

SENSITIVITY ANALYSIS OF SOLIDIFICATION PROCESS WITH RESPECT TO THE GEOMETRICAL PARAMETERS OF CASTING AND MOULD

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SUMMARY

In the paper the sensitivity analysis of solidification process with respect to the casting and mould thicknesses (problem 1D) is presented. The aim of investigations is to estimate the change of temperature in the system considered due to a change of position of contact surface between casting and mould sub-domains.

Key words: solidification, numerical modelling, shape sensitivity analysis

1. SOLIDIFICATION MODEL

The 1D casting-mould system is considered - Figure 1. The temperature field in casting sub-domain determines the energy equation

$$0 < x < L_1 : \quad C(T) \frac{\partial T(x, t)}{\partial t} = \lambda \frac{\partial^2 T(x, t)}{\partial x^2} \quad (1)$$

where $C(T)$ is the substitute thermal capacity [1], λ is the thermal conductivity.
A temperature field in mould sub-domain describes the equation of the form

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$$L_1 < x < L: \quad c_m \frac{\partial T_m(x, t)}{\partial t} = \lambda_m \frac{\partial^2 T_m(x, t)}{\partial x^2} \quad (2)$$

On the contact surface between casting and mould the continuity condition

$$x = L_1 : \quad \begin{cases} -\lambda \frac{\partial T(x, t)}{\partial x} = \lambda_m \frac{\partial T_m(x, t)}{\partial x} \\ T(x, t) = T_m(x, t) \end{cases} \quad (3)$$

is assumed. The remaining boundary conditions are following

$$\begin{cases} x = 0 : \quad \lambda \frac{\partial T(x, t)}{\partial x} = 0 \\ x = L : \quad -\lambda_m \frac{\partial T_m(x, t)}{\partial x} = 0 \end{cases} \quad (4)$$

For the moment $t=0$:

$$T(x, 0) = T_0 \quad T_m(x, 0) = T_{m0} \quad (5)$$

2. SHAPE SENSITIVITY ANALYSIS

Among the different sensitivity problems, especially important are the shape sensitivity ones [2]. We assume that $b=L_1$ is the shape design parameter - Figure 1. This parameter corresponds to the position of contact surface between casting and mould sub-domains.

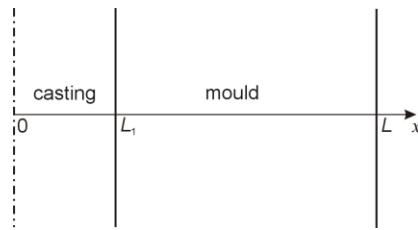


Fig. 1. Domain considered
Rys. 1. Rozpatrywany obszar

Using the concept of material derivative we can write [2]

$$\frac{DT}{Db} = \frac{\partial T}{\partial b} + \frac{\partial T}{\partial x} v \quad (6)$$

where $T(x, t)$ is the temperature, $v = v(x, b)$ is the velocity associated with design parameter b .

If the direct approach of sensitivity method is applied [2] then the governing equations are differentiated with respect to shape parameter b . So, the differentiation of equation (1) gives [3]

$$\begin{aligned} \frac{\mathbf{D} C(T)}{\mathbf{D} b} \frac{\partial T(x, t)}{\partial t} + C(T) \frac{\partial U(x, t)}{\partial t} = \\ \lambda \left[\frac{\partial^2 U(x, t)}{\partial x^2} - 2 \frac{\partial^2 T(x, t)}{\partial x^2} \frac{\partial v}{\partial x} - \frac{\partial T(x, t)}{\partial x} \frac{\partial^2 v}{\partial x^2} \right] \end{aligned} \quad (7)$$

Assuming that the substitute thermal capacity is described by staircase function [1] one obtains

$$C(T) \frac{\partial U(x, t)}{\partial t} = \lambda \frac{\partial^2 U(x, t)}{\partial x^2} - 2C(T) \frac{\partial T(x, t)}{\partial t} \frac{\partial v}{\partial x} - \lambda \frac{\partial T(x, t)}{\partial x} \frac{\partial^2 v}{\partial x^2} \quad (8)$$

where $U(x, t) = DT / Db$ is the sensitivity function.

In similar way we differentiate the equation (2) and then

$$c_m \frac{\partial U_m(x, t)}{\partial t} = \lambda_m \frac{\partial^2 U_m(x, t)}{\partial x^2} - 2c_m \frac{\partial T_m(x, t)}{\partial t} \frac{\partial v}{\partial x} - \lambda_m \frac{\partial T_m(x, t)}{\partial x} \frac{\partial^2 v}{\partial x^2} \quad (9)$$

where $U_m(x, t) = DT_m / Db$.

