# STRENGTHENING OF COMPRESSION MEMBERS OF A LATTICE GIRDER

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Abstract

**The paper presents the method of calculation of the critical load of the axially compressed, closely spaced, built-up bar composed of three branches, made of cold-formed steel sections. Main branch with rectangular hollow cross-section is connected to two additional branches with open channel cross-section by unilateral BOM-type bolts. Flexibility of these types of joints does not provide the full participation of strengthening additional branches in the transmitting the load, affecting the value of buckling resistance of the built-up bar. The proposed analytical model permits to state the value of the critical buckling load of the considered bar, according to Euler. The Mathematica system has been applied to solve the task.**

Streszczenie

**Przedstawiono metodę obliczania obciążenia krytycznego ściskanego osiowo, bliskogałęziowego pręta złożonego z trzech gałęzi, wykonanych z kształtowników giętych na zimno. Gałąź główna o przekroju z rury prostokątnej połączona jest z dwoma dodatkowymi gałęziami o przekrojach otwartych ceowych, za pomocą łączników jednostronnych typu BOM. Podatność tego typu połączeń nie zapewnia pełnego udziału dodatkowych gałęzi wzmacniających w przenoszeniu obciążenia, wpływając na wartość obciążenia krytycznego pręta złożonego. Zaproponowany model analityczny pozwala określić wartość obciążenia krytycznego wyboczenia sprężystego wg Eulera dla analizowanego pręta. Do rozwiązania zadania użyto systemu obliczeniowego Mathematica.**

Keywords: **Unilateral bolts; Strengthening of steel structures; Critical buckling load according to Euler.**

## 1. GENESIS OF THE PROBLEM

During the exploitation of steel structures, they often need to be strengthened. It is usually caused by a change of live loads, but sometimes it result form design errors, consisting in the improper statement or incorrect combination of the loads. Indeed, very often designers do not follow principles, resulting from the analysis of influence lines. Especially it concerns brace members in steel lattice girders, loaded in an asymmetrical way with variable actions. Then, compressive forces can appear in the bars originally designed to tension.

The enlargement of the buckling resistance of such compressed bar, can be achieved through the use of additional branches-straps – increasing values of the radius of gyration of the strengthened bar – connected longitudinally with strengthened element between supporting points [1]. Then the additional branches don't need to reach supporting nodes, provided that the conditions of the carrying capacity of cross-sections of the strengthened bar near to nodes, are satisfied.

In every case, strengthening of the existing building structure is a difficult logistical and technical problem, because strengthening works should be performed quickly and safely. Usually it is not possible to interrupt the technological process, and in most cases it is unacceptable to conduct welding work.

In this case, it is preferable to use the modern systems of blind bolts, enabling rapid assembly of the construction members at the building site [2, 3]. Flexibility of these types of joints does not provide the full participation of strengthening branches in the transfer of the load, and consequently it is necessary to accurately recognize their cooperation with the strengthened element.

Our aim is to provide a method for estimating the level of that cooperation. It also proposed a method for calculating the critical buckling load of the closely spaced built-up axially compressed bar, according to the Euler, in which strengthened and strengthening branches are flexibly interconnected between supporting nodes by means of BOM-type blind bolts.

## 2. THE DESIGN ISSUE

There is considered the problem of lightweight steel lattice girder with parallel chords and brace members eccentrically connected in the nodes (Fig. 1). In order to preserve the same geometry of the nodes, all brace members have been designed with the hollow rectangular sections RP 100/50×4, with a steel S235JR. To keep conditions of the carrying capacity of the most of compressed brace members, it was necessary to strengthen them with the straps made of cold-formed steel sections U 40/80/40×4 (compare Fig. 1). The walls of adjacent elements have been connected by unilateral BOM-R16-4 bolts [2, 3]. The rotational stiffness of the supporting nodes of the brace members was neglected in the consideration – the pinned connections have been adopted in the nodes.

