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IDENTIFICATION OF BOUNDARY HEAT FLUX USING THE SENSITIVITY COEFFICIENTS

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SUMMARY

In the paper the method of boundary heat flux identification is presented. The algorithm is constructed on the basis of the least squares criterion in which the sensitivity coefficients are introduced. The method can be useful for the analysis of continuous casting technology. In the final part the example of computations is shown.

Key words: boundary elements method, least squares criterion

1. FORMULATION OF THE PROBLEM

The following 2D inverse problem is considered – c.f. Figure 1

$$\begin{cases} x \in \Omega: & \nabla^2 T(x) = 0 \\ x \in \Gamma_1: & q(x) = -\lambda \, \partial T(\mathrm{x}) / \partial n = ? \\ x \in \Gamma_2: & T(\mathrm{x}) = T_b \\ x \in \Gamma_3: & q(\mathrm{x}) = q_b \\ \xi^i \in \Omega: & T_i^d - \text{known}, \quad i = N+1, N+2, \ldots, N+M \end{cases} \tag{1}$$

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where: T_b is a given boundary temperature, q_b is a known boundary heat flux, $\partial T / \partial n$ denotes the normal derivative at the boundary point. The aim of investigations is to determine the boundary heat flux on Γ_1. Additionally, the information concerning the values of temperature at the set of internal points from the domain considered is also given.

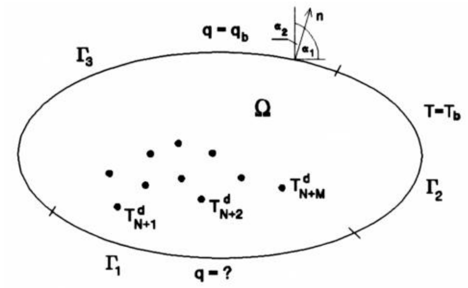

Fig. 1. Problem considered
Rys.1. Rozważane zadanie

2. BOUNDARY ELEMENT METHOD

The boundary $\Gamma=\Gamma_1\cup\Gamma_2\cup\Gamma_3$ is divided into N constant boundary elements – Figure 2. At first, we consider the direct problem, in which the all boundary conditions are known, this means we arbitrary impose the values of $q_j=q_j^*$ for $j=1, 2, \ldots, N_1$.

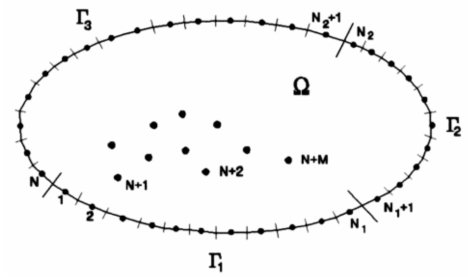

Fig. 2. Discretization of boundary Γ
Rys. 2. Dyskretyzacja brzegu Γ

The boundary element method [1, 2] leads to the following system of equations

$$\sum_{j=1}^{N} G_{ij} q_j = \sum_{j=1}^{N} H_{ij} T_j \tag{2}$$

where [1]

$$G_{ij} = \frac{1}{2\pi\lambda}\int_{\Gamma_j} \ln\frac{1}{r_{ij}} d\Gamma_j \tag{3}$$

and

$$H_{ij} = \begin{cases} \frac{1}{2\pi}\int_{\Gamma_j} \frac{d_{ij}}{r_{ij}^2} d\Gamma_j, & i \neq j \\ -0.5, & i = j \end{cases} \tag{4}$$

At the same time

$$r = \sqrt{(x_1^j - \xi_1^i)^2 + (x_2^j - \xi_2^i)^2}\,, \quad d = (x_1^j - \xi_1^i)\cos\alpha_1 + (x_2^j - \xi_2^i)\cos\alpha_2 \tag{5}$$

In equation (5) $\xi = (\xi_1, \xi_2)$ is the observation point. The system of equations (2) allows to find the missing boundary temperatures and heat fluxes at the points resulting from the boundary discretization (c.f. Figure 2). In the second stage of computations the internal temperatures can be found using the formula

$$T_i = \sum_{j=1}^{N} H_{ij} T_j - \sum_{j=1}^{N} G_{ij} q_j \tag{6}$$

The solution obtained in this way we denote by T_i^*.

3. SOLUTION OF INVERSE PROBLEM

As it was mentioned, we want to find the boundary heat fluxes q_j, $j=1, 2, \ldots, N_1$ at selected set of pints along the Γ_1. In order to solve the problem, we differentiate the mathematical model (1) with respect to q_k, $k=1, 2, \ldots, N_1$ [3]. We obtain

$$\begin{cases} x \in \Omega: & \nabla^2 \left(\frac{\partial T}{\partial q_k} \right) = 0 \\ x \in \Gamma_1: & \frac{\partial q_j}{\partial q_k} = \begin{cases} 1, & j = k \\ 0, & j \neq k \end{cases}, \qquad j = 1,\ 2,\ \ldots,\ N_1 \\ x \in \Gamma_2: & \frac{\partial T_j}{\partial q_k} = \frac{\partial T_b}{\partial q_k} = 0\,, \qquad j = N_1 + 1,\ N_1 + 2,\ \ldots,\ N_2 \\ x \in \Gamma_3: & \frac{\partial q_j}{\partial q_k} = \frac{\partial q_b}{\partial q_k} = 0\,, \qquad j = N_2 + 1,\ N_2 + 2,\ \ldots,\ N \end{cases} \tag{7}$$

or

$$\begin{cases} x\in\Omega: \quad \nabla^2 Z_k = 0 \\ x\in\Gamma_1: \quad Z_k^j = \begin{cases}1, & j=k\\ 0, & j\neq k\end{cases}, \quad j=1,2,\dots,N_1 \\ x\in\Gamma_2: \quad Z_k^j = 0, \qquad j=N_1+1, N_1+2,\dots,N_2 \\ x\in\Gamma_3: \ -\lambda\dfrac{\partial Z_k^j}{\partial n} = 0, \quad j=N_2+1, N_2+2,\dots,N \end{cases} \tag{8}$$

