

## ESTIMATION OF CAST STEEL THERMAL CONDUCTIVITY ON THE BASIS OF COOLING CURVES FROM THE CASTING DOMAIN

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### SUMMARY

In the paper the thermal processes proceeding in the system casting-mould-environment are discussed. Heat transfer in nonhomogeneous domain considered is described by the system of partial differential equations (energy equations) supplemented by the adequate boundary, initial, physical and geometrical conditions. If the parameters appearing in governing equations are known then the direct problem is considered, while if the part of them is unknown then the inverse problem should be formulated. In particular in the paper presented the problem of thermal conductivity of casting material identification analyzed (the thermal conductivity of casting is assumed to be a constant value). In order to solve the inverse problem formulated the additional information concerning the cooling curves at selected set of points from the domain considered must be given. The inverse problem is solved using the least square criterion in which the sensitivity coefficients appear. On the stage of numerical computations the boundary element method is applied. In the final part of the paper the results of computations are shown.

*Keywords:* solidification process, inverse problem, parameter estimation method, boundary element method

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## 1. FORMULATION OF THE INVERSE PROBLEM

The 1D casting-mould system is considered. Transient temperature field in casting sub-domain determines the energy equation

$$0 < x < L_1 : \quad C(T) \frac{\partial T(x, t)}{\partial t} = \lambda \frac{\partial^2 T(x, t)}{\partial x^2} \quad (1)$$

where  $C(T)$  is the substitute thermal capacity [1],  $\lambda$  is the thermal conductivity,  $T$  is the temperature,  $x$  is the spatial co-ordinate and  $t$  is the time.

A temperature field in mould sub-domain describes the equation

$$L_1 < x < L : \quad c_m \frac{\partial T_m(x, t)}{\partial t} = \lambda_m \frac{\partial^2 T_m(x, t)}{\partial x^2} \quad (2)$$

where  $\lambda_m$  is the thermal conductivity and  $c_m$  is the volumetric specific heat of mould. On the contact surface between casting and mould the continuity condition

$$x = L_1 : \quad \begin{cases} -\lambda \frac{\partial T(x, t)}{\partial x} = -\lambda_m \frac{\partial T_m(x, t)}{\partial x} \\ T(x, t) = T_m(x, t) \end{cases} \quad (3)$$

is assumed. For  $x = 0$  (axis of symmetry) the no-flux condition is accepted. For the outer surface of the system the heat transfer process is determined by condition

$$x = L : \quad q_m(x, t) = -\lambda_m \frac{\partial T_m(x, t)}{\partial x} = \alpha [T_m(x, t) - T^\infty] \quad (4)$$

where  $\alpha$  is the heat transfer coefficient,  $T^\infty$  is the ambient temperature. For the moment  $t = 0$  the initial temperature distribution is known

$$T(x, 0) = T_0(x) \quad T_m(x, 0) = T_{m0}(x) \quad (5)$$

Additionally, the values  $T_{di}^f$  at the selected set of points  $x_i$  from casting sub-domain for times  $t^f$  are known, namely

$$T_{di}^f = T_d(x_i, t^f), \quad i = 1, 2, \dots, M, \quad f = 1, 2, \dots, F \quad (6)$$

## 2. METHOD OF SOLUTION

At first, the governing equations (1) – (5) are differentiated with respect to the thermal conductivity  $\lambda$  and then

$$\begin{aligned}
 0 < x < L_1 : \quad C(T) \frac{\partial Z(x, t)}{\partial t} &= \lambda \frac{\partial^2 Z(x, t)}{\partial x^2} + \frac{C(T)}{\lambda} \frac{\partial T(x, t)}{\partial t} \\
 L_1 < x < L : \quad c_m \frac{\partial Z_m(x, t)}{\partial t} &= \lambda_m \frac{\partial^2 Z_m(x, t)}{\partial x^2} \\
 x = L_1 : \quad \begin{cases} Z(x, t) = Z_m(x, t) \\ -\lambda \frac{\partial Z(x, t)}{\partial x} = -\lambda_m \frac{\partial Z_m(x, t)}{\partial x} + \frac{\partial T(x, t)}{\partial x} \end{cases} & \\
 x = 0 : \quad \frac{\partial Z(x, t)}{\partial x} &= 0 \\
 x = L : \quad -\lambda_m \frac{\partial Z_m(x, t)}{\partial x} &= \alpha Z_m(x, t) \\
 t = 0 : \quad Z(x, 0) = 0, \quad Z_m(x, 0) = 0 &
 \end{aligned} \tag{7}$$