In order to determine the effectiveness of the strengthening, numerical model was applied. The numerical model permits to state, respectively: values of the shear forces acting on bolts on the length of the strengthening and the values of the axial forces carried by individual branches of the built-up member. Then, the method of determining of the critical buckling load according to Euler was proposed.

## 3. THE COMPUTATIONAL MODEL

An analytical solution of the problem of the distribution of the total load $N$ over three cooperation elements (branches) in the built-up member, is a set of $m$ nonlinear equations, which permits to state the values of the shearing forces acting on individual bolts $N_i$ ($i = 1, 2,..., m$) in the $i$-th level of the joint, according to [4, 5], in the form:

$$f\left(\frac{N_i}{r}\right) = i\frac{N \cdot a}{E \cdot A} - \frac{2a}{E}\left(\frac{1}{A} + \frac{1}{\Delta A}\right)\left(\sum_{k=1}^{i} N_k \cdot k + i\sum_{k=i+1}^{m} N_k\right) \quad (1)$$

where:

$A$ – cross-sectional area of the strengthened part of the cross-section,

$\Delta A$ – cross-sectional area of the strengthening part of the cross-section,

$E$ – Young's modulus,

$N$ – axial compressive force acting on element with cross-sectional area $A$,

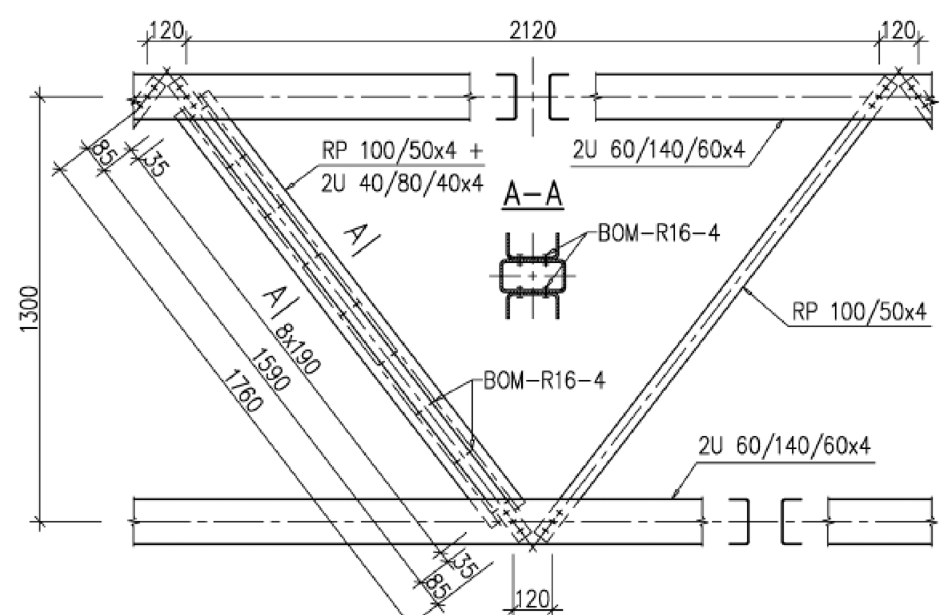


**Figure 1.**
**Fragment of lightweight lattice girder with reinforcement of compressive brace member**

$N_i$, $N_k$ – load exerted on the $i$-th and $k$-th level in the connection of the strengthened section with the one strengthening branch,

$a = L_i$ ($i$=1÷$m$) – spacing between connections along the built-up element (Fig. 2),

$k$ – index: 1, 2, 3…$m$,

$m$ – number of levels of the joints at half the length $L/2$ of the built-up bar (see. Fig. 2),

$r$ – number of bolts in the $i$-th connection of the one strengthening branch with strengthened element,

$f\left(\frac{N_i}{r}\right)$ – anti-function of the relation (3), in the form: (2)

$$f\left(\frac{N_i}{r}\right) = f(S_i) = -\frac{1}{b_s}\ln\left(1-\frac{N_i}{r\cdot a_s}\right) \tag{2}$$

where:

