$$

The occurrence of a vertical curvature is associated with the bending of the pipeline in the vertical plane, whereas the bending moment in the longitudinal direction is:

$$M = K \cdot EI \tag{7}$$

where EI is pipeline rigidity.

Detailed information concerning the determination of longitudinal forces and bending moments caused by horizontal deformations and a vertical curvature in rigid and flexible pipelines is given in [15], [18].

5. NUMERICAL MODELLING OF THE IMPACT OF MINING DEFORMATION ON PIPELINES

The presented numerical analysis of the impact of mining deformation of land was carried out with Z_Soil acad. ver. 11.03 software. A model of the pipeline – soil system according to a discrete approach is the subject of the analysis. A rigid concrete pipe and a PVC flexible pipe was analysed in variants. The both pipes were modelled according to their flexibility, which is often used in case of constructional material. A large-area elastic-plastic model was applied for soil modelling with Hardening Soil – Small isotropic strengthening [2], [21], [22], available in Z_Soil software. The model is representing the relationship between soil stiffness and such factors as the state of effective stresses, plastic flow and changes in volume during plastic flow and changes in the stiffness together with a rising amplitude of deviator strain for small strains. The analysis programme includes the modelling of the effect of horizontal deformations on an inflexible and flexible pipeline laid parallel to the front of exploitation (2D type models) and modelling the effect of overall interaction of horizontal deformations and a vertical curvature (3D type model) on a rigid pipeline laid perpendicular to the front of exploitation.

5.1. Pipeline laid parallel to the front of exploitation

Characteristic of the 2D model of the pipeline – soil system (Fig. 2):

The model is representing the pipeline in homogenous soil at the depth of 2.0 m. The pipeline diameter is ϕ500 mm, the thickness or walls: 19.1 mm (PVC-U pipe) and alternatively 75 mm (concrete pipe). The soil mass dimensions: 6 m x 4 m. The number of nodes: 503, the number of *quad* type elements (soil mass): 448, the number of *beam* type elements (pipe): 28, the number of *Boundary Conditions* supports: 57. Loads on the pipeline represent a uniformly distributed load of the surcharge of 100 kN/m^2 (t = 0 to t = 5) and horizontal deformations of, respectively, compressive and tensile nature, with the extreme intensiveness of ε_{max} = ±9 mm/m (t = 5 to t = 185), corresponding to the category V of the mining area. The time intervals used in the analysis are of the conventional character, they reflect a difference in a relatively rapid activity of external loads and a significantly slower activity of horizontal soil deformations. Their activity was simulated by introducing kinematic excitations in the supports with the value of $\Delta_i = \varepsilon_{max} \cdot x_i$, where x_i is the distance of the i-th support to the model's axis of symmetry.

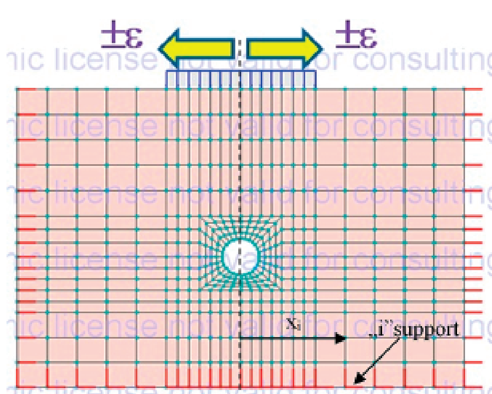

Figure 2.
View of the 2D numerical model of pipe-soil system

Multiple specific material soil parameters have to be introduced into the analysis of the constitutive soil model HS-Small. Values of some of them are given below:

Young's modulus odc./obc. E_{ur}^{ref} = 21 MPa, tangential Young's modulus E_0^{ref} = 41.5 MPa, model parameters: H = 9.13 MPa, M = 0.861, internal friction angle ϕ = 24°, cohesion c = 10 kPa, OCR = 3.5.

The following material parameters of the analysed pipelines were assumed:

- concrete pipe: Young's modulus E = 30000 MPa, bulk density γ = 24 kN/m^3, Poisson's ratio ν = 0.2.
- PVC pipe: Young's modulus E = 4000 MPa, bulk density γ = 14 kN/m^3, Poisson's ratio ν = 0.4.

