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Abstract

In this paper the following statement have been stated: – For adequate research and engineering numerical analyses application of definite, unique criteria for creation of the numerical building-subsoil interactive systems as well as for assessment of reliability of the existing analyses results are required. The task brought to the method allows determination of adequate numerical dimensions of the subsoil model. Those are equivalent to areas of the subsoil response to a loading (realized by foundations of different rigidity and sizes). The global calibration process to obtain a loading subsoil response have been used. For critical state models the calibration process and criterion of the material-response allow determining adequate H_{ustal} value for subsoil model, for each (B)-(P) system. For the elastic and elastic-perfectly plastic models the calibration process and criterion of the material-response allow taking out deviations of the solutions, evaluated by the contact settlements.

Streszczenie

Właściwe badawcze i inżynierskie zastosowanie analiz numerycznych wymaga wprowadzenia jednoznacznych kryteriów postępowania, przydatnych zarówno w procesie tworzenia modeli obliczeniowych interaktywnych układów budowla-podłoże gruntowe, jak również przy ocenie wiarygodności wyników istniejących analiz. Tak postawione zadanie doprowadziło do stworzenia metody ustalania adekwatnych wymiarów numerycznych modeli podłoża, równoważnych obszarom odpowiedzi podłoża na obciążenie przekazywane z budowli.

W metodzie tej posłużono się procesem globalnego kalibrowania odpowiedzi podłoża na obciążenie. Dla modeli stanu krytycznego zastosowanie procesu kalibrowania oraz kryterium odpowiedzi materiału pozwala na określanie wartości H_{ustal} , odpowiadającej adekwatnej wysokości modelu podłoża dla dowolnego układu (B)-(P). Dla modeli sprężystych oraz sprężysto-idealnie plastycznych zastosowanie procesu kalibrowania oraz kryterium sprężystej odpowiedzi materiału pozwolą na oszacowanie w układzie (B)-(P) przewidywanego błędu rozwiązania, ocenianego w osiadaniach.

Keywords: Numerical modelling of subsoil; Critical state models; Calibration of numerical models; Building-subsoil contact tasks.

1. INTRODUCTION

Boundary value tasks describing reactions of soil formation to loads caused by man's investment activity or resulting from action of the nature forces are considered to be the most difficult computational tasks of the continuum mechanics. Sensitivity of the subsoil response to loading path implies a range of specific behaviours of the ground medium confirmed in thorough tests. Properties of ground mechanical behaviour include among others:

- diversified reaction to loading and unloading,
- inelastic isotropy forced by loadings,
- cyclic accumulation of dilatational deformations,
- expansion joint etc.

While considering generally the problem of modelling of any building – subsoil ((B)-(P)) contact tasks, we can see that what mainly decides about accuracy of the analysis results is proper idealization of ground behaviour description.



Improvement of such description accuracy is related to the use of adequately developed soil constitutive model. The range of numerical realisations related to advanced modelling has been extending rapidly since 90's. Information regarding range of applications of particular groups of models in the analyses of various problems in the field of soil mechanics include among other things works by *Gens and Potts* [1], *Duncan* [2], *Gryczmański* [3], *Wood* [4], *Kisiel, Dmitruk* and *Lysik* [5], *Kwiatek* [6].

In the analysed contact problems (B)-(P) the application of analyses with the use of developed models of both sub-systems begins to be considered as standard procedure. Such an approach results from currently common drive to "exchange of tools" i.e. replacement of classical verified methods of calculating with generally understood numerical modelling. Due to the above, with the tendency for more and more developed, spatial modelling of a building construction, the mechanism of modelling the "the whole", in which subsoil usually becomes a solid employed in totally arbitrary way – Fig. 1, comes into being automatically. The problem refers to both subsoil area as well as employed constitutive relations of soil.



Geotechnical boundary value task transformed into structural mechanics one

In the discussed problem of interaction taking place in the building-subsoil system, the accuracy of reconstruction of the researched reality with the use of model depends not only on the available mathematical apparatus i.e. preciseness of the reality description. To the large extend such an accuracy – as shown in the paper – is related to ability of calibration of the modelled computational system (B)-(P), most favourably in relation to tests in situ creating so called basic task.

The process of system calibration is related to both:

- employment of appropriate model computational area (P), representing subsoil, as well as
- selection of appropriate soil constitutive model.

The purpose of the research was to create the unique criteria enabling assessment of the adequacy of the constructed computational systems (B)-(P) by meeting the following conditions:

- (I) possibly best reconstruction of base task in subsoil model (P) and
- (II) possibility of model behaviour assessment with analysis of actual task.

The criteria, as shown later can be used during assessment of an adequacy of the existing numerical analyses results for hereby considered static load.