$a_s$, $b_s$ – parameters of the function (3), describing the physical dependence occurring between the load $S_i$ acting on individual bolt in the $i$-th level of the joint and mutual displacement $\delta_i$ of the joined walls in the axis of the bolt, determined on the basis of experimental research [6] in the form of an exponential function (Fig. 3):

$$S_i = a_s(1-e^{-b_s\cdot\delta_i}) \tag{3}$$

Distribution of the total load $N$ over cooperating branches in the subsequent sections $L_i$ on the length of the built-up element can be determined form the expressions:

$$N^{A}_{i,i+1} = N - 2\sum_{k=i+1}^{m} N_k \tag{4}$$

$$N^{\Delta A}_{i,i+1} = 2\sum_{k=i+1}^{m} N_k \tag{5}$$

where:

$N^{A}_{i,i+1}$ $N^{\Delta A}_{i,i+1}$ – axial compressive forces within $i,i+1$ section, respectively in the: strengthened ($A$) and strengthening ($\Delta A$) branches.

Calculation model of the analyzed built-up member is shown in Figure 2. Coefficients of instantaneous stiffness $k_i$ at various levels of the connections $i$($i$ = 0÷$m$) were determined for the resulting curve (3), according to Figure 3.

## 4. CRITICAL BUCKLING LOAD OF THE BUILT-UP MEMBER

Critical buckling load of the considered built-up

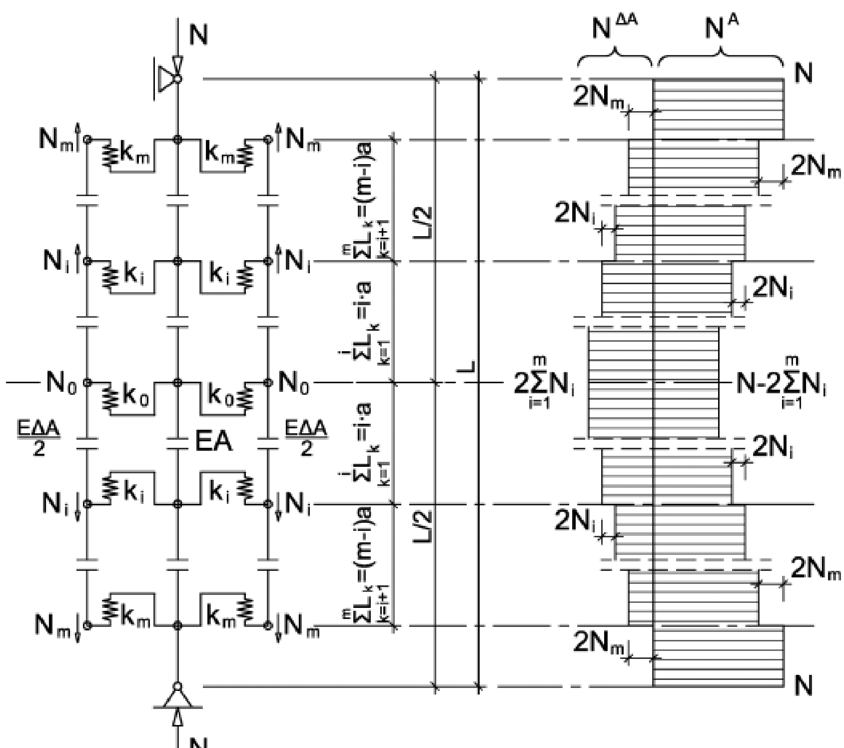


**Figure 2.**
**Calculation model of a considered built-up bar with distribution of the load in branches: reinforced $N^A$ and reinforcing $N^{\Delta A}$**