The selected analysis results of the pipe-soil system model are presented as displacement maps (Fig. 3). The acting surcharge's load and horizontal deformations of the compressive or tensile nature are marked in the figure schematically.

The acting surcharge load causes a local deformation of land surface (the maximum vertical displacement value at t = 5 is -0.048 m). The deformation is increasing as horizontal deformations are growing of the tensile character, causing soil loosening (the maximum land depression at t = 185 is -0.12 m). The activity of horizontal deformations of the compressive nature causes land compaction (the maximum land depression at t = 185 is -0.042 m). The activity of horizontal deformations of the compressive nature causes gradual reduction of the soil mass width (the maximum value of horizontal displacements is -0.027 m), and in the case of acting deformations of the tensile character, the soil mass width is increasing (the maximum value of horizontal displacements is +0.027 m).

One can observe on the basis of an analysis of vertical and horizontal distributions of displacements shown in Fig. 3 that a rigid, concrete pipe does not deform during the activity of a surcharge load and the growth of deformations of the compressive or tensile character. The pipe is then displacing entirely and, as a rigid inclusion, causes only disrupted distribution of land displacements in its surrounding. The value of its vertical displacements caused by a surcharge load is -0.0057 m, and it rises to -0.032 m during the activity of horizontal deformations of the tensile character. Horizontal displacements of the compressive character do not cause significant displacements of the pipe.

