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Application of Kaplan-Meier analysis in reliability evaluation of products cast from aluminium alloys

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Abstract

The article evaluates the reliability of AlSi17CuNiMg alloys using Kaplan-Meier-based technique, very popular as a survival estimation tool in medical science. The main object of survival analysis is a group (or groups) of units for which the time of occurrence of an event (failure) taking place after some time of waiting is estimated. For example, in medicine, the failure can be patient's death. In this study, the failure was the specimen fracture during a periodical fatigue test, while the survival time was either the test duration to specimen failure (complete observations), or the test end time (censored observations). The parameters of theoretical survival function were estimated with procedures based on the method of least squares, while typical survival time distribution followed either an exponential or two-parameter Weibull distribution. The goodness of fit of a model survival function was estimated with an incremental chi-square test, based on the values of the log likelihood ratio. The effect of alloy processing history on the run of a survival function was examined. The factors shaping the alloy processing history included: mould type (sand or metal mould), alloy modification process, and heat treatment type (solution heat treatment and ageing).

Keywords: Computer-Aided Casting Production, Kaplan-Meier Survival Analysis, Reliability

1. Introduction

Parts operating under changing loads undergo after some time a degradation process, even if the breaking stresses are lower than the tensile strength or yield point [1]. Because of safety issues, the fatigue strength of materials is of utmost importance for the automotive industry and aviation.

When estimating the reliability of the examined AlSiCuNiMg alloys one can use the techniques directly relating to a survival analysis based on the Kaplan-Meier methods, widely used in medical science. The main object of interest in the survival analysis [2] is a group (or groups) of units for which the time of occurrence of an event is determined. In many cases this is failure that takes place after certain time of waiting. In medicine, for example, the failure can be patient's death. In this study, the failure was the specimen fracture in a periodical fatigue test, while the survival time was either the test time to specimen failure (complete observations with the survival time equal to T), or the test end time (censored observations).

In spite of the fact that the life-time tables (the time to specimen failure) describe with sufficient accuracy the specimen „survival" rate during the test, an analysis of the survival function shape is indispensable for the specimen life estimation [3].

2. Description of problem

The parameters of the theoretical survival function have been estimated with the method of least squares. The typical survival time distribution follows either an exponential or two-parameter Weibull distribution.

The exponential distribution is defined by the following formula:

$$f(x) = \lambda \cdot \qquad -\lambda \cdot \tag{1}$$

where λ is the parameter.

The two-parameter Weibull distribution assumes the following form:

$$f(x) = \frac{}{b}\left(\quad\right) \qquad \left[\;\left(\quad\right)\;\right] \text{ dla } b > 0 \text{ i } c > 0 \tag{2}$$

where: b – is the scale parameter and c – is the shape parameter.

The goodness of model fit was estimated with an incremental chi-square test based on the values of the log likelihood ratio, which verifies the null hypothesis of identity between the theoretical and empirical distributions.

The theoretical distributions, i.e. the exponential distribution and Weibull distribution, were estimated using the method of weighted least squares since, through appropriate transformations, both these distributions can be reduced to their linear counterparts. Using this method, the quantity

$$\sum_{i=} \qquad\qquad \rfloor \tag{3}$$

was minimalised. In the process of estimation, three weights were used, namely:

- Weight 1 = 1, which corresponds to the method of least squares,
- Weight 2 = $1/\nu_i$, where ν_i is the variance of a risk estimator,
- Weight 3 = n_i/h_i, where n_i and h_i are the number of units exposed to risk in the i-th interval and the width of the i-th inverval, respectively.

Yet, the applied method of estimation based on survival tables has a drawback, i.e. an arbitrary grouping of observation time into the time intervals of an equal length. The Kaplan-Meier method is free from this drawback. In this method, the parameters of a survival function are estimated directly from the continuous-time survival analysis. Thus, the searched probability estimation is the product of all successive conditional probabilities estimated separately, which enables censored observations to be included in the estimation. Finally, an estimator, called product limit estimator, is obtained. Another important fact is that the Kaplan-Meier function is independent of the assumed distribution.

So, the survival function, called Kaplan-Meier curve, determines the probability that the specimen will „live" for a time longer than the assumed time t, which means that it shall not fail before the time t has elapsed. If the survival function is denoted as S(t), then its definition will be $S(t)=P(T>t)$, where P is the function determining given probability.

3. Test method

The Kaplan-Meier survival tests were carried out on specimens taken from the AlSi17CuNiMg alloy bars cast to metal moulds in non-modified condition, in modified and heat treated-condition (precipitation hardening), and in non-heat-treated condition. The technique by which the fatigue specimens of the examined hypereutectic AlSiCuNiMg alloy were taken, prepared and heat treated was selected in such a way as to ensure the best possible repeatability of the results. The material for specimens originated from one melt of the same history. Care was taken to machine all specimens in a series in the same very careful way to the roughness level of PN-73/M-04251. The way in which the specimens were taken and prepared for tests was in accordance with the PN-76/H-04325 Standard. The dimensions of the specimens were consistent with the PN-74/H-04327 Standard for an axial tensile and compression test.