where  $Z(x, t) = \partial T(x, t) / \partial \lambda$  and  $Z_m(x, t) = \partial T_m(x, t) / \partial \lambda$  are the sensitivity functions. Next, the least squares criterion is applied [2, 3, 4]

$$S(\lambda) = \sum_{i=1}^M \sum_{f=1}^F \left( T_i^f - T_{di}^f \right)^2 \tag{8}$$

where  $T_i^f = T(x_i, t^f)$  is the calculated temperature at the point  $x_i$  for time  $t^f$ .

Differentiating the criterion (8) with respect to the unknown thermal conductivity  $\lambda$  and using the necessary condition of minimum, one obtains

$$\frac{\partial S}{\partial \lambda} = 2 \sum_{i=1}^M \sum_{f=1}^F \left( T_i^f - T_{di}^f \right) \frac{\partial T_i^f}{\partial \lambda} \Bigg|_{\lambda=\lambda^k} = 0 \tag{9}$$

where  $k$  is the number of iteration,  $\lambda^k$  for  $k = 0$  is the arbitrary assumed value of  $\lambda$ , while  $\lambda^k$  for  $k > 0$  results from the previous iteration.

Function  $T_i^f$  is expanded in a Taylor series about known values of  $\lambda^k$ , this means

$$T_i^f = \left(T_i^f\right)^k = \frac{\partial T_i^f}{\partial \lambda} \Bigg|_{\lambda=\lambda^k} (\lambda^{k+1} - \lambda^k) \quad (10)$$

or

$$T_i^f = \left(T_i^f\right)^k + \left(Z_i^f\right)^k (\lambda^{k+1} - \lambda^k) \quad (11)$$

Putting (11) into (9) one has

$$\sum_{i=1}^M \sum_{f=1}^F \left[ \left(Z_i^f\right)^k \right]^2 (\lambda^{k+1} - \lambda^k) = \sum_{i=1}^M \sum_{f=1}^F \left(Z_i^f\right)^k \left[ T_{di}^f - \left(T_i^f\right)^k \right] \quad (12)$$

this means

$$\lambda^{k+1} = \lambda^k + \frac{\sum_{i=1}^M \sum_{f=1}^F \left(Z_i^f\right)^k \left[ T_{di}^f - \left(T_i^f\right)^k \right]}{\sum_{i=1}^M \sum_{f=1}^F \left[ \left(Z_i^f\right)^k \right]^2}, \quad k = 0, 1, \dots, K \quad (13)$$

This equation allows to find the values  $\lambda^{k+1}$ . The iteration process is stopped when the assumed accuracy is achieved.

For each iteration the basic problem and additional ones connected with the sensitivity function have been solved using the 1<sup>st</sup> scheme of the boundary element method [5, 6] supplemented by the artificial heat source procedure [7].

### 3. EXAMPLE OF COMPUTATIONS

The casting-mould system of dimensions  $2L_1 = 0.02$  [m] (casting) and  $0.03$  [m] (mould) has been considered. The following input data have been introduced:  $C(T) = 5.175$  [MJ/m<sup>3</sup>K] for  $T < 1470$  °C,  $C(T) = 61.4$  for  $T \in [1470, 1505]$ ,  $C(T) = 5.74$  for  $T > 1505$ ,  $\lambda_m = 2.6$  [W/mK],  $c_m = 1.75$  [MJ/m<sup>3</sup>K], pouring temperature  $T_0 = 1550$  °C, initial mould temperature  $T_{m0} = 30$  °C, heat transfer coefficient  $\alpha = 10$  [W/m<sup>2</sup>K], ambient temperature  $T^\infty = 30$  °C.

