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SIMULATION OF THE CHARACTERISTICS OF THE BALL MOVEMENT ON A BEAM BY THE USE OF GENETIC ALGORITHM

Summary. Article aims to show the possibility of simulating the characteristics of different technical systems in a way of genetic evolution. Genetic algorithms (GA) are widely used in computer science, mathematics and technology and broadly understood engineering that the authors describe in papers. In the literature were shown examples of GA used to solve logistic problems, structural or design in engineering, computer science and even biology or chemistry. In this paper, the authors seek to demonstrate the applicability of GA methods in simulation of the system characteristics of dynamic systems.

SYMULACJA CHARAKTERYSTYK RUCHU KULKI PO RÓWNOWAŻNI POPRZEZ ZASTOSOWANIE ALGORYTMU GENETYCZNEGO

Streszczenie. Artykuł ma na celu ukazanie możliwości symulowania charakterystyk różnych systemów technicznych w drodze genetycznej ewolucji. Algorytmy genetyczne (GA) są szeroko stosowane w informatyce, matematyce oraz szeroko rozumianej inżynierii, co opisują autorzy w pracach.

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W literaturze wykazano przykłady GA wykorzystywane do rozwiązywania problemów logistycznych, strukturalnych, projektowania inżynierskiego, informatyki i nawet biologii czy chemii. W tym artykule autorzy starają się wykazać przydatność metod GA w symulacji charakterystyk układów dynamicznych.

1. Defining the problem

In Figure 1 is presented a system in which a free ball is on the moving beam. A ball movement along the beam is unaffected by external factors. Beam can rotate in a vertical plane through an angle φ around the center of the positioned support.

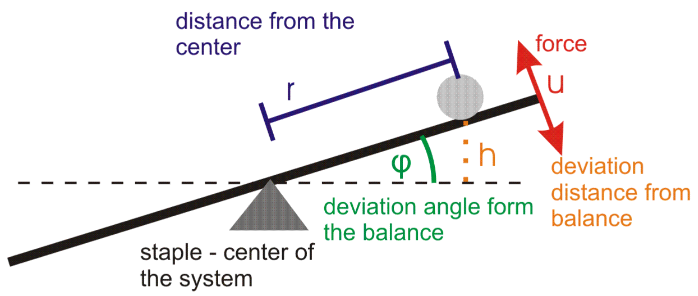

Fig. 1. The model of the ball moving along the movable beam
Rys. 1. Model kulki poruszającej się po ruchomej belce

The beam is affected by torque, located in the center of rotation by the set support. This paper discusses the possibility to simulate the characteristics of the ball moving along a rotating beam through the designed genetic algorithm, similar problem is discussed in literature [1, 3, 6]. This simulation will be possible by establishing an appropriate criterion function for the GA simulation method and setting initial population of individuals.

1.1. Description of the system and simulation method

In the literature [2, 4, 5, 10] authors describe many possible ways to adapt GA to simulate different systems. Other interesting examples are shown in [7, 8, 9]. The equation of the force $u(t)$ depends on the angular acceleration of the rotating beam angle relative to the distance, which is similar to literature [1, 3, 6]. The system that was studied is described by four characteristics:

$r(t)$	– ball distance from the center of the beam where is placed a support,
$\frac{dr(t)}{dt}$	– speed of movement of a ball on the beam,
$\varphi(t)$	– beam deflection angle relative to the center of the support,
$\frac{d\varphi(t)}{dt}$	– derivative of angular acceleration.

