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# IDENTIFICATION OF CAST IRON SUBSTITUTE THERMAL CAPACITY

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SUMMARY

In the paper the inverse problem consisting in estimation of cast iron substitute thermal capacity is presented. In order to solve the inverse problem formulated it is assumed that the cooling curves at selected set of points from the casting domain are given. The algorithm bases on the least squares criterion in which the sensitivity coefficients appear. In the final part of the paper the results of computations are shown.

*Key words: solidification process, inverse problem, parameter estimation method*

## 1. DIRECT PROBLEM

The 1D casting-mould system is considered. Transient temperature field in casting sub-domain determines the energy equation

$$0 < x < L_1 : \; C(T)\,\frac{\partial T(x,t)}{\partial t} = \frac{\partial}{\partial x}\left[\lambda(T)\,\frac{\partial T(x,t)}{\partial x}\right] \tag{1}$$

where $C(T)$ is the substitute thermal capacity [1] of cast iron - Figure 1, $\lambda(T)$ is the thermal conductivity, $T$ is the temperature, $x$ is the spatial co-ordinate and $t$ is the time. A temperature field in mould sub-domain is described by equation

$$L_1 < x < L : \; c_m\,\frac{\partial T_m(x,t)}{\partial t} = \lambda_m\,\frac{\partial^2 T_m(x,t)}{\partial x^2} \tag{2}$$

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where $\lambda_m$ is the thermal conductivity and $c_m$ is the volumetric specific heat of mould. On the contact surface between casting and mould the continuity condition

$$x = L_1 : \quad \begin{cases} -\lambda \dfrac{\partial T(x, t)}{\partial x} = -\lambda_m \dfrac{\partial T_m(x, t)}{\partial x} \\ T(x, t) = T_m(x, t) \end{cases} \tag{3}$$

is assumed. For $x=0$ (axis of symmetry) and $x=L$ (outer surface of the system) the no-flux conditions are accepted. For the moment $t=0$ the initial temperature distribution is known

$$T(x, 0) = T_0(x) \qquad T_m(x, 0) = T_{m0}(x) \tag{4}$$

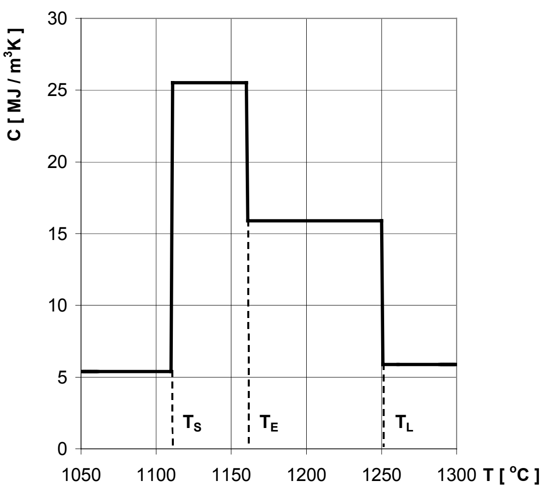


Fig. 1. Substitute thermal capacity of cast iron.
Rys. 1. Zastępcza pojemność cieplna żeliwa.

## 2. INVERSE PROBLEM

If the parameters appearing in governing equations are known then the direct problem is considered. If part of them is unknown then the inverse problem should be formulated. In particular, in this paper the problem of cast iron substitute thermal capacity identification is presented.

In order to solve the inverse problem formulated the additional information concerning the cooling curves at the selected set of points from the domain considered must be given. So, it is assumed that the values $T_{di}^f$ at the sensors $x_i$ from casting sub-domain for times $t^f$ are known, namely

$$T_{di}^f = T_d\,(x_i\,,\,t^f)\ ,\quad i = 1,\,2,\,\ldots\,,\,M\ ,\quad f = 1,\,2,\,\ldots\,,\,F \tag{5}$$

The substitute thermal capacity of cast iron can be expressed as follows (c.f. Figure 1)

$$C(T) = \begin{cases} p_1\ , & T > T_L \\ p_2\ , & T_E < T \le T_L \\ p_3\ , & T_S < T \le T_E \\ p_4\ , & T \le T_S \end{cases} \tag{6}$$

where $p_e$, $e$ = 1, 2, 3, 4 are the unknown constant parameters. Using the parameter estimatiom method the values of $p_e$ should be identified.