One can notice that the problems described by (8) are correctly posed and can be treated as the direct ones. So, we can use the same algorithm as in chapter 2 once for every sensitivity coefficient Z_k. In this way we determine the set of sensitivity coefficients at internal points ξ^i for which the temperatures are known (measured) – see Figure 2. We denote

$$Z_k^i = \frac{\partial T_i}{\partial q_k}, \quad i = N+1,\ N+2,\ \dots, N+M \tag{9}$$

Now, we expand the temperature T_i into a Taylor series in the vicinity of point (T_i^*, q_k^*) taking into account the first and second component:

$$T_i = T_i^* + \sum_{k=1}^{N_1} Z_k^i (q_k - q_k^*), \quad i = N+1,\ N+2,\ \dots, N+M \tag{10}$$

In order to solve the problem (1), the least squares criterion is applied

$$S = \sum_{i=N+1}^{N+M} \left(T_i - T_i^d\right)^2 \to \quad MIN \tag{11}$$

Differentiating the criterion (11) with respect to the unknown heat fluxes q_j and using the necessary condition of minimum one obtains

$$\frac{\partial S}{\partial q_l} = 2\sum_{i=N+1}^{N+M}\left(T^i - T_i^d\right)\frac{\partial T^i}{\partial q_l} = 0, \quad l = 1,\ 2,\ \dots,\ N_1 \tag{12}$$

Putting (10) to (12) we have

$$\sum_{i=N+1}^{N+M}\sum_{k=1}^{N_1} Z_k^i Z_l^i q_k = \sum_{i=N+1}^{N+M} Z_l^i\left(T_i^d - T_i^*\right) + \sum_{l=N+1}^{N+M}\sum_{k=1}^{N_1} Z_k^i Z_l^i q_k^* \tag{13}$$

where $l=1, 2, \ldots, N_1$. This system of equations allows to find the boundary heat fluxes $q_1, q_2, \ldots, q_{N1}$.

4. EXAMPLE OF COMPUTATIONS

The symmetrical fragment of continuous casting mould is considered – c.f. Figure 3 The thickness of the cooper wall equals 5 cm, the diameter of cooling pipe equals 2 cm. The thermal conductivity of the material is assumed to be λ=330 [W/mK]. The boundary conditions and also the temperatures at internal points are marked in Figure 3. These values of temperature approximately correspond to the solution of direct problem for which the heat flux between casting and continuous casting mould equals $-3\cdot 10^5$ [W/m^2], this means $q_j = -3\cdot 10^5$, $j = 1, 2, \ldots, 5$ - c.f. Figure 4.

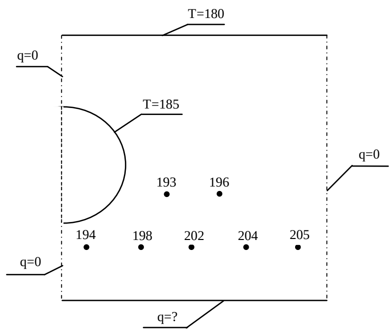

Fig. 3. Boundary conditions and internal temperatures
Rys. 3. Warunki brzegowe i temperatury wewnętrzne

In Figure 4 the discretization of the domain is shown.

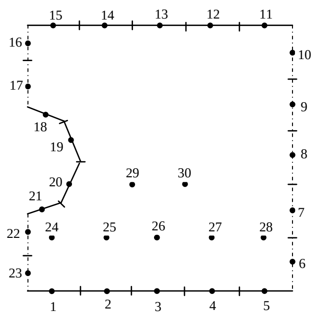

Fig. 4. Discretization
Rys. 4. Dyskretyzacja

The solution of the inverse problem gives the following results:
- the mean heat flux $q_m = -296728.4$ [W/m^2],

- the temperatures at internal points T_{24} = 194.00, T_{25} = 198.03, T_{26} = 202.06, T_{27} = =204.04, T_{28} = 205.02, T_{29} = 192.89, T_{30} = 195.85.

The good exactness of boundary heat flux and internal temperature identification shows that the algorithm presented is quite effective and can be used for the numerical solution of boundary inverse problems.

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ODTWORZENIE BRZEGOWEGO STRUMIENIA CIEPŁA Z WYKORZYSTANIEM WSPÓŁCZYNNIKÓW WRAŻLIWOŚCI

STRESZCZENIE

W pracy przedstawiono metodę identyfikacji brzegowego strumienia ciepła na fragmencie brzegu ograniczającego analizowany obszar ciała stałego. Może być to np. krystalizator urządzenia COS pracującego w warunkach pseudoustalonego przepływu ciepła. Algorytm skonstruowano na podstawie kryterium najmniejszych kwadratów, w którym wykorzystano współczynniki wrażliwości. Jak wspomniano, metoda może być użyteczna przy analizie technologii odlewania ciągłego. W końcowej części pracy pokazano przykład obliczeń numerycznych.

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