State of the system can be described by a vector of variables

$$x = (r, v, \varphi, \omega)^T = \left(r, \frac{dr}{dt}, \varphi, \frac{d\varphi}{dt}\right)^T. \tag{1}$$

The equation of the state in the illustrated arrangement is expressed in a matrix form

$$\begin{bmatrix} \frac{dr}{dt} \\ \frac{d^2r}{dt^2} \\ \frac{d\varphi}{dt} \\ \frac{d^2\varphi}{dt^2} \end{bmatrix} = \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} x_2 \\ x_1 \cdot x_4^2 + 32.14 \cdot \sin x_3 \\ x_4 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \cdot u. \tag{2}$$

Because in the system, there is a movement of components, changes in the characteristics shall be described on the basis of principles known from the dynamics. It is important to take into an account the energy values which are achieved during ball's movement over the beam. In this arrangement the potential energy of ball during the movement will be vital. Potential energy of the moving ball is expressed in a model by the equation

$$E_p = m \cdot g \cdot h. \tag{3}$$

At the same time the dependence of h on the distance from the center of the system describes the relationship

$$\sin|\varphi| = \frac{h}{|r|} \to h = |r| \cdot \sin|\varphi| \text{ and } h < h_{dop}. \tag{4}$$

Thus, the formula (2) we obtain the following equation of ball's acceleration

$$\frac{d^2r}{dt^2} = r\omega^2 + 32.14 \cdot \sin\varphi. \tag{5}$$

The equation of the proposed criterion function for GA simulation of the system in Figure 1 will depend on the potential energy value set by equation (3). Because in this arrangement the ball and gravitational mass are invariant, the control equation will in fact depend on the value of the swing beam height h above the level described by equation (4). Genetic algorithm simulating the characteristics is shown in Figure 2.

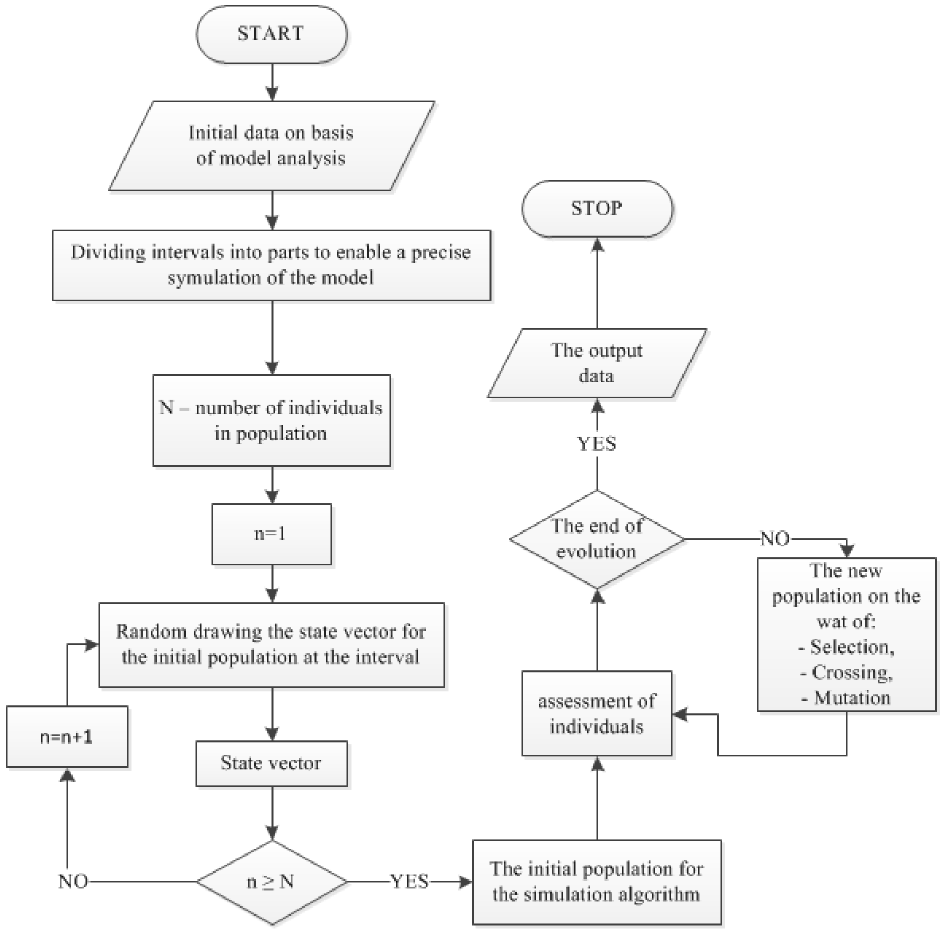

Fig. 2. Algorithm for the genetic simulation of the characteristics of the beam with free moving ball
Rys. 2. Algorytm genetycznej symulacji charakterystyk kulki poruszającej się po ruchomej belce