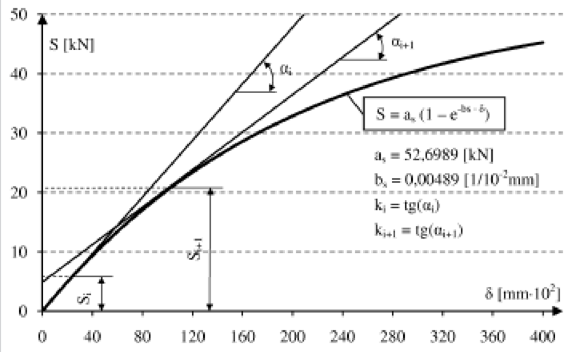


**Figure 3.**
**Relation S – δ in the connection of the walls about thickness 4.0 mm [6]**

member was determined as for perfectly straight element (without imperfections) with pinned connections in both supporting nodes and with equivalent modulus of elasticity $E_i$, which changes along the longitudinal axis of the bar (Fig. 4). Due to horizontal symmetry of the bar, its behavior is considered at the length *L/2*. For determining the critical buckling load *N*, a system of differential equations was applied, in the form [7, 8]:

$$\frac{d^2}{dx^2}\left(E_i I_i \frac{d^2 y_i}{dx^2}\right) + N \frac{d^2 y_i}{dx^2} = 0 \qquad (6)$$

where:

$E_i I_i$ – bending stiffness of the bar in the section $L_i$, about the weak *z-z* axis of the cross-section (see Fig. 4),

$y_i$ – the value of deflection in *i*-th cross-section of the bar.

Assuming that: $k_i^2 = N/E_i I_i$ the general solution of the equation (6) is obtained, in the form:

$$y_i = A_i \cos k_i x + B_i \sin k_i x + C_i x + D_i \qquad (7)$$

where:

$A_i, B_i, C_i, D_i$ – integration constants, determined from the boundary conditions for *i*-th section of the deflected bar (see Fig. 4):

$$y_1'\left(\sum_{i=1}^{m} L_i\right) = 0 \qquad (8a)$$

$$y_{i=1\div(m-1)}\left(\sum_{k=i+1}^{m} L_k\right) - y_{i+1}\left(\sum_{k=i+1}^{m} L_k\right) = 0 \qquad (8b)$$

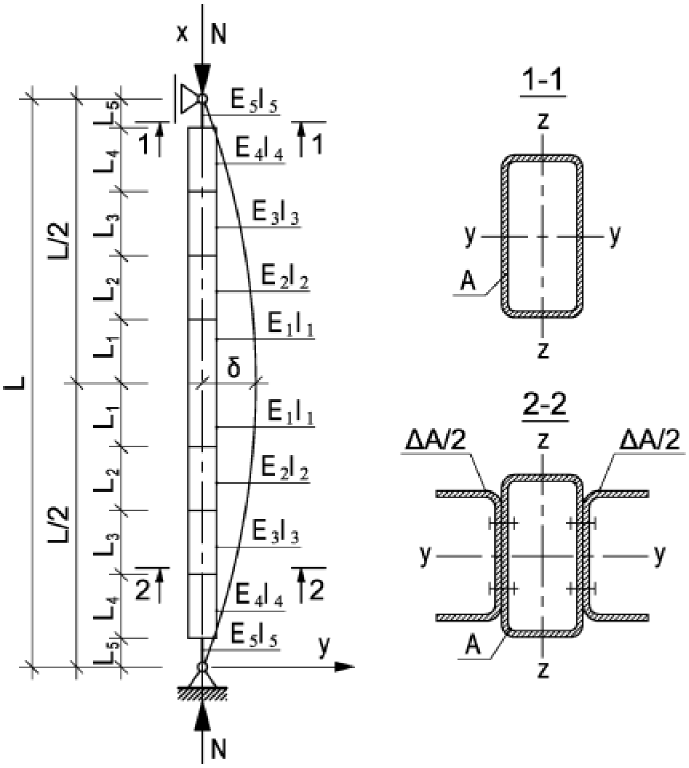


**Figure 4.**
**Computational model of the axially compressed bar with various – along the length of the bar – values of the modulus of elasticity**

$$y'_{i=1\div(m-1)}\left(\sum_{k=i+1}^{m} L_k\right) - y'_{i+1}\left(\sum_{k=i+1}^{m} L_k\right) = 0 \quad (8c)$$