In order to identify the value of  $\lambda$  the courses of cooling curves (c.f. equation (7)) at the points  $x_1 = 0$  [m] (axis of symmetry),  $x_2 = 0.004$  [m] and  $x_3 = 0.0075$  [m] have been taken into account. They result from the direct problem solution under the assumption that  $\lambda = 35$  [W/mK]. Figure 2 illustrates the solution of inverse problem for different initial values of  $\lambda^0$ . It is visible that the iteration process for the assumed initial values of parameter  $\lambda$  is convergent and the solution close to the exact value is obtained after 15 iterations.

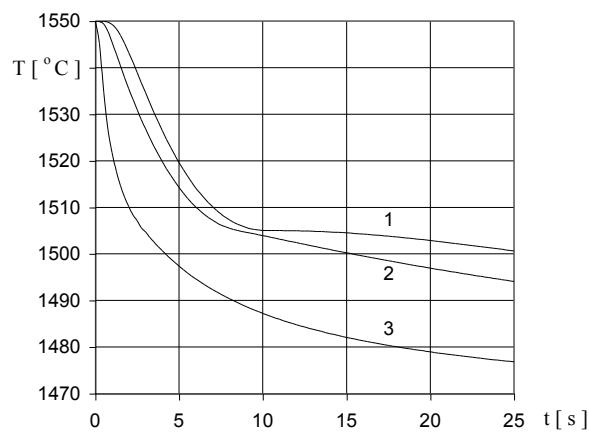


Fig. 1. Cooling curves  
Rys. 1. Krzywe stygnięcia

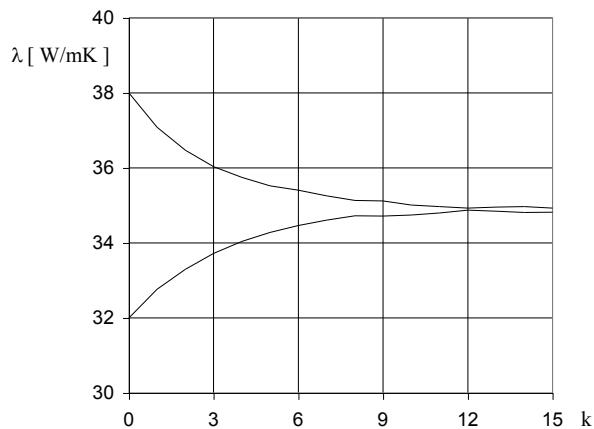


Fig. 2. Estimation of  $\lambda$   
Rys. 2. Estymacja parametru  $\lambda$

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## OSZACOWANIE WSPÓŁCZYNNIKA PRZEWODZENIA CIEPŁA STALIWA NA PODSTAWIE KRZYWYCH STYGNIĘCIA Z OBSZARU ODLEWU

### STRESZCZENIE

W artykule analizowano procesy cieplne zachodzące w układzie odlew-formatożycie. Przepływ ciepła w obszarze niejednorodnym opisuje układ równań różniczkowych (równań energii) uzupełniony odpowiednimi warunkami brzegowymi, początkowymi, fizycznymi i geometrycznymi. Jeśli parametry termofizyczne występujące w opisie matematycznym procesu są znane, wówczas rozpatruje się tzw. zadanie bezpośrednie, natomiast, jeśli część z nich nie jest znana, to formułuje się odpowiednie zadanie odwrotne. W przedstawionym artykule zajmowano się problemem identyfikacji stałego współczynnika przewodzenia ciepła odlewu. Do rozwiązania tak sformułowanego zadania odwrotnego wykorzystano dodatkową informację dotyczącą przebiegu krzywych stygnięcia w wybranych punktach odlewu. Problem odwrotny rozwiązyano stosując kryterium najmniejszych kwadratów, w którym pojawiają się współczynniki wrażliwości. Na etapie obliczeń numerycznych wykorzystano metodę elementów brzegowych. W końcowej części przedstawiono wyniki obliczeń.

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