Genetic simulation is made on the basis of selecting the initial population of a given size, in which part of the chromosome consists of the distance r from the center of beam and beam deflection angle φ. Randomly selected individuals belong to given intervals of possible values. Then from among the various combinations of randomly selected population of individuals is selected a pair of r and φ satisfying the criterion of genetic algorithm. This way is similar to suggested in literature [10]. Built in this way system introduces a linear control equation for small values of the distance from the center $|r| < 1.525$ and angle of deviation from horizontal $|\varphi| < \frac{\pi}{4}$:

$$u = -0.0418 \cdot r - 0.1851 \cdot \frac{dr}{dt} + 6.4679 \cdot \varphi + 3.7331 \frac{d\varphi}{dt}. \qquad (6)$$

2. Simulation results

The simulation method of system characteristic values is made for random values of the specified range. In the designed genetic simulation system are given random values: the level of GA criterion method, minimum value of radius r, the radius r max value, min value of the angle φ, max value of the angle φ, the number of iterations in the method. For such initial conditions is formulated search operation to find the right piece of solution space and the setting of values that match the given criterion, minimized in carrying out the method of GA. Sample results from the simulations are shown in Figure 3–6. For Figure 3 GA simulation was performed on the input data: $ball_{mass} = 3.28$, $level = 0.1$, $r_{min} = 0$, $r_{max} = 20$, $\varphi_{min} = 0$, $\varphi_{max} = 60$, and a) iterations=400, b) iterations=800. The GA method returned the following results for Figure 3a: $h_{max} = 10\sqrt{3}$,$h_{dop} = 1.73205$, $r = 14.8354$,$\varphi = 2.05639$,$h = 0.53234$,$E_p = 10.4445$ The GA method returned the following results for Figure 3b: $h_{max} = 10\sqrt{3}$, $h_{dop} = 1.73205$, $r = 2.50333$, $\varphi = 5.35769$, $h = 0.233744$, $E_p = 4.58606$.

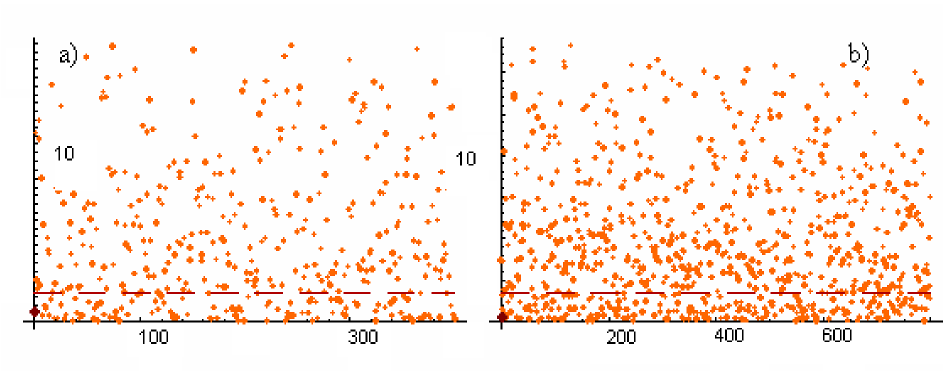

Fig. 3. The results of GA method on a random population of a) 400 individuals, b) 800 individuals

Rys. 3. Wyniki symulacji GA dla losowej populacji a) 400 osobników, b) 800 osobników