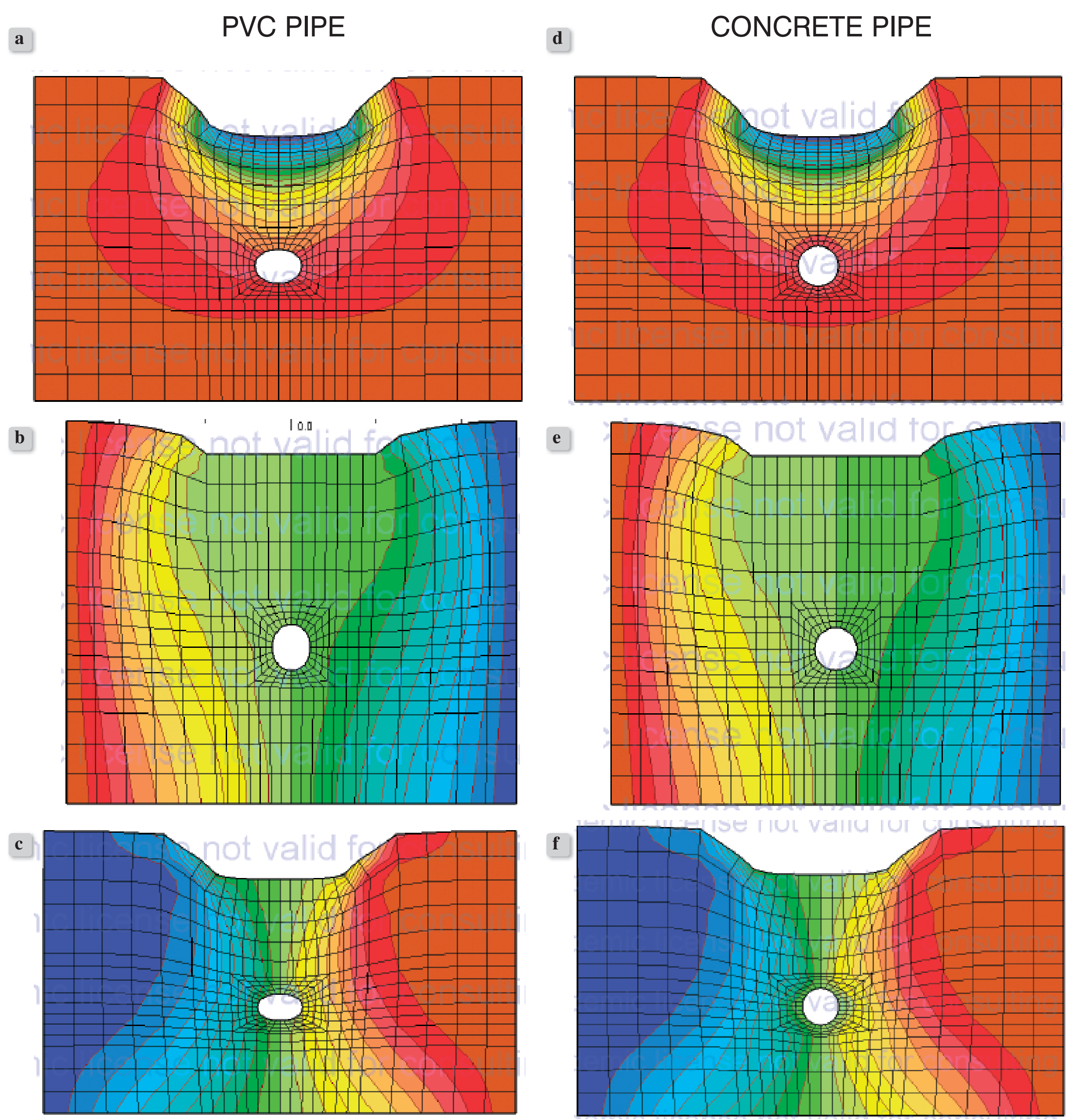

Figure 3.
Maps of displacements (t = 185): a) PVC pipe – the map of vertical displacements (acting surcharge load), b) PVC pipe – the map of horizontal displacements (the effect of horizontal deformations of compressive character), c) PVC pipe – the map of horizontal displacements (the effect of horizontal deformations of tensile character), d) concrete pipe – the effect of vertical deformations (acting surcharge load), e) concrete pipe – the effect of horizontal deformations (the effect of horizontal deformations of tensile character)

The maps of displacements (Fig. 3) illustrate, however, clear changes in the lateral shape of the cross section of the PVC-U flexible pipe which, under the influence of an acting surcharge load, is ovalising along the horizontal axis, and is then ovalising vertically during the activity of horizontal deformations of the compressive nature. Soil loosening (activity of horizontal deformations of the tensile character) causes on the other hand growth in horizontal ovalisation of a PVC-U pipe.

The progress described above of the process of changing the PVC-U pipe shape during a simulation of an acting surcharge load (the first 5 calculation steps) and of horizontal deformations of the compressive and tensile character (the next 180 calculation steps) is presented by means of diagrams of displacements of the four main points of PVC pipe (Fig. 4) situated in the cross section at the intersection of the vertical and horizontal axis with the pipe contour (397 – highest point, 404 – lowest point, 431 – side point on the right side, 458 – side point on the left side).

