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IDENTIFICATION OF THERMOPHYSICAL PARAMETERS OF THE MOULD

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SUMMARY

In the paper the inverse problem consisting in the estimation of thermophysical parameters of the mould in the system casting-mould-environment is presented. On the basis of the knowledge of heating curves at selected points from the mould the thermal conductivity and volumetric specific heat of the mould are identified. In order to solve the problem the least squares criterion in which the sensitivity coefficients appear has been used, on the stage of numerical computations the boundary element method has been applied. In the final part of the paper the results of computations are shown.

Key words: solidification, numerical modelling, inverse problem, parameter estimation

1. FORMULATION OF THE PROBLEM

The 1D casting-mould system is considered. Transient temperature field in casting sub-domain determines the energy equation

$$0 < x < L_1 : \quad C(T)\frac{\partial T(x,\, t)}{\partial t} = \lambda \frac{\partial^2 T(x,\, t)}{\partial x^2} \tag{1}$$

where $C(T)$ is the substitute thermal capacity [1], λ is the thermal conductivity, T is

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the temperature, x is the spatial co-ordinate and t is the time.
The substitute thermal capacity for cast steel is defined as follows

$$C(T) = \begin{cases} c_S & T < T_S \\ c_P & T_S \le T < T_L \\ c_L & T \ge T_L \end{cases} \tag{2}$$

where c_S, c_P, c_L are the volumetric specific heats for liquid, mushy zone and solid state, T_S and T_L correspond to solidus and liquidus temperatures, respectively [1].
A temperature field in mould sub-domain describes the equation of the form

$$L_1 < x < L: \quad c_m \frac{\partial T_m(x, t)}{\partial t} = \lambda_m \frac{\partial^2 T_m(x, t)}{\partial x^2} \tag{3}$$

where the values of thermal conductivity λ_m and volumetric specific heat c_m are unknown. On the contact surface between casting and mould the continuity condition

$$x = L_1: \quad \begin{cases} -\lambda \dfrac{\partial T(x, t)}{\partial x} = -\lambda_m \dfrac{\partial T_m(x, t)}{\partial x} \\ T(x, t) = T_m(x, t) \end{cases} \tag{4}$$

is assumed. For the outer surface of the system the heat transfer process is determined by condition

$$x = L: \quad q_m(x, t) = -\lambda_m \frac{\partial T_m(x, t)}{\partial x} = \alpha \left[T_m(x, t) - T^{\infty} \right] \tag{5}$$

where α is the heat transfer coefficient, T^{∞} is the ambient temperature.
For the moment $t = 0$:

$$T(x, 0) = T_0(x) \qquad T_m(x, 0) = T_{m0}(x) \tag{6}$$

Additionally, the values $T_{d\,i}^{\,f}$ at the selected set of points x_i from mould sub-domain for times t^f are known, namely

$$T_{d\,i}^{\,f} = T_d(x_i, t^f), \quad i = 1, 2, \ldots, M, \quad f = 1, 2, \ldots, F \tag{7}$$

2. SOLUTION OF THE INVERSE PROBLEM

In order to solve the inverse problem, the least squares criterion is applied [2]

$$S(\lambda_m, c_m) = \sum_{i=1}^{M}\sum_{f=1}^{F}\left(T_i^f - T_{di}^f\right)^2 \tag{8}$$

where $T_i^f = T(x_i, t^f)$ is the temperature at the point x_i for time t^f.

Differentiating the criterion (8) with respect to the unknown thermal conductivity λ_m and volumetric specific heat c_m and using the necessary condition of minimum, one obtains

$$\begin{cases} \dfrac{\partial S}{\partial \lambda_m} = 2\sum_{i=1}^{M}\sum_{f=1}^{F}\left(T_i^f - T_{di}^f\right)\dfrac{\partial T_i^f}{\partial \lambda_m}\Bigg|_{\lambda_m=\lambda_m^k} = 0 \\ \dfrac{\partial S}{\partial c_m} = 2\sum_{i=1}^{M}\sum_{f=1}^{F}\left(T_i^f - T_{di}^f\right)\dfrac{\partial T_i^f}{\partial c_m}\Bigg|_{c_m=c_m^k} = 0 \end{cases} \tag{9}$$

where k is the number of iteration, λ_m^k, c_m^k for $k = 0$ are the arbitrary assumed values of λ_m, c_m, while λ_m^k, c_m^k for $k > 0$ result from the previous iteration.