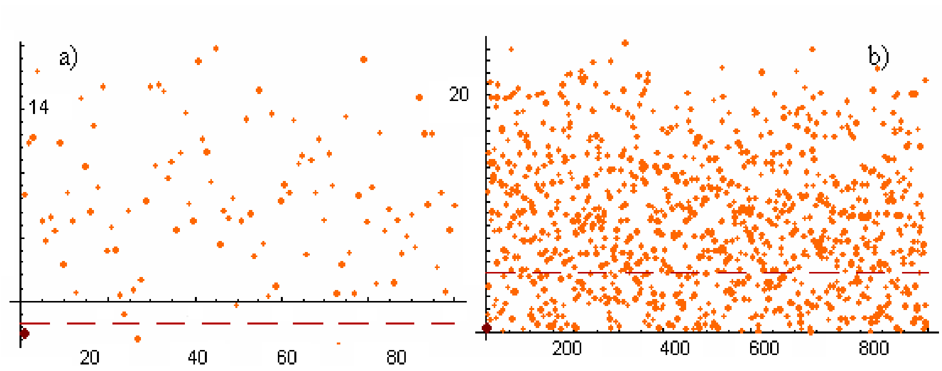

Fig. 4. The results of GA method on a random population of a) 100 individuals, b) 1000 individuals
Rys. 4. Wyniki symulacji GA dla losowej populacji a) 100 osobników, b) 1000 osobników

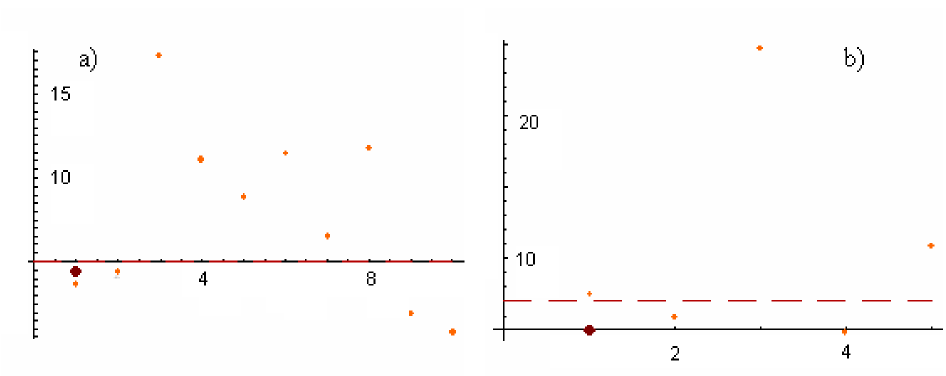

Fig. 5. The results of GA method on a random population of a) 10 individuals, b) 5 individuals
Rys. 5. Wyniki symulacji GA dla losowej populacji a) 10 osobników, b) 5 osobników

Charts posted in Figure 3 and Figure 4 show that the GA simulation method used in the system is capable to choose correctly (bold value point) from random initial population of an individuals the one that meets the inflicted criterion (horizontal dashed line). The simulation procedure for different input data obtains satisfying results for the search criterion, as shown in the illustrations. All received results meet the criterion score, which means that the GA simulation method can be successfully applied to simulate the characteristics. GA simulation method works fine for large and small populations of individuals as is shown in Figure 3 and Figure 4. As a result of the simulation at different time points from baseline were obtained points belonging to the state space of the system. Simulation of the entire space of solutions of the given model was made in the ranges of variables.

For the radius distance r from the center: $|r| \in \langle 0, 60\rangle \rightarrow |r_1| \in \langle 0, 6\rangle, |r_2| \in \langle 6, 12\rangle$, $|r_3| \in \langle 12, 18\rangle, |r_4| \in \langle 18, 24\rangle$, $|r_5| \in \langle 24, 30\rangle, |r_6| \in \langle 30, 36\rangle$, $|r_7| \in \langle 36, 42\rangle$, $|r_8| \in \langle 42, 48\rangle$, $|r_9| \in \langle 48, 54\rangle$, $|r_{10}| \in \langle 54, 60\rangle$.