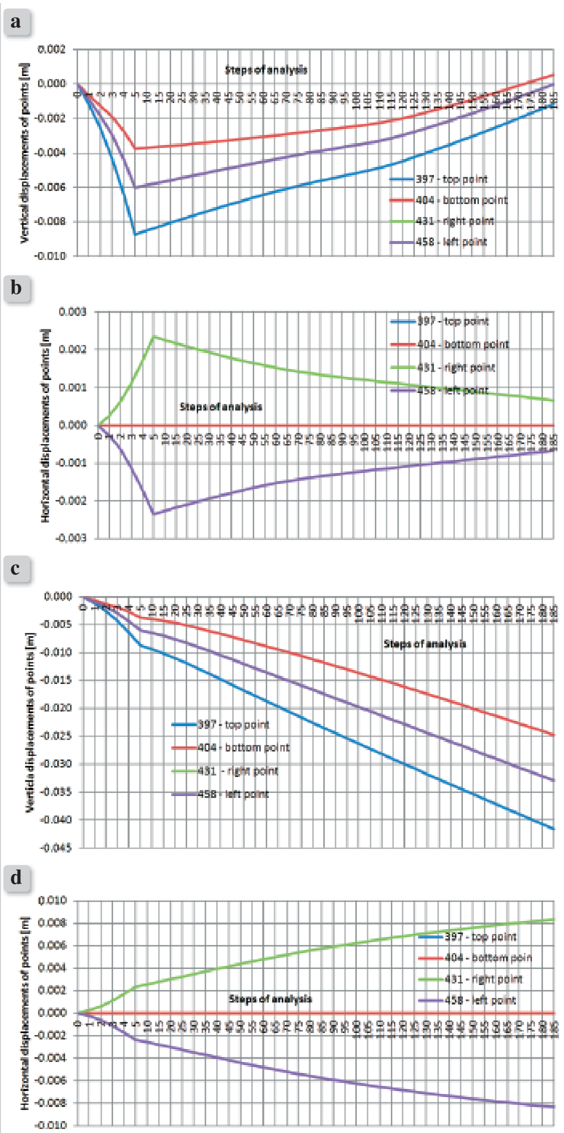

Figure 4.
Diagrams of displacements of the main points of PVC-U pipe: a) vertical displacements – the effect of horizontal deformations of compressive character, b) horizontal displacements – the effect of horizontal deformations of compressive character, c) vertical displacements – the effect of horizontal deformations of tensile character, d) horizontal displacements – the effect of horizontal deformations of tensile character

5.2. Pipeline laid perpendicular to the front of exploitation

<u>Characteristic of the 3D model of the pipeline – soil system (Fig. 5):</u>

The model is representing a section of a concrete 30 m long pipeline with the diameter of ϕ500 mm laid in homogenous soil at the depth of 2.0 m. The thickness of the pipeline wall is 75 mm. The number of nodes: 16 431, the number of *Continuum* type elements (soil mass): 13 440, the number of Shell type elements (pipe): 840, the number of *Boundary Conditions* supports: 2 827. The soil mass dimensions: 6 m x 4 m x 30 m. Loads on the pipeline are represented by a uniformly distributed load of the surcharge of 100 kN/m^2 (t = 0 to t = 5), horizontal tensile deformations with the intensiveness of ε_{max} = ±9 mm/m and a horizontal curvature with the radius of R = 4 km (t = 5 do t = 185), corresponding to the category V of the mining area. In the numerical model, the effects are represented by vertical and horizontal kinematic excitations applied appropriately in the nodes located on the lower and all vertical, external planes of the model. The load was applied non-symmetrically in relation to the longitudinal axis of the pipe in order to illustrate possibilities of considering any situation

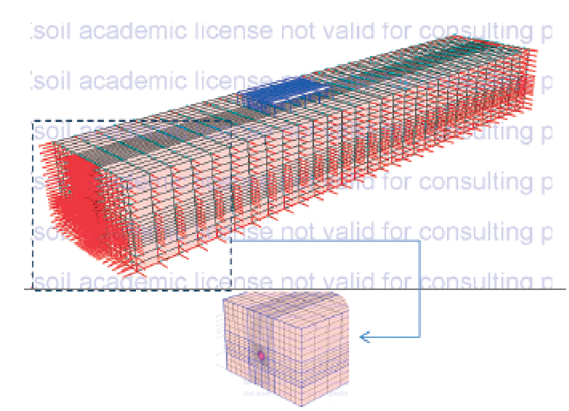

Figure 5.
View of the numerical model 3D of pipeline-soil system

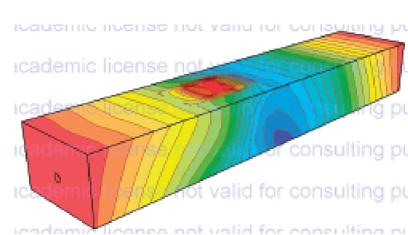

Figure 6.
Maps of resultant displacements in the soil mass caused by the activity of a surcharge load, horizontal deformations of the tensile character and mining land curvature (t = 185)