$$y_m \;\; 0 \;= 0 \quad (8d)$$

$$V_1\left(\sum_{i=1}^{m} L_i\right) = 0 \quad (8e)$$

$$-V_{i=1\div(m-1)}\left(\sum_{k=i+1}^{m} L_k\right) + V_{i+1}\left(\sum_{k=i+1}^{m} L_k\right) = 0 \quad (8f)$$

$$-M_{i=1\div(m-1)}\left(\sum_{k=i+1}^{m} L_k\right) + M_{i+1}\left(\sum_{k=i+1}^{m} L_k\right) = 0 \quad (8g)$$

$$M_m \;\; 0 \;= 0 \quad (8h)$$

where:

$$M_i(x_i) = -E_i \cdot I_i \cdot y''_i(x) \quad (9)$$

$$V_i(x_i) = -E_i \cdot I_i \cdot y'''_i(x) - N \cdot y'_i(x) \quad (10)$$

The condition of stability is obtained when comparing the main determinant of set of 20 equations in the form (8) to zero, due to the parameters $A_i$, $B_i$, $C_i$, $D_i$.

An expression which permits to state equivalent values of Young's modulus $E_i$ ($i = 1\div4$) of the built-up bar with cross-section $A_{\text{tot}} = A + \Delta A$ (see. Fig. 4) in the section $L_i$, has the form:

$$\frac{\delta_i}{\delta_{i,zast}} = \frac{N \cdot L_i}{E \cdot A_{tot}} \cdot \frac{E_i A_{tot}}{N \cdot L_i} \Rightarrow E_i = E\frac{\delta_i}{\delta_{i,zast}} \quad (11)$$

where:

$\delta_i$ – change of the length of the bar with built-up cross-section $A_{tot}$, in the section $L_i$, assuming the full cooperation of the strengthening section in transmitting the axial force $N$.

$\delta_{i,zast}$ – change of the length of the bar with built-up cross-section $A_{tot}$, in the section $L_i$, assuming limited – due to flexible joints of the connected sections – cooperation of the strengthening branches in transmitting the axial force $N$.

The value $\delta_{i,zast}$, may be obtained from expression:

$$\delta_{i,zast} = \frac{N \cdot L_i}{E \cdot A_{tot,i}} \quad (12)$$

where:

$A_{tot,i}$ – equivalent cross-sectional area of the built-up bar in the section $L_i$, defined by the expression:

$$A_{tot,i} = A + \Delta A_{i,zast} \quad (13)$$

in which:

$\Delta A_{i,zast}$ – equivalent cross-sectional area of the strengthening element in the section $L_i$, given with the following formula:

$$\Delta A_{i,zast} = \frac{\Delta N_i}{\dfrac{\Delta A}{A+\Delta A}} \Delta A = \Delta N_i \;\; A + \Delta A \quad (14)$$

where:

$\Delta N_i$ – share of the strengthening part of the bar $\Delta A$ in transmitting the axial force $N$, in the section $L_i$, determined from expression:

$$\Delta N_i = 2\sum_{k=i}^{m} N_k / N \quad (15)$$

## 5. CALCULATION EXAMPLE

Numerical calculations of the considered brace member of the lattice girder, were performed using Mathematica program, assuming following data: $A = 10.95$ [cm$^2$] and $\Delta A = 2 \cdot 5.74 = 11.48$ [cm$^2$] according to [9]; $a_s = 52.6989$ [kN]; $b_s = 0.00489$ [1/10$^{-2}$mm]; $r = 2$; $a = L_1 \div L_4 = 190$ [mm]; $L_5 = 60$ [mm]; $E = 210$ [GPa].