Fig. 1 shows conversion of geotechnical task into strictly structural mechanics task so characteristic for contemporary numerical analyses. Such a procedure may lead, with lack of appropriate unequivocal indications regarding subsoil modelling, to great threats resulting from improper assessment of sub-systems interaction.

Potts, one of the leading contemporary geotechnicians in his work of characteristic title "Numerical analysis: a virtual dream or practical reality" [7] reminds that analysis is only a part of the designing process. However, there is no doubt that in the future numerical analyses will play decisive role in this process. Nevertheless, for this to happen further works regarding problems of various level of complexity, creating leading paths for future better practice, are required [8].

2. PHILOSOPHY OF CALIBRATION OF NUMERICAL MODELS

Numerical analysis of boundary problems representing work of building-subsoil systems (or foundationsubsoil) are naturally limited by lack of clear conditions for transition from analyses of boundary problems representing tests in situ to analyses of actual engineering problems.

Solution to the above referred problem of possible ambiguity of the received results require response to the following questions:

- 1. Can in situ tests be reproduced in any employed subsoil (P) model?
- 2. How can in situ tests be used to calibrate subsoil (P) model and simultaneously the whole computational system (B)-(P)?

Presented problem is related to task of identification of parameters of used soil constitutive model. Generally, such identification is a result of:

- process of local calibration of model (which is con-

ventionally related to carrying out of appropriate tri-axial tests), or

global calibration, i.e. generally speaking, assessment of model parameters based on field tests results and sometimes on monitoring of actual settlements of the building structures [9,10,11].

The above shows that adequate numerical model (P) should implement full response of the subsoil to the loading path employed during trial loading i.e. reconstruct:

- 1. phenomena recorded on the surface (dependence loading-settlement), as well as
- 2. phenomena taking place in the loaded soil massive.

Therefore, let's confront the responses of:

- actual homogenous subsoil to trial loading applied during in situ tests to the prepared rigid foundations – Fig. 3 [12], and
- numerical models of subsoil (working in axial symmetry state) to loading transferred from testing slab Fig. 2.

At first possibilities were considered as to responses to soil constitutive models most frequently used in engineering and research analyses – of elastic-perfectly plastic models (e-p) with associated flow rule.



Figure 2a presents two "smoothed" dependencies load-settlement, hypothetically obtained in the in situ tests in bedded subsoils of different layers rigidity. Possibility of the same, initial system response to load expressed by angle α . It is a result of linear-elastic, in initial phase, behaviour of soil medium.

Employment of elastic model (e) or elastic-perfectly plastic models (e-p) of constitutive relations of classical elasticity for subsoil analysis is equivalent to acceptance of below effects incompliant with actual woks of subsoil:

- 1. displacement homogenization of response in model elastic work phase,
- 2. unlimited propagation of deforming and stressing effects of medium loading.

Effect (1) results in the fact that obtained numerical responses of both subsoils – α_1 i α_2 in Fig. 2b – differ from the value α obtained in in situ tests (Fig. 2a). Effect (2) results in the fact that with the same paths of loading we revive the same numerical responses (measured with α value) in two different subsoils – Fig. 2c, first created from homogenisation of layers m_1 and m_2 , and the second one created from homogenisation of layers *m*₁ and *m*₂. It becomes obvious that actual subsoil requirements cannot be satisfied in any way by the above considered numerical models.

Therefore, the process of direct calibration of classical models of subsoil (e) and (e-p), based on reconstruction of in situ test results, is impossible. Proposals of assessment of anticipated results accura-



cy in systems (B)-(P) using the above soil models (with appropriate subsystem computational area (P)) has been presented in the following chapter 3.

Fig. 3 presents part of field tests carried out in 2001 by Swedish Geotechnical Institute and Published in the form of report by Larsson [9], (http://www.swedgeo.se). We can see the picture of coming to the end, at depth, soil "internal response" to load transmitted by the foundation. Area of soil response to load is local and impact of deeper and deeper located soil layers rigidity emerges successively, as applied load Q increases. Models that the paper finds to be "adapted" to reconstruct the above referred phenomena are critical state models - Cam-Clay and Modified Cam-Clay [4,13,14]. The models use empirically noted logarithmical dependencies of porosity index and effective mean stress normal consolidation state as well as in the distressing first phase. Characteristic feature of critical state models (confirmed in tests results presented under chapter 4), extremely essential during creation of adequate numerical computational models, is uniqueness of the area of structure and subsoil cooperation - Fig. 9.

3. COMPUTATIONAL AREA OF ELASTIC (e) AND ELASTIC – PERFECTLY PLAS-TIC (e-p) MODELS

Commonly used definitions "analysis on elastic subsoil" or "analysis on elastic half space" are actually not defined in numerical calculations. Area range of response to loading in numerical models (e) and (e-p) is a purely symbolic quantity. The subject to control is only external response of the model to load; condition (I) in chapter 1.

