

**IDENTIFICATION OF EXTERNAL HEAT FLUX
IN THE SYSTEM CASTING-MOULD-ENVIRONMENT**E. MAJCHRZAK¹, D. JANISZ², G. KAŁUŻA³^{1,2,3} Department for Strength of Materials and Computational Mechanics
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SUMMARY

In the paper the inverse problem consisting in the identification of external heat flux in the system casting-mould-environment is presented. On the basis of the knowledge of heating curves at selected points from the mould the time dependent value of boundary heat flux is identified. In order to solve the problem the sequential function specification method [1, 2] has been used, on the stage of numerical computations the boundary element method has been applied. In the final part the results of computations are shown.

Key words: solidification, numerical modelling, inverse problem

1. FORMULATION OF THE PROBLEM

The 1D casting-mould system is considered. Transient temperature field in casting sub-domain determines the energy equation

$$0 < x < L_1 : C(T) \frac{\partial T(x,t)}{\partial t} = \lambda \frac{\partial^2 T(x,t)}{\partial x^2} \quad (1)$$

where $C(T)$ is the substitute thermal capacity [3], λ is the thermal conductivity, T is

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the temperature, x is the spatial co-ordinate and t is the time.

The substitute thermal capacity for cast steel is defined as follows

$$C(T) = \begin{cases} c_S & T < T_S \\ c_P & T_S \leq T < T_L \\ c_L & T \geq T_L \end{cases} \quad (2)$$

where c_S , c_P , c_L are the volumetric specific heats for liquid, mushy zone and solid state, T_S and T_L correspond to solidus and liquidus temperatures, respectively [3]. A temperature field in mould sub-domain describes the equation of the form

$$L_1 < x < L: \quad c_m \frac{\partial T_m(x,t)}{\partial t} = \lambda_m \frac{\partial^2 T_m(x,t)}{\partial x^2} \quad (3)$$

On the contact surface between casting and mould the continuity condition

$$x = L_1: \quad \begin{cases} -\lambda \frac{\partial T(x,t)}{\partial x} = -\lambda_m \frac{\partial T_m(x,t)}{\partial x} \\ T(x,t) = T_m(x,t) \end{cases} \quad (4)$$

is assumed. For the outer surface of the system the heat transfer process is determined by condition

$$x = L: \quad q_m(x,t) = -\lambda_m \frac{\partial T_m(x,t)}{\partial x} = q(t) \quad (5)$$

and course of function $q(t)$ is unknown.

For the moment $t=0$:

$$T(x,0) = T_0(x) \quad T_m(x,0) = T_{m0}(x) \quad (6)$$

Additionally, the values T_{di}^f at the selected set of points x_i from mould sub-domain for times t^f are known, namely

$$T_{di}^f = T_d(x_i, t^f), \quad i = 1, 2, \dots, M \quad f = 1, 2, \dots, F \quad (7)$$

In order to solve the inverse problem the sequential function specification method [1, 2] is applied.

2. SEQUENTIAL FUNCTION SPECIFICATION METHOD

In the sequential function specification method [1, 2] the sensitivity coefficients are used. In order to calculate them, the governing equations are differentiated with respect to the unknown boundary heat flux. So, for casting sub-domain one has (c.f. equation (1))

$$0 < x < L_1: C(T) \frac{\partial Z(x,t)}{\partial t} = \lambda \frac{\partial^2 Z(x,t)}{\partial x^2} \quad (8)$$

while for mould sub-domain (c.f. equation (3))

$$L_1 < x < L: c_m \frac{\partial Z_m(x,t)}{\partial t} = \lambda_m \frac{\partial^2 Z_m(x,t)}{\partial x^2} \quad (9)$$

where

$$Z(x,t) = \frac{\partial T(x,t)}{\partial q}, \quad Z_m(x,t) = \frac{\partial T_m(x,t)}{\partial q} \quad (10)$$

are the sensitivity functions.

A differentiation of the boundary and initial conditions with respect to q gives
– continuity conditions (c.f. equation (4))

$$x = L_1: \begin{cases} -\lambda \frac{\partial Z(x,t)}{\partial x} = -\lambda_m \frac{\partial Z_m(x,t)}{\partial x} \\ Z(x,t) = Z_m(x,t) \end{cases} \quad (11)$$

– Neumann condition (c.f. equation (5))

$$x = L: -\lambda_m \frac{\partial Z_m(x,t)}{\partial x} = 1 \quad (12)$$

– initial condition

$$t = 0: Z(x,t) = 0, \quad Z_m(x,t) = 0 \quad (13)$$

The additional problem can be solved directly - it is correctly posed because both the differential equations determining the distribution of sensitivity functions in the sub-domains considered and also the boundary initial conditions are known.

Owing to the discrete nature of temperature data (7) the unknown function $q(t)$ must also be expressed in a discrete form, for example

$$t \in [t^{f-1}, t^f] : q^f = q(t^f), \quad f = 1, 2, \dots, F \quad (14)$$

It is assumed that the heat flux is known at times t^1, t^2, \dots, t^{f-1} and we want to determine the heat flux q^f at time t^f . Of course, some measured temperature histories are given at interior locations x_i , namely $T_{di}^1, T_{di}^2, \dots, T_{di}^{f-1}$, $i = 1, 2, \dots, M$. This variant of function specification method is called the sequential approach [1, 2]. Additionally, we assume that the temperature histories are known for R future intervals, namely

$$T_{di}^{f+r-1} = T_d(x_i, t^{f+r-1}), \quad r = 1, 2, \dots, R, \quad i = 1, 2, \dots, M \quad (15)$$

and the heat flux is constant over R future steps, namely $q^f = q^{f+1} = \dots = q^{f+R-1}$.