The fatigue tests were carried out under the conditions of a symmetric (oscillating) stress cycling, where the maximum σ_{max} and minimum σ_{min} stresses were equal but opposite in signs: $\sigma_{max} = -\sigma_{min.}$ To the changing load of each type, e.g. tensile, compressive, the respective states of the changing stress were ascribed. The time when the stress followed a continuous mode of changes during one single cycle was called stress cycle. The stress cycle was described with changes in the normal stresses σ, as these were the only stresses applied in the investigations [1].

The fatigue strength was tested under a loading of $\sigma_{max} = -\sigma_{min} = 150$ [MPa], using normal-running fatigue test device, which was a mechanical resonance pulsator operating at a 40 Hz oscillation frequency.

Tests were carried out in a mode such that after a preset time of loading (one hour and a half minimum) the test was interrupted and the load withdrawn. If the specimen failed (also before the lapse of preset time), the result expressed as a test time gave "complete information". If the specimen did not fail, the result (also expressed as a test time) was considered "censored information".

4. Statistical analysis

The goodness of fit between the exponential model and two-parameter Weibull model was estimated with an incremental chi-square test, based on the values of a reliability logarithm, which verified the null hypothesis of identity between the theoretical and empirical distributions. The results of this verification, depicted in Fig. 1, clearly indicate that in each case, i.e. for alloy non-modified and non-heat-treated (Fig. 1a, 1e), modified and non-heat-treated (Fig. 1b, 1f), non-modified and heat-treated (Fig. 1c, 1g), and modified and heat-treated (Fig. 1d, 1h), the best model for a description of the survival time distribution is Weibull model.

Figure 2 shows the results of a survival function estimation done for the four subgroups of the examined specimens. The survival times were next compared with each other.

The significance of the difference was estimated through verification of a hypothesis of the form Ho: $S_1(t) = S_2(t) = S_3(t) = S_4(t)$ for all t, that is, in absence of general differences between these survival curves. The value of the chi-square statistic comparing the survival curves was 13,279, which for the three degrees of freedom gave the significance level of p = 0,0041. Hence there are premises to reject the null hypothesis and accept that the survival curves describing different processing histories of the AlSi17CuNiMg alloy differ in a statistically significant way.