For this purpose we can use appropriately formulated material response criterion. Criterion makes use of the fact that in the considered models (working in the elastic range) module E is directly interconnected with subsoil settlements s.

$$E_{od} = Q \cdot D (1 - v^2) \cdot 1/s_n \tag{1}$$

where E_{od} – here becomes parameter of material response,

 s_n – is numerical value of settlement in the selected point *i*, obtained in n the model of the material parameter E_d introduced in calculations and equal to module of deformation $E_o = E_d$.

Therefore, the process of computational model calibration will involve reconstruction of a given basic task including control of dependency loading-settle-



Figure 4.

Ambiguity in the elastic model that arise during a basic task reconstruction

ment. The problem is presented in Fig. 4. To reconstruct basic task (here understood as result of in situ test within the rage of soil elastic work) in subsequently enlarged soil models (P) appropriately increased values of module E_o were driven. Condition $\alpha_{numer} = \alpha_{in \ situ}$ was met in solutions to system research plate – subsoil.

If constitutive subsoil model does not reconstruct phenomena taking place in the loaded soil massif (condition II in chapter 1) unique numerical prediction of behaviour of actual system structure – subsoil is impossible.

There is, at most, possibility of impact assessment of selection of subsoil model area dimensions on the solution results of actual (B)-(P) system.



Figure 5 confirms the above statements. It confronts:

- Standard settlements values, and
- Values determined numerically for subsoil models (2D) of values comparable to standard values *z_{norm}*, required for the selected values of loading *Q*.

Load was transmitted to subsoil via rigid continuous footing of width B=1m footed at the depth of 0.5m. Elastic (e) solutions are accompanied by solutions for elastic-perfectly plastic Coulomb-Mohr (C-M) subsoil.

Generally it can be stated that standard requirements (z_{norm}) cannot be unique criterion for subsoil model adequate height selection in analysis of boundary problems (2D).

4. COMPUTATIONAL AREA OF MODI-FIED CAM-CLAY MODEL (MCC)

For critical state model we do not have analytical solutions for boundary value problems, so there is no data available to arrange a priori of the area of subsoil model response to loading. Therefore we will carry out procedure of subsoil model calibration using, created for this purpose, below referred criterion of material response.

In the model void ratio *e* is directly interconnected with stress *p* (or σ_v) existing in the given point of subsoil. It can be expressed by two states recorded symbolically in the form of pairs of quantities determining the following:

- 1) {(p),(e)} or {(σ_v (γ ·z),(e)} state (1), of natural soil deposition;
- 2) { $(\sigma_v + d\sigma_v (dQ)), (e+de)$ } state (2), created after applying external loading Q.

Therefore, calibration of computational area for critical state models can be related to analysis of subsoil model internal response to external load, causing transition of soil from state (1) to state (2).

To simplify this study, the process was related below to the state of system axial symmetry. The course of the process is assessed by means of effect of blanking at depth value de – of porosity (e) index change.

Figure 6a presents state of natural deposition of soil (1) which can be treated as matrix to which state caused by loading is "applied", where (NC) – symbol of naturally consolidated soil, (OC) – pre-consolidated soil. We now consider soil (OC), pre-consolidated in the range of total value of external loading Q – Fig. 6b.



a) State (1) matrixes for the subsoil (NC) and (OC) obtained for the MCC (NC) and MCC (OC) models, b) the subsoil preconsolidated for whole range of the loading value

Using for realisation of state (2) analogy to method Cc (Cs) will be presented as:

$$e_{z} - e_{1} = \Delta e = C_{s} \cdot log\left(\frac{\sigma_{1}}{\sigma_{o}}\right) = \frac{C_{s}}{ln(10)} ln\left(\frac{\sigma_{1}}{\sigma_{o}}\right) = \kappa ln\left(\frac{\sigma_{1}}{\sigma_{o}}\right) \quad (2)$$

where: $\sigma_0 = \gamma \cdot z$ - vertical component of stress in state (1)), $\sigma_I = \gamma \cdot z + Q$ - sum of in situ stresses as well as value of anticipated external loading Q, e_I - void ratio value corresponding to stress σ_I .

Then, elementary shortening of symbolic subsoil layer at any z depth (at external loading Q) in MCC model will amount to:

$$\frac{\Delta h}{h} = \frac{1}{1 + e_z} \cdot \kappa \cdot ln \left(\frac{\gamma \cdot z + Q}{\gamma \cdot z} \right)$$
(3)

We find the evaluation of elementary shortening of layer at depth to be assessment measurement of computational area height of adequate subsoil model.