For the considered bar, system of the 4 nonlinear equations ($i = 1\div4$), according to (1) – which allows to determine the values of shearing forces acting on blind bolts at various levels of joints – have been formulated, in the form:

$$-\frac{1}{b_s}\ln\left(1-\frac{N_1}{r \cdot a_s}\right) = \frac{N \cdot a}{E \cdot A} - \frac{2a}{E}\left(\frac{1}{A}+\frac{1}{\Delta A}\right) N_1 + N_2 + N_3 + N_4 \quad (16a)$$

$$-\frac{1}{b_s}\ln\left(1-\frac{N_2}{r \cdot a_s}\right) = 2\frac{N \cdot a}{E \cdot A} - \frac{2a}{E}\left(\frac{1}{A}+\frac{1}{\Delta A}\right) N_1 + 2(N_2 + N_3 + N_4) \quad (16b)$$

$$-\frac{1}{b_s}\ln\left(1-\frac{N_3}{r \cdot a_s}\right) = 3\frac{N \cdot a}{E \cdot A} - \frac{2a}{E}\left(\frac{1}{A}+\frac{1}{\Delta A}\right) N_1 + 2N_2 + 3(N_3 + N_4) \quad (16c)$$

$$-\frac{1}{b_s}\ln\left(1-\frac{N_4}{r \cdot a_s}\right) = 4\frac{N \cdot a}{E \cdot A} - \frac{2a}{E}\left(\frac{1}{A}+\frac{1}{\Delta A}\right) N_1 + 2N_2 + 3N_3 + 4N_4 \quad (16d)$$

Then the distribution of the axial force over cooperating branches in the built-up member, according to relations (4) and (5), have been determined.

In the next step, value of the critical load for the considered bar was defined, on the basis of the procedure described in the preceding paragraph.

Table 1 shows the results of numerical analysis for two levels of the load $N$. The upper limit of the axial force $N = 313.33$ kN, is a design compression resistance $N_{c,Rd}$ [10 , 11] of cross-section of the strengthened part (RP 100/50×4) of the bar.

**Table 1.**
**Results of numerical analysis of the built-up member**

| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|
| $N$ [kN] | $N_1$ [kN] | $N_2$ [kN] | $N_3$ [kN] | $N_4$ [kN] | $\frac{\Delta N}{N} 100\%$ | $\frac{E \cdot \Delta A}{E(\Delta A + A)} 100$ | $N_{cr,Euler}$ [kN] |
| 1 | 0.01374 | 0.02980 | 0.05097 | 0.08080 | 35.06 | 51.18 | 1389.81 |
| 313.33 | 4.50 | 9.51 | 15.66 | 23.52 | 33.95 | | 1369.01 |

Comparing the results in columns 6 and 7, it is evident that the actual total participation of the strengthening part of the bar $\Delta A$ – flexibly connected with strengthened bar – in transmitting the axial load (column 6), calculated in the section $L_1$ (see. Fig. 2) – is smaller respectively 16.12% and 17.23%, than it would result from its stiffness at axial compression, assuming a continuous and rigid joints of the straps (column 7).

As the load $N$ increases, the percentage share of strengthening part of the cross-section decreases, due to decreasing instantaneous stiffness $k_i$ of the joints (see. Fig. 3, 4). The critical buckling load of the considered built-up member, according to Euler, has the value $N_{cr,Euler} = 1369.01$ kN, which represents approximately 81% of the critical buckling load of the bar with the cross-section $A_{tot,}$ assuming full cooperation of the strengthening part of the cross-section in transmitting of the load.

## 6. CONCLUSIONS

Presented method of the calculation of buckling resistance of the perfectly straight bar, strengthened on certain length by additional branches, flexibly connected with the strengthening axially compressed bar between the supporting points, allows to estimate efficiency of applied strengthening.

The results of the analysis indicate a significant influence of flexibility of the joints on the value of the critical buckling load, according to Euler. They are important to the slender bars when the elastic buckling decides about failure of the bar.

On the basis of presented method it is possible to formulate condition of the carrying capacity of the real, burdened with imperfections bar.

The results of theoretical analysis will be verified experimentally and will be essential for the proper design of the experiment. In the next stage of the study, the authors take issue: how to take into account in the analysis of the built-up bar the influence of imperfections and the existing state of stresses in the strengthened branch.

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