Function T_i^f is expanded in a Taylor series about known values of λ_m^k, c_m^k, this means

$$T_i^f = \left(T_i^f\right)^k + \frac{\partial T_i^f}{\partial \lambda_m}\Bigg|_{\lambda_m=\lambda_m^k}(\lambda_m^{k+1} - \lambda_m^k) + \frac{\partial T_i^f}{\partial c_m}\Bigg|_{c_m=c_m^k}(c_m^{k+1} - c_m^k) \tag{10}$$

or

$$T_i^f = \left(T_i^f\right)^k + \left(Z_{im1}^f\right)^k(\lambda_m^{k+1} - \lambda_m^k) + \left(Z_{im2}^f\right)^k(c_m^{k+1} - c_m^k) \tag{11}$$

where $\left(Z_{im1}^f\right)^k$, $\left(Z_{im2}^f\right)^k$ are the sensitivity coefficients. Putting (11) into (9) one has

$$\begin{bmatrix} \sum_{i=1}^{M}\sum_{f=1}^{F}\left[\left(Z_{im1}^f\right)^k\right]^2 & \sum_{i=1}^{M}\sum_{f=1}^{F}\left(Z_{im1}^f\right)^k\left(Z_{im2}^f\right)^k \\ \sum_{i=1}^{M}\sum_{f=1}^{F}\left(Z_{im2}^f\right)^k\left(Z_{im1}^f\right)^k & \sum_{i=1}^{M}\sum_{f=1}^{F}\left[\left(Z_{im2}^f\right)^k\right]^2 \end{bmatrix}\begin{bmatrix} \lambda_m^{k+1} - \lambda_m^k \\ c_m^{k+1} - c_m^k \end{bmatrix} =$$

$$\begin{bmatrix} \sum_{i=1}^{M}\sum_{f=1}^{F}\left(Z_{im1}^{f}\right)^{k}\left[T_{di}^{f}-\left(T_{i}^{f}\right)^{k}\right] \\ \sum_{i=1}^{M}\sum_{f=1}^{F}\left(Z_{im2}^{f}\right)^{k}\left[T_{di}^{f}-\left(T_{i}^{f}\right)^{k}\right] \end{bmatrix} \tag{12}$$

This system of equations allows to find the values λ_m^{k+1}, c_m^{k+1}. The iteration process is stopped when the assumed accuracy is achieved. In order to determine the sensitivity coefficients, the governing equations should be differentiated with respect to λ_m and c_m, respectively. So, the following additional problems should be solved

$$\begin{aligned}
&0 < x < L_1: \ C(T)\frac{\partial Z_1(x,t)}{\partial t} = \lambda \frac{\partial^2 Z_1(x,t)}{\partial x^2} \\
&L_1 < x < L: \ c_m \frac{\partial Z_{m1}(x,t)}{\partial t} = \lambda_m \frac{\partial^2 Z_{m1}(x,t)}{\partial x^2} + \frac{c_m}{\lambda_m}\frac{\partial T_m(x,t)}{\partial t} \\
&x = L_1: \ \begin{cases} Z_1(x,t) = Z_{m1}(x,t) \\ W_1(x,t) = W_{m1}(x,t) + \dfrac{1}{\lambda_m} q_m(x,t) \end{cases} \\
&x = 0: \ W_1(x,t) = 0, \quad x = L: \ W_{m1}(x,t) = \alpha Z_{m1}(x,t) - \frac{1}{\lambda_m} q(t) \\
&t = 0: \ Z_1(x,0) = 0, \quad Z_{m1}(x,0) = 0
\end{aligned} \tag{13}$$

and

$$\begin{aligned}
&0 < x < L_1: \ C(T)\frac{\partial Z_2(x,t)}{\partial t} = \lambda \frac{\partial^2 Z_2(x,t)}{\partial x^2} \\
&L_1 < x < L: \ c_m \frac{\partial Z_{m2}(x,t)}{\partial t} = \lambda_m \frac{\partial^2 Z_{m2}(x,t)}{\partial x^2} - \frac{\partial T_m(x,t)}{\partial t} \\
&x = L_1: \ \begin{cases} Z_2(x,t) = Z_{m2}(x,t) \\ W_2(x,t) = W_{m2}(x,t) \end{cases} \\
&x = 0: \ W_2(x,t) = 0, \quad x = L: \ W_{m2}(x,t) = \alpha Z_{m2}(x,t) \\
&t = 0: \ Z_2(x,0) = 0, \quad Z_{m2}(x,0) = 0
\end{aligned} \tag{14}$$

where $W_1 = -\lambda \partial Z_1 / \partial x$, $W_{m1} = -\lambda_m \partial Z_{m1} / \partial x$ and $W_2 = -\lambda \partial Z_2 / \partial x$, $W_{m2} = -\lambda_m \partial Z_{m2} / \partial x$.