Then let's employ a priori certain function $\Phi_i(z)$. The function describes distribution of vertical components of stresses in subsoil, under the axis of circular foundation of any diameter D_i .

For model, standard distribution of function $\Phi_i(z)$ [15] individual shortening of symbolic subsoil layer changing along with the depth z will amount to:

$$\frac{\Delta h}{h} = \frac{\kappa}{2 \cdot (1 + e_z)} ln \left(\frac{\gamma \cdot z + Q}{\gamma \cdot z} \right) \cdot \left[1 + \frac{4 \cdot \eta_i^2 \cdot (1 - 4 \cdot \eta_i^2)}{\left[1 + 4 \cdot \eta_i^2 \right]^2} \right]$$
(4)

where: e_z – initial value of void ratio determined based on matrix:

$$e_{z} = e_{cs} - (\lambda - \kappa) \cdot ln \left(\frac{p_{co}}{2}\right) - \kappa \cdot ln(p_{in})$$
(5)

with $p_{in} = \frac{1}{3} \left(\gamma \cdot z + q^* \right) \cdot \left(1 + 2K_o^{NC} \right)$ and $p_{co} = \frac{9 \cdot \left(1 - K_o^{NC} \right)^2 + M^2 \cdot \left(1 + 2K_o^{NC} \right)^2}{3 \cdot M^2 \cdot \left(1 + 2K_o^{NC} \right)} \cdot \left(\gamma \cdot z + q^* \right)$

The following parameters were employed in analyses of soil behaviour determined under the MCC model ([8],[16],[17]): $\lambda = 0.066$, $\kappa = 0.0074$, M = 1.2, $e_{cs} = 1.788$, $\nu = 0.15$, as well as $\gamma = 20 \ kN/m^3$ and $K_O^{(NC)} = 0.45$.

To find soil to be pre-consolidated for employed value of load $Q \le 400 \ kPa$ and to prevent path of stresses $(p,q)^{l}$ under the middle of anticipated loading surfaces from entering the surface of SBS state the following was determined for $q^*=210 \ kPa$: $p_{co}=232 \ kPa$ (at the depth of 0.75 m) distribution profile $K_0^{(OC)}=1.74$ (up to depth 2.5 m), below linearly changeable to the value $K_0^{(OC)}=0.51$ (at the depth of 40 m).



a) Function (4) – diagrams for different foundations D_i , b) Function (4.25) sensitivity to loading Q variations

Let us consider now dependencies (4) as functions representing state (2) of loaded subsoil MCC (OC), of given parameters of model. The functions in system $(\Delta h/h, H)$, for foundations of subsequent diameters D_i , with employed level of load Q have been presented in Fig. 7a. Figure 7b in return gives a picture of function (4) sensitiveness to change of value of external load Q.

Figure 8 is a brief look at work of subsoil determined under critical state model MCC. In Fig. 8a foundation D=2 m is subjected to increasing load, accompanied by clear stabilisation of area of subsoil settlement under foundation. Fig. 8b shows comparison of character of settlement functions obtained in half space of the MCC (OC) model in the axes of successively enlarged foundations (D=1, D=2 and D=4 m) with elastic settlements functions (e). Functions (e) were determined numerically in the elastic half space with the values of deformation modulus obtained from the compliance of settlements of the following foundations in MCC (OC) and elastic half space, at $Q=100 \ kPa$.



Figure 8.

a) Settlement distribution under the foundation centre D=2m, b) Settlement distribution (to a depth of the subsoil model) for MCC (OC) and (e) models



Figure 9.

a) Basic task (subsoil – group B-models), b) General task (subsoil – group B-models)

Characteristic phenomenon recorded on the MCC model surface is lack of proportions between increase of settlements value and increase of the diameter.

5. CONCLUSION

Characteristic feature of critical state models, very essential during creation of adequate numerical computational models, is clear-cut nature of the area of building and subsoil cooperation. Fig. 9 sums it up symbolically Reconstruction of basis task (e.g. in situ test) in MCC model allows statement that soil stays inactive outside the zone of model displacement response to loading. The phenomenon will obviously repeat in actual computational system building – subsoil, if subsoil model meets the condition $H>H_{ustal}$. The method of determining area range of model response to loading – H_{ustal} is based on functions $\Delta h/h$, type (4).

As these functions are the picture of external load Q impact decay, with increasing depth, they also become diagrams used to determine appropriate height of computational area of subsoil model (according to employed assessment measurement in [%]).

Presented deliberations would require (to consider them to be complete assessment of critical state models usefulness in analysis of building – subsoil contact problems) additional presentation of effectiveness of assessment of subsoil response area to loading in case of both full spatial work (3D) as well as state (2D) of analysed systems.

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