For the beam deflection angle φ from the equilibrium: $|\varphi| \in \langle 0^o, 45^o\rangle \rightarrow |\varphi_1| \in \langle 0^o, 5^o\rangle, |\varphi_2| \in \langle 10^o, 15^o\rangle, |\varphi_3| \in \langle 0^o, 5^o\rangle$, $|\varphi_4| \in \langle 15^o, 20^o\rangle, |\varphi_5| \in \langle 20^o, 25^o\rangle$, $|\varphi_6| \in \langle 25^o, 30^o\rangle, |\varphi_7| \in \langle 30^o, 35^o\rangle$, $|\varphi_8| \in \langle 35^o, 40^o\rangle, |\varphi_9| \in \langle 40^o, 45^o\rangle$, $|\varphi_{10}| \in \langle 45^o, 50^o\rangle$.

As a result of the GA simulation method of characteristics within these limits, the state space system is described in points. The simulation was presented in Tables 1 and 2, where GA points belong to the selected sample interval (7) for an examined population of 100 random individuals

$$|r_1| \in \langle 0, 6\rangle \, , |\varphi_1| \in \langle 0^o, 5^o\rangle \, . \tag{7}$$

Sample GA simulation results for samples p1, p5, p24 and p29 are shown in Figure 6.

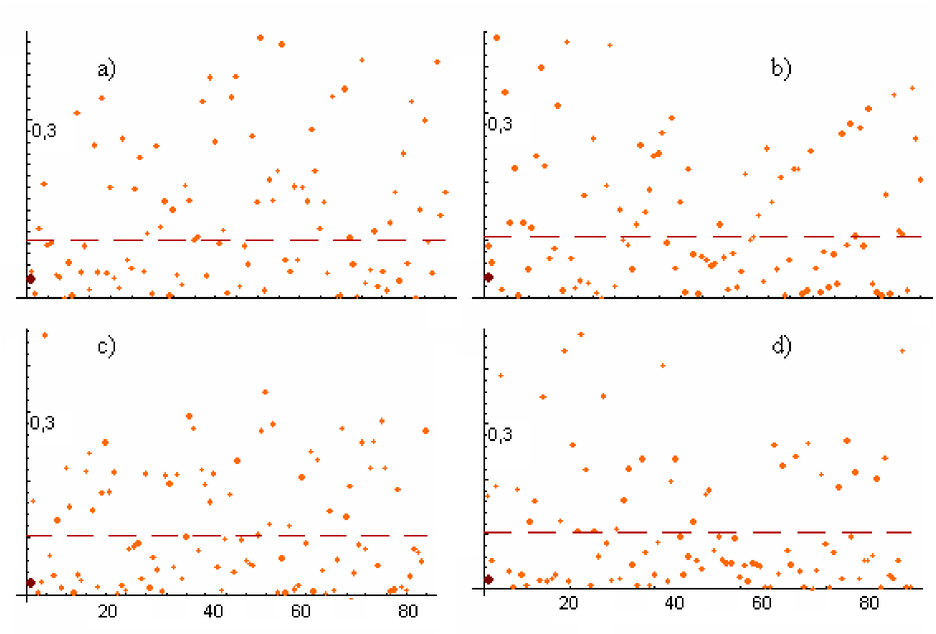

Fig. 6. The results of the GA simulation of the radius r distance for individuals from Tables 1 and 2

Rys. 6. Wyniki symulacji GA dla kąta r dla osobników z Tabel 1 i 2

Table 1
The GA simulation results of the state space of ball-moving beam

No.	distance r	angle φ	distance h
p1	3,77864	0,513081	0,0338371
p2	5,66504	0,0288124	0,00284929
p3	5,82408	0,885581	0,0900151
p4	2,45845	0,840313	0,0360549
p5	0,301246	3,82935	0,021187
p6	2,46475	0,739747	0,318215
p7	5,45165	0,94974	0,0903629
p8	3,11129	0,799739	0,0434262
p9	4,95611	0,222542	0,01925
p10	4,95629	0,489267	0,0423228
p11	2,05189	1,24557	0,044605
p12	5,23578	1,0713	0,0978918
p13	0,841443	3,07167	0,0450888
p14	5,0771	0,58899	0,521907
p15	3,78506	0,428856	0,0283307
p16	4,00831	0,938484	0,0656517
p17	2,6394	1,37169	0,0631825
p18	0,735383	4,44276	0,0569651
p19	2,48533	1,38818	0,0602093
p20	5,84666	0,159814	0,016308
p21	4,29777	0,767234	0,0575488
p22	2,13995	0,980079	0,0366033
p23	4,74358	0,261974	0,021689
p24	0,104587	2,29734	0,0539602
p25	3,37079	0,102251	0,00601554
p26	3,00705	1,45932	0,0765812
p27	0,354082	3,1298	0,0193322
p28	0,458989	0,964478	0,00772595
p29	2,41018	1,94091	0,0816298
p30	3,46645	1,00892	0,0610374
average:	3,200911	1,23059308	0,070659453