3. RESULTS OF COMPUTATIONS

For each iterations the basic problem and additional ones connected with the sensitivity analysis have been solved using the 1st scheme of the boundary element method supplemented by the artificial procedure [3].

The 1D casting-mould system of dimensions $2L_1$=0.02 [m] (casting) and 0.03 [m] (mould) has been considered. The following input data have been introduced: λ = 35 [W/mK], c_S = $5.175\cdot10^6$ [J/m^{3}K], c_P = $1.118\cdot10^8$, c_L = $5.74\cdot10^6$, pouring temperature T_0 =1570 °C, liquidus temperature T_L = 1505 °C, solidus temperature T_S = 1470 °C, initial mould temperature T_{m0} =30 °C, heat transfer coefficient α = 10 [W/m^{2}K], ambient temperature T^∞ = 30 °C.

In order to identify the values of λ_m and c_m the courses of heating curves (c.f. equation (7)) at the points x_1 = 0.03 [m], x_2 = 0.033 [m] and x_3 = 0.036 [m] have been taken into account. They result from the direct problem solution under the assumption that λ_m=2.6 [W/mK] and c_m=1.75 [MJ/m^{3}K]. Figure 2 illustrates the solution of inverse problem for initial values λ_m^0=3 [W/mK] and c_m^0=1.75 [MJ/m^{3}K]. It is visible that the iteration process for the assumed initial values of parameters is convergent and the exact solution is obtained after 6 iterations.

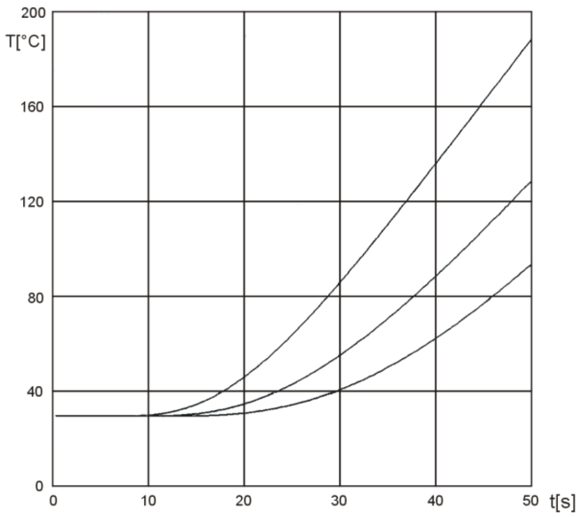

Fig. 1. Heating curves
Rys. 1. Krzywe nagrzewania

The testing computations show that one can assume the values of λ_m^0 and c_m^0 for which the iteration process is unfortunately not convergent. So, it seems that the initial values of parameters discussed should be rather close to the real ones.

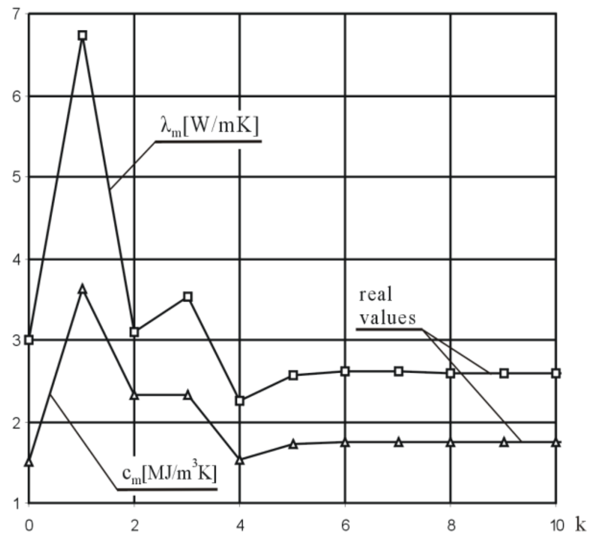

Fig. 2. Inverse problem solution
Rys. 2. Rozwiązanie zadania odwrotnego

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IDENTYFIKACJA PARAMETRÓW TERMOFIZYCZNYCH MASY FORMIERSKIEJ

STRESZCZENIE

W pracy rozważano zadanie odwrotne polegające na odtworzeniu wartości parametrów termofizycznych masy formierskiej. Problem rozwiązano wykorzystując kryterium najmniejszych kwadratów, w którym pojawiają się tzw. Współczynniki wrażliwości. Dodatkowo założono, że znane są krzywe nagrzewania w kilku punktach masy formierskiej. Na etapie obliczeń numerycznych zastosowano metodę elementów brzegowych uzupełnioną procedurą sztucznego źródła ciepła. W końcowej części artykułu pokazano przykład identyfikacji parametrów masy formierskiej.

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