Differentiation of equations (3), (4), (5) leads to the following conditions

$$x = L_1 : \begin{cases} -\lambda \left[\frac{\partial U(x, t)}{\partial x} - \frac{\partial T(x, t)}{\partial x} \frac{\partial v}{\partial x} \right] = \lambda_m \left[\frac{\partial U_m(x, t)}{\partial x} - \frac{\partial T_m(x, t)}{\partial x} \frac{\partial v}{\partial x} \right] \\ U(x, t) = U_m(x, t) \end{cases} \quad (10)$$

and

$$\begin{cases} x = 0 : \quad \lambda \frac{\partial U(x, t)}{\partial x} = 0 \\ x = L : \quad -\lambda_m \frac{\partial U_m(x, t)}{\partial x} = 0 \end{cases} \quad (11)$$

while

$$t = 0 : \quad U(x, 0) = 0 \quad U_m(x, 0) = 0 \quad (12)$$

In order to realize the shape sensitivity analysis of solidification process with respect to the mould thickness, the following definition of velocity associated with design parameter $b=L_1$ can be accepted

$$v = \begin{cases} \frac{x}{b}, & 0 \leq x \leq L_1 \\ \frac{L-x}{L-b}, & L_1 \leq x \leq L \end{cases} \quad (13)$$

The equations connected with the sensitivity functions $U(x, t)$ and $U_m(x, t)$ have the following form (c.f. equation (8))

$$0 < x < L_1 : C(T) \frac{\partial U(x, t)}{\partial t} = \lambda \frac{\partial^2 U(x, t)}{\partial x^2} - \frac{2}{b} C(T) \frac{\partial T(x, t)}{\partial t} \quad (14)$$

and (c.f. equation (9))

$$L_1 < x < L : c_m \frac{\partial U_m(x, t)}{\partial t} = \lambda_m \frac{\partial^2 U_m(x, t)}{\partial x^2} - \frac{2c_m}{L-b} \frac{\partial T_m(x, t)}{\partial t} \quad (15)$$

On the contact surface between casting and mould we have

$$x = L_1 : \begin{cases} -\lambda \frac{\partial U(x, t)}{\partial x} + \frac{\lambda}{b} \frac{\partial T(x, t)}{\partial x} = \lambda_m \frac{\partial U_m(x, t)}{\partial x} + \frac{\lambda_m}{L-b} \frac{\partial T_m(x, t)}{\partial x} \\ U(x, t) = U_m(x, t) \end{cases} \quad (16)$$

The remaining conditions (11) and (12) are not changing.

3. RESULTS OF COMPUTATIONS

The basic and additional problems have been solved using the 1 st scheme of the boundary element method supplemented by artificial heat source method [4]. The 1D casting-mould system of dimensions $2L_1 = 0.02$ [m] (casting) and 0.03 [m] (mould) has been considered. The following input data have been introduced: $\lambda = 35$ [W/mK], $\lambda_m = 2.6$, $c_s = 5.175 \cdot 10^6$ [J/m³K], $c_p = 1.118 \cdot 10^8$, $c_L = 5.74 \cdot 10^6$, $c_m = 1.75 \cdot 10^6$, pouring temperature $T_0 = 1570$ °C, liquidus temperature $T_L = 1505$ °C, solidus temperature $T_S = 1470$ °C, initial mould temperature $T_{m0} = 30$ °C. In Figure 2 the sensitivity functions $U(x, t)$, $U_m(x, t)$ multiplied by the change of the parameter considered, this means $\Delta L_1 = 0.1 L_1$ for times 5, 10, 15, ..., 60 [s] are shown.

Figures 3 and 4 illustrate the changes of temperature due to a change of parameter L_1 .