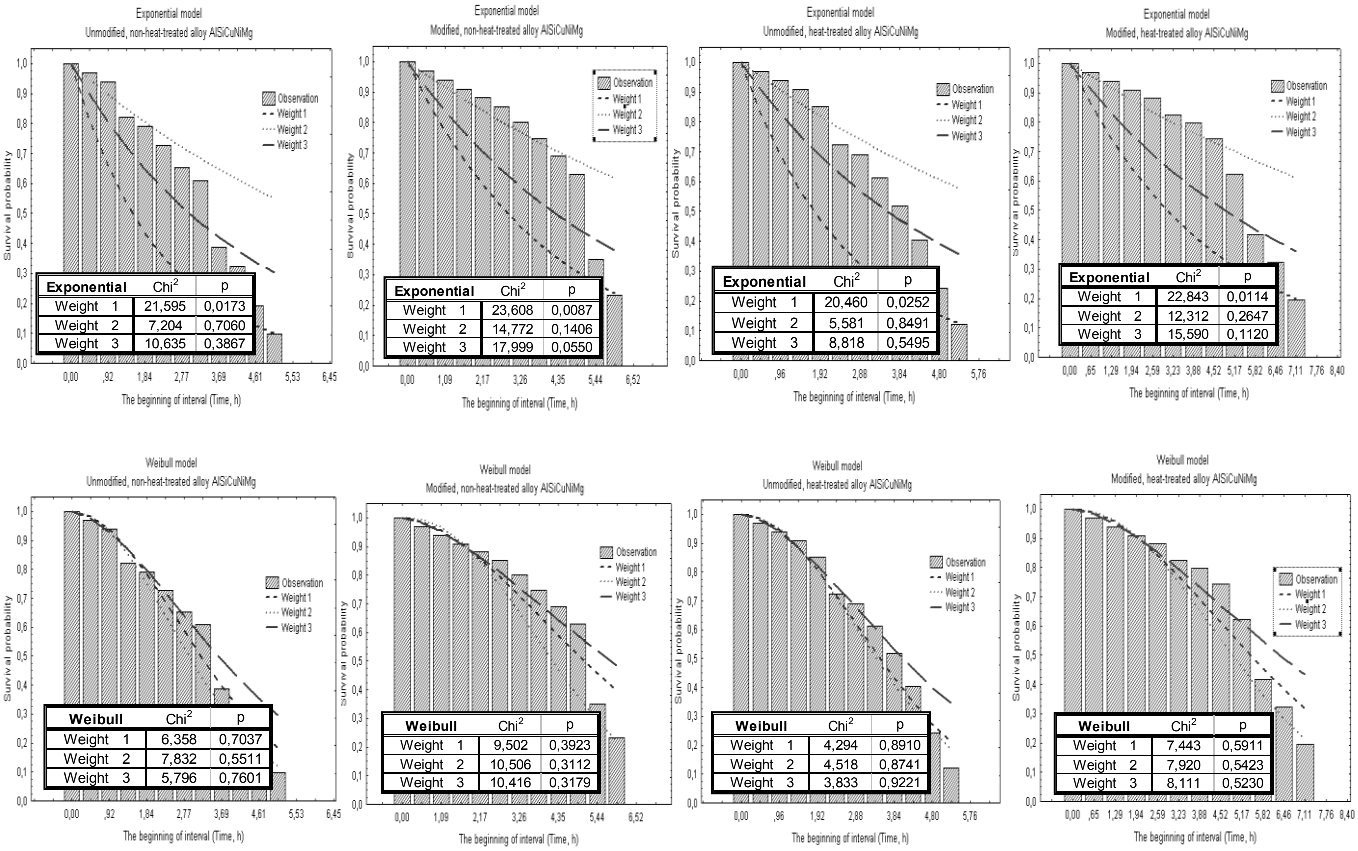

Fig. 1. The survival time distribution estimated according to models: exponential (a÷d) and Weibull (e÷h)

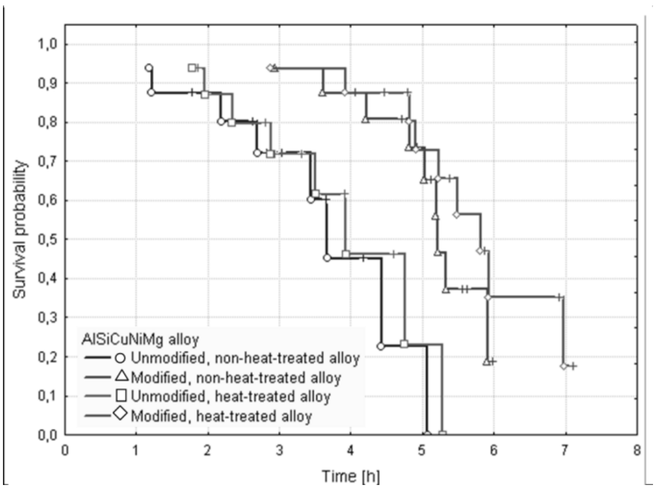

Fig. 2. The survival curves plotted for an AlSi17CuNiMg alloy in function of its processing history

The results of the reliability and damage intensity evaluation are an important tool in the determination of casting quality and enable us to estimate in a reliable way the casting life-time on performance. This is particularly true in the case of elements operating under the conditions of changing dynamic loads.

From an analysis of the plotted Kaplan-Meier survival curves it follows that the longest life-time (survival rate) can offer the specimens of an AlSi17CuNiMg alloy after modification and heat treatment. To confirm this conclusion, the chosen percentiles of the survival function were estimated in 4 groups, as shown in Fig. 3.

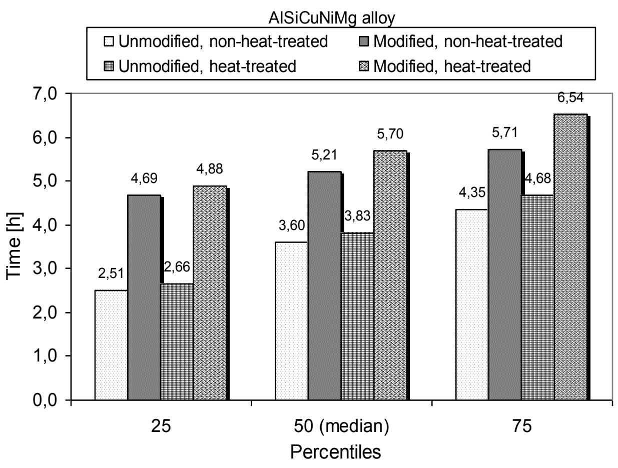

Fig. 3. The values of the selected percentiles of a survival curve plotted for the AlSi17CuNiMg alloy in function of its processing history

From Figure 3 it follows that 50% of the specimens made from a non-modified and non-heat-treated alloy have „survived" under the adopted conditions of fatigue testing only 3,60 hours, while in the modified and heat-treated condition the survival rate was extended to as much as 5,70 hours.

The first task of a correctly conducted Kaplan-Meier survival analysis is to estimate the goodness of fit of the model itself, using for this purpose an incremental chi-square test based on the values of the log likelihood ratio. The test verifies the null hypothesis and tells us if the empirical distribution is consistent with a reference theoretical distribution, e.g. a Weibull or exponential distribution. Yet, the estimators computed from the Weibull distribution, though characterised by a good fit, have one main drawback. They are taken from the life-time tables, and therefore depend on an arbitrarily selected set of numbers and widths of the survival time intervals. Therefore, as a next step, the survival functions should be estimated with the Kaplan-Meier method, since in this estimation the results function independent of the data grouping system. Next, the survival times should be compared with each other and in reference to, e.g. the alloy processing history.

The final stage should cover detailed analysis of the survival curves to estimate the life-time and selected percentiles of a survival function referred to the alloy processing history.

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