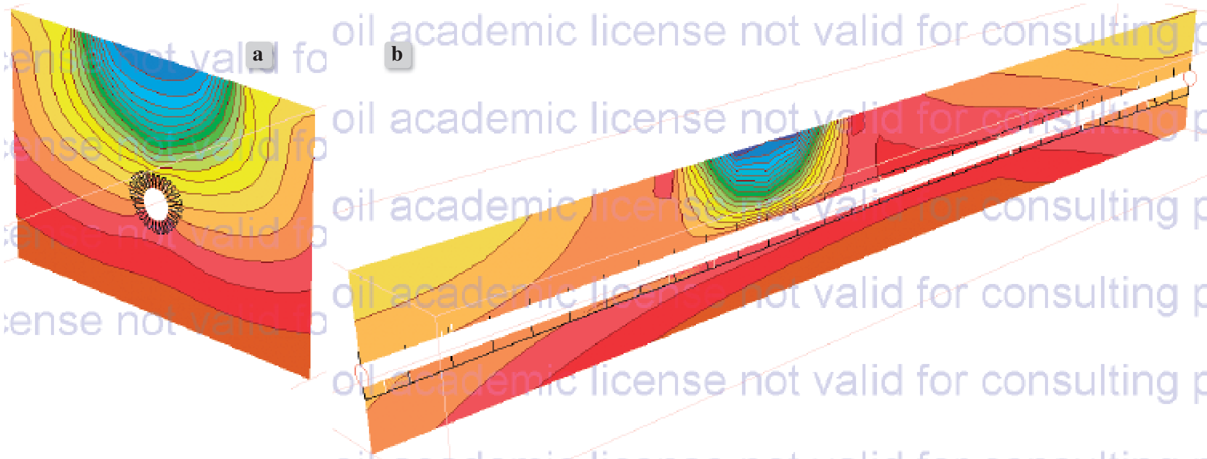

Figure 7.
Maps of vertical displacements in the soil mass caused by the activity of a surcharge load, horizontal deformations of the tensile character and mining land curvature (t = 185): a) in cross section in the activity area of surcharge's load (z = 11.0 m), b) in cross section along the pipe axis

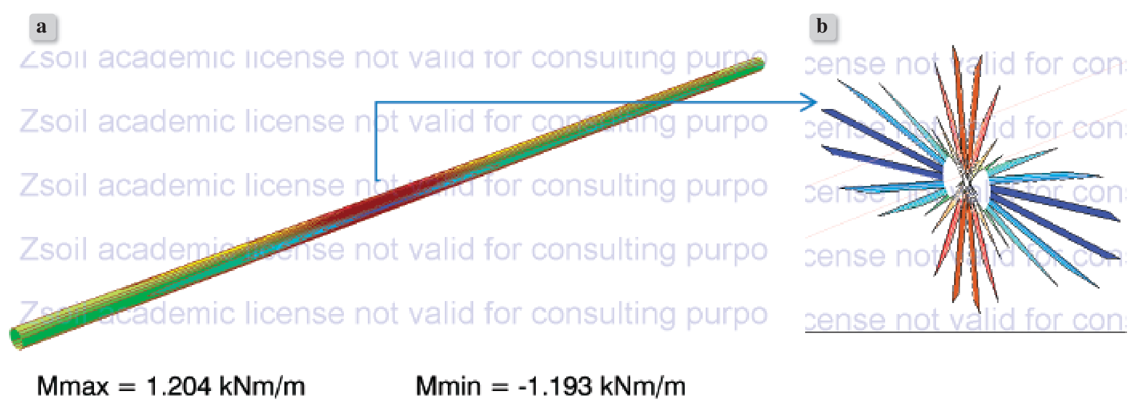

Figure 8.
Distribution of circumferential bending moments (t = 185) caused by the activity of a surcharge load, horizontal deformations of the tensile character and mining land curvature: a) distribution map of circumferential bending moments in the pipe coating, b) diagram of circumferential bending moments in cross section in the activity area of surcharge's load (z = 11.0 m)

of a surcharge load in a 3D model. The material parameters of the pipe and soil are identical as in a 2D model (item 5.1).

The results of the 3D analysis made in Z_Soil software are very comprehensive, they may illustrate the distribution of displacements, stresses and strains of the soil mass and pipe coating. The software also enables to generate diagrams of generalised internal forces in characteristic sections along and perpendicular to the pipe axis. The selected results of the analysis are provided below showing the distribution of displacements in the soil mass caused by the activity of a surcharge load, horizontal deformations of the tensile character and a vertical land curvature (Fig. 6), maps of displacements in the soil mass in characteristic sections – a lateral and longitudinal section (Fig. 7) and distribution maps in the coating and diagrams of circumferential (Fig. 8) and longitudinal (Fig. 9) bending moments in the pipeline (Fig. 8). The listed distributions of displacements and maps and diagrams of bending moments refer to the last stage of the analysis (t =185).