## 3. METHOD OF SOLUTION

In order to solve the inverse problem the least squares criterion is applied [2, 3]

$$S\left(\,p_1\,,\,p_2\,,\,p_3\,,\,p_4\right) = \frac{1}{M\,F}\sum_{i=1}^{M}\,\sum_{f=1}^{F}\left(T_i^f - T_{di}^f\right)^2 \tag{7}$$

where $T_{di}^f$ (c.f. equation (5)) and $T_i^f = T(x_i\,,\,t^f)$ are the measured and estimated temperatures, respectively, for the sensor $x_i$, $i$ =1, 2,..., $M$ and for time $t^f$. The estimated temperatures are obtained from the solution of the direct problem (c.f. chapter 1) by using the current available estimate for the unknown parameters.

Differentiating the criterion (7) with respect to the unknown parameters $p_e$ and using the necessary condition of optimum one obtains the following system of equations

$$\frac{\partial S}{\partial p_e} = \frac{2}{M\,F}\sum_{i=1}^{M}\,\sum_{f=1}^{F}\left(T_i^f - T_{di}^f\right)\left(Z_{ei}^f\right)^k = 0\ ,\quad e = 1,\,2,\,3,\,4 \tag{8}$$

where

$$\left(Z_{ei}^f\right)^k = \left.\frac{\partial T_i^f}{\partial p_e}\right|_{p_e = p_e^k} \tag{9}$$

are the sensitivity coefficients, $k$ is the number of iteration, $p_e^0$ are the arbitrary assumed values of $p_e$, while $p_e^k$ for $k > 0$ result from the previous iteration.

Function $T_i^f$ is expanded in a Taylor series about known values of $p_l^k$, this means

$$T_i^f = \left(T_i^f\right)^k + \sum_{l=1}^{4} \left(Z_{li}^f\right)^k \left(p_l^{k+1} - p_l^k\right) \tag{10}$$

Putting (10) into (8) one obtains ($e$=1, 2, 3, 4)

$$\sum_{i=1}^{M} \sum_{f=1}^{F} \sum_{l=1}^{4} \left(Z_{li}^f\right)^k \left(Z_{ei}^f\right)^k \left(p_l^{k+1} - p_l^k\right) = \sum_{i=1}^{M} \sum_{f=1}^{F} \left[T_{di}^f - \left(T_i^f\right)^k\right] \left(Z_{ei}^f\right)^k \tag{11}$$

This system of equations allows to find the values of $p_e^{k+1}$. The iteration process is stopped when the assumed number of iterations $K$ is achieved.
It should be pointed out that in order to obtain the sensitivity coefficients, the governing equations should be differentiated with respect to $p_e$ [4]. So, for each time step the basic problem and four additional problems connected with the sensitivity functions should be solved.

## 4. RESULTS OF COMPUTATIONS

The casting-mould system of dimensions $2L_1$ = 0.03 [m] (casting) and 0.045 [m] (mould) has been considered. The following input data have been introduced: $T_L$=1250 °C, $T_E$=1160 °C, $T_S$=1110 °C, $\lambda(T)$=20 [W/mK] for $T>T_L$, $\lambda(T)$=30 [W/mK] for $T\in[T_S, T_L]$, $\lambda(T)$=40 [W/mK] for $T<T_S$, $\lambda_m$ = 1 [W/mK], $c_m$=1.75 [MJ/m$^3$ K], pouring temperature $T_0$ = 1300 °C, initial mould temperature $T_{m0}$ = 20 °C.

In order to identify the values of $p_e$ the courses of cooling curves (c.f. equation (5)) at the points $x_1$ =0 [m] (axis of symmetry), $x_2$ =0.009 [m] and $x_3$ =0.00148 [m] have been taken into account - Figure 2. They result from the direct problem solution under the assumption that $p_1$=5.88 [MJ/m$^3$ K], $p_2$=15.89 [MJ/m$^3$ K], $p_3$=25.52 [MJ/m$^3$ K], $p_4$=5.4 [MJ/m$^3$ K].

For each iteration the basic problem and additional ones connected with the sensitivity coefficients determination have been solved using the explicit scheme of finite differences method [1] (mesh step $h$=0.0003 [m], time step $\Delta t$=0.002 [s]).