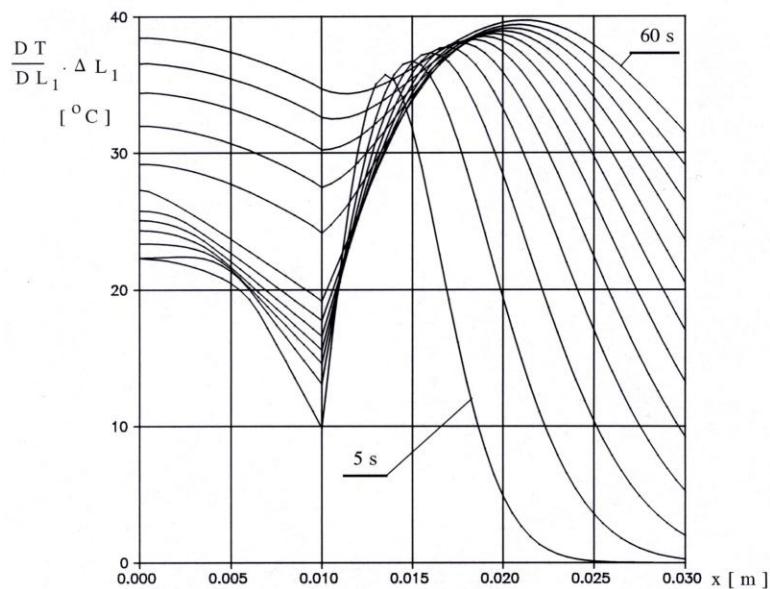


Fig. 2. Sensitivity function
Rys. 2. Funkcja wrażliwości

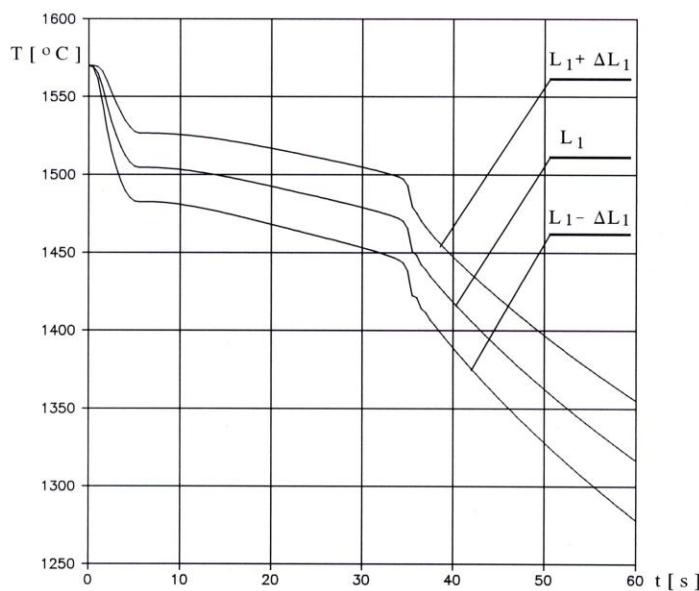


Fig. 3. Cooling curves (axis of symmetry)
Rys. 3. Krzywe stygnięcia (oś symetrii)

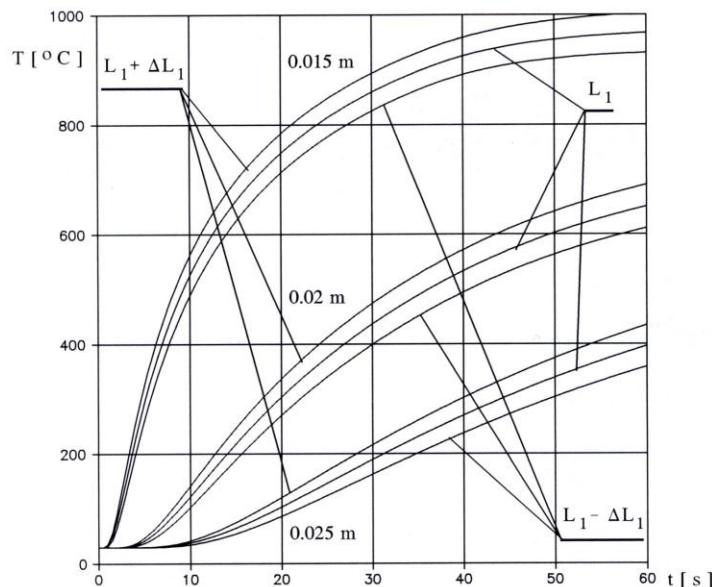


Fig. 4. Heating curves (mould)
Rys. 4. Krzywe nagrzewania (forma)

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ANALIZA WRAŻLIWOŚCI PROCESU KRZEPNIĘCIA ZE WZGLĘDU NA PARAMETRY GEOMETRYCZNE ODLEWU I FORMY

STRESZCZENIE

W pracy przedstawiono analizę wrażliwości procesu krzepnięcia ze względu na grubość odlewów i formy odlewniczej (zadanie 1D). Głównym celem badań było oszacowanie zmian temperatury w rozważanym układzie spowodowanych zmianą położenia granicy kontaktu między odlewem i formą.

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