Function $T_i^{f+r-1} = T(x_i, t^{f+r-1})$ is expanded in a Taylor series about arbitrary but known value of heat flux q^{*f}

$$T_i^{f+r-1} = T_i^{*f+r-1} + \left. \frac{\partial T_i^{f+r-1}}{\partial q^f} \right|_{q^f = q^{*f}} (q^f - q^{*f}) \quad (16)$$

where T_i^{*f+r-1} is the temperature at time t^{f+r-1} and location x_i obtained under the assumption that for $t \in [t^{f-1}, t^{f+r-1}]$ the heat flux equals $q^f = q^{f+1} = \dots = q^{f+R-1} = q^{*f}$. We introduce the sensitivity coefficients and then

$$T_i^{f+r-1} = T_i^{*f+r-1} + Z_i^{f+r-1} (q^f - q^{*f}) \quad (17)$$

In order to solve the inverse problem, the least squares method is applied [1, 2]

$$S(q^f) = \sum_{i=1}^M \sum_{r=1}^R (T_i^{f+r-1} - T_{di}^{f+r-1})^2 \rightarrow \text{MIN} \quad (18)$$

Putting (17) into (18) differentiating the criterion (18) with respect to the unknown heat flux q^f and using the necessary condition of minimum, one obtains

$$q^f = q^{*f} + \frac{\sum_{i=1}^M \sum_{r=1}^R (T_{di}^{f+r-1} - T_i^{*f+r-1}) Z_i^{f+r-1}}{\sum_{i=1}^M \sum_{r=1}^R (Z_i^{f+r-1})^2} \quad (19)$$

In Figure 1 the idea of the inverse problem solution for transition $t^{f-1} \rightarrow t^f$ and $R = 4$ is shown.

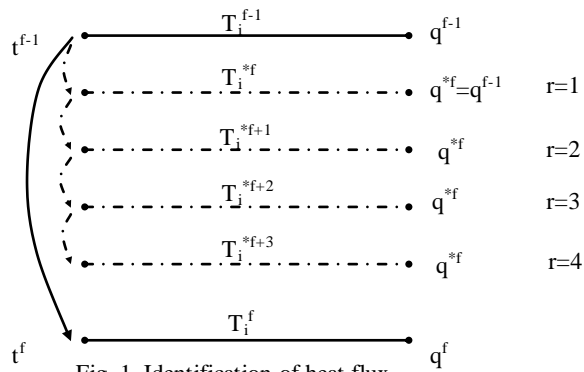


Fig. 1. Identification of heat flux
Rys. 1. Identyfikacja strumienia

The basic problem for the arbitrary assumed value of unknown boundary heat flux and the additional problem connected with sensitivity functions have been solved using the boundary element method [4].

3. RESULTS OF COMPUTATIONS

The 1D casting-mould system of dimensions $2L_1=0.02$ [m] (casting) and 0.03 [m] (mould) has been considered. Figure 2 illustrates the course of heating curves (c.f. equation (7)) at the points $x_1 = 0.03$ [m], $x_2=0.033$ [m] and $x_3=0.036$ [m].

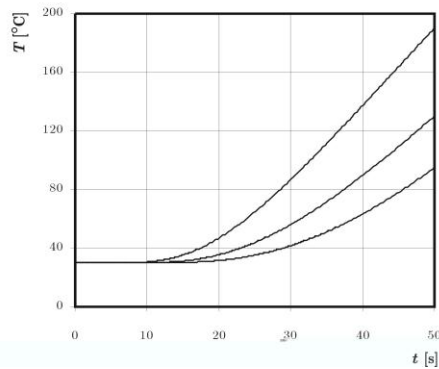


Fig. 2. Heating curves
Rys. 2. Krzywe nagrzewania

The following input data have been introduced: $\lambda=35$ [W/mK], $\lambda_m=2.6$ [W/mK], $c_S=5.175 \cdot 10^6$ [J/m³K], $c_P=1.118 \cdot 10^8$, $c_L=5.74 \cdot 10^6$, $c_m=1.75 \cdot 10^6$, pouring temperature $T_0=1570$ °C, liquidus temperature $T_L=1505$ °C, solidus temperature $T_S=1470$ °C, initial mould temperature $T_{m0}=30$ °C. In Figure 3 the inverse problem solution is shown.

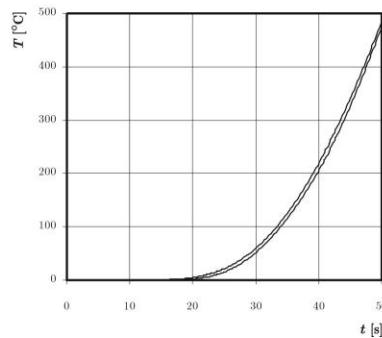


Fig. 3. Real and identified heat fluxes
Rys. 3. Rzeczywisty i odtworzony strumień

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IDENTYFIKACJA BRZEGOWEGO STRUMIEIA CIEPŁA W UKŁADZIE ODLEW-FORMA-OTOCZENIE

STRESZCZENIE

W pracy przedstawiono metodę rozwiązania zadania odwrotnego polegającą na odtworzeniu brzegowego strumienia ciepła w układzie odlew-forma-otoczenie. Przebieg tego strumienia ciepła identyfikowano na podstawie znajomości krzywych nagrzewania w kilku punktach masy formierskiej. Do rozwiązania zadania wykorzystano sekwencyjną metodę specyfikacji funkcji [1, 2], a na etapie obliczeń numerycznych zastosowano metodę elementów brzegowych. W końcowej części przedstawiono wyniki obliczeń.

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