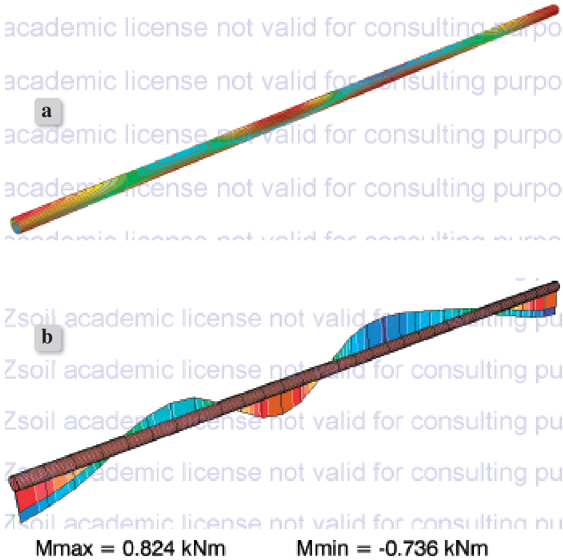

Figure 9.
Distribution of longitudinal bending moments along the pipe axis (t = 185) caused by the activity of a surcharge load, horizontal deformations of the tensile character and mining land curvature:
a) distribution map of longitudinal bending moments in the pipe coating, b) diagram of longitudinal bending moments in the cross section along the pipe axis

Due to the acting surcharge's load (t = 5), the soil mass surface undergoes spatial deformation, and the maximum depression then is 0.047 m. The depressions rise to 0.145 m in the last stage of the analysis (t = 185), in which horizontal deformations of the tensile character and a convex land curvature is acting. This signifies soil loosening caused by mining impacts. The distribution of displacements in the soil (Fig. 7b) illustrates a considerable effect of mining impacts on the soil mass deformation. The non-uniform effort state of the pipeline coating in the circumferential and longitudinal direction is the result of loading the upper surface of soil mass and the changes taking place in the soil as mining impacts are growing (Fig. 8 and 9).

6. SUMMARY

The effort state of buried pipelines laid in the area where mining impacts occur are estimated approximately by means of classical computational methods. The ground space is represented by a system of vertical and horizontal pressures, whereas the spatial pipe construction is most often analysed in a selected cross section as a pipe ring in the flat state of strain or as a bar. This issue can be captured more realistically by employing the FEM method for constructing a pipeline – soil system model and by performing a numerical analysis of this system [6], [20]. The effects of horizontal deformations and/or a vertical curvature are represented in such analyses by kinematic excitations, applied in the external nodes of the model, while the whole analysis reflects the curve of deformation growth over time. The use of computer software (e.g. Z_Soil) in FEM analyses with incorporated advanced constitutive models of soil (e.g. HS-Small) enables to analyse the processes taking place in the ground space and in pipe coating (maps of deformations, displacements, stresses and strains). An example of such an analysis is presented in the article and two characteristics positions of a rigid and flexible pipeline relative to the front of exploitation are analysed. The comprehensive results of the analyses are presented in sections as graphics for the purpose of a qualitative analysis. The outcomes of the analysis are presented in sections as graphics, primarily to illustrate qualitatively the phenomena occurring in the ground and in the pipe coating as loads and mining impacts are acting.

The maps of displacements (2D model) show a varied reaction of flexible and rigid pipelines on the acting surcharge's load and horizontal deformations of the compressive or tensile nature (a pipeline laid parallel to the front of exploitation). A progressing deformation (ovalisation) of a flexible pipeline and the displacement of a rigid pipeline without deformation is visible. The spatial deformation of the soil mass and pipeline and its effort state (3D model) is the effect of the activity of a surcharge's load, horizontal deformations of the tensile nature and a vertical land curvature. A strongly non-uniform distribution of ground particles' displacement signifies that the soil mass has adapted to the kinematic excitations applied at its peripheries, representing horizontal deformations of the tensile character and a convex land curvature.

The stress state in the coating of the analysed pipe is caused by the acting bending moments and axial forces in the circumferential and longitudinal direction. A 3D analysis allows to determine the stress state in the pipeline coating caused by the aggregate activity of the surcharge load, situated in any way in relation to the pipeline axis and the simultaneous activity of horizontal deformations and mining land curvature, which is undoubtedly more realistic. This is not feasible in analytical methods. The particular effects are analysed separately and the results are superimposed.

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