In Figures 3 and 4 the results of inverse problem solution for undisturbed cooling curves are shown. In the first variant the initial values of identified parameters equal $p_1^0 = p_2^0 = p_3^0 = p_4^0$ = 5 [MJ/m$^3$K] - Figure 3, while in the second variant $p_1^0 = p_2^0 = p_3^0 = p_4^0$=15 [MJ/m$^3$K].

It is visible that for these initial values of estimated parameters $p_e$, $e$ =1, 2, 3, 4 the iteration process is convergent and the solution close to the exact values is obtained after the several iterations.

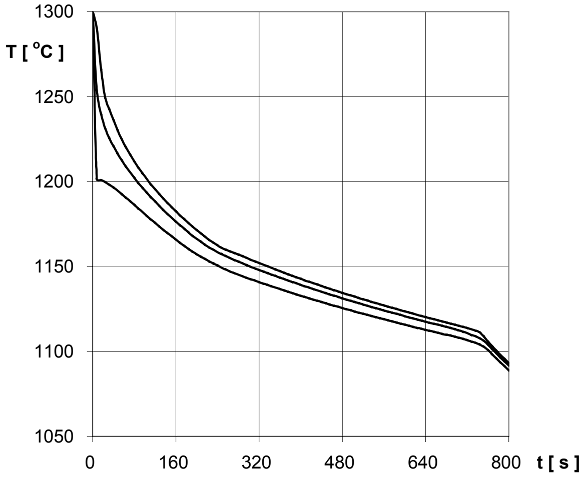


Fig. 2. Cooling curves.
Rys. 2. Krzywe stygnięcia.

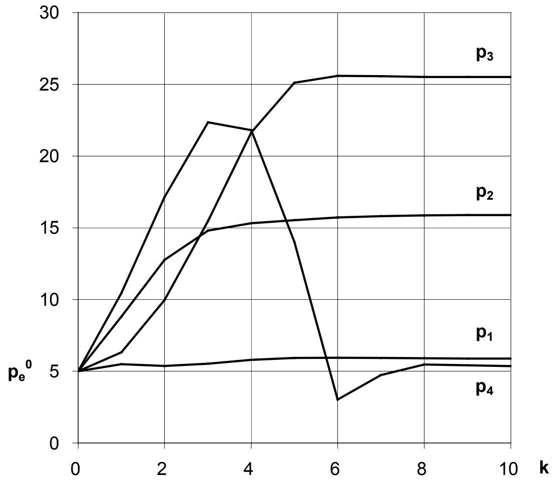


Fig. 3. Results of identification for $p_e^0$=5 [MJ/m$^3$ K].
Rys. 3. Wyniki identyfikacji dla $p_e^0$=5 [MJ/m$^3$ K].

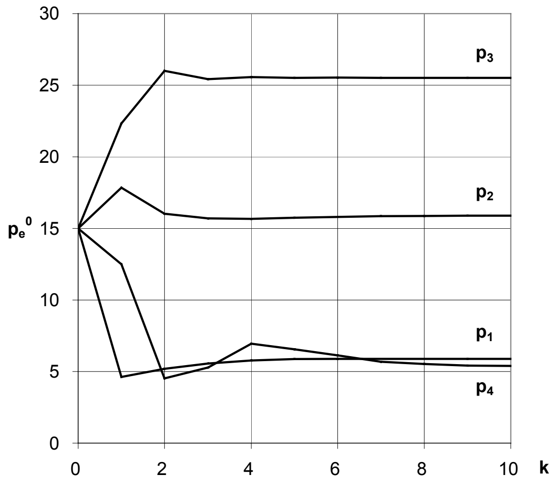


Fig. 4. Results of identification for $p_e^0$=15 [MJ/m$^3$ K].
Rys. 4. Wyniki identyfikacji dla $p_e^0$=15 [MJ/m$^3$ K].

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## IDENTYFIKACJA ZASTĘPCZEJ POJEMNOŚCI CIEPLNEJ ŻELIWA


STRESZCZENIE

W artykule przedstawiono zadanie odwrotne polegające na identyfikacji zastępczej pojemności cieplnej żeliwa. Założono, że znane są krzywe stygnięcia w kilku punktach z obszaru odlewu. Problem rozwiązano wykorzystując kryterium najmniejszych kwadratów zawierające współczynniki wrażliwości. W końcowej części przedstawiono wyniki obliczeń.


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