Table 2
The GA simulation results of the state space of ball-moving beam

No.	speed v	potential energy E_p	force u
p1	1,74486	1,08877	24,756
p2	1,73179	0,091681	30,555
p3	1,87715	2,8964	18,9357
p4	5,22055	1,16013	7,46553
p5	9,33296	0,647355	4,95827
p6	9,71965	1,02391	5,53421
p7	6,21511	2,90759	8,32363
p8	3,71128	1,39732	7,38395
p9	2,16911	0,619402	38,1335
p10	1,6582	1,36181	29,2215
p11	2,54836	1,43525	12,1144
p12	1,77305	3,14985	26,1302
p13	1,75543	1,45081	30,9961
p14	12,168	1,67933	3,58418
p15	2,00164	0,91159	34,7268
p16	6,6076	2,11246	8,12653
p17	2,47873	2,03301	13,3993
p18	8,04984	1,83295	5,88972
p19	2,14251	1,93734	13,6544
p20	2,47039	0,524738	14,4291
p21	0,967324	1,85173	7,63429
p22	2,51785	1,17778	44,9939
p23	2,16751	0,697883	38,0194
p24	226,1	1,73627	-40,5964
p25	2,77491	0,193561	49,6348
p26	2,08995	2,46414	14,3865
p27	1,76611	0,61205	31,5956
p28	1,33086	0,248596	9,56159
p29	102,147	2,62658	-16,7179
p30	1,35887	1,96399	20,6139
average:	14,28655313	1,461142533	16,58145667

The results presented in Tables 1 and 2 were analyzed. The analysis was designed to assess the quality of the simulation and check if the GA simulation is able to simulate the operation of the system at any point in the test state space. Analysis of the results was presented at the following figures.

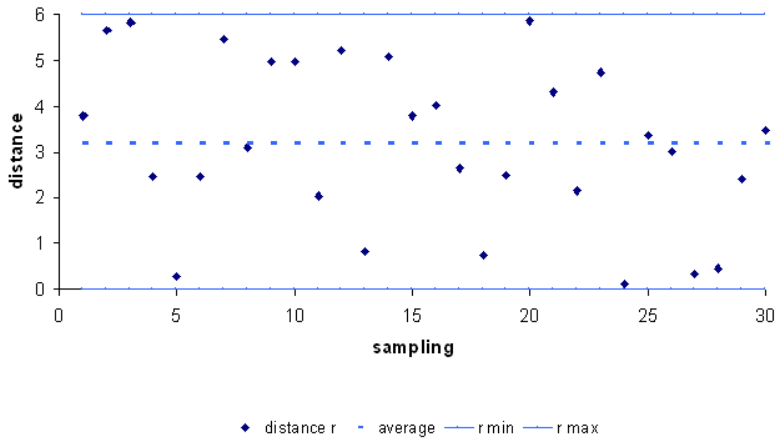

Fig. 7. The results of the GA simulation of the r distance for individuals from Tables 1 and 2

Rys. 7. Wyniki symulacji GA dla odległości r dla osobników z Tabel 1 i 2

In Figure 7 and Figure 8 are shown the distribution of a population of random points in the interval described by equation (6). Analysis of the data contained in the illustrations shows that the GA simulation method is able to cover the test range. GA simulation system with random arbitrary points of the set range can simulate the operation of the described system in any position. The results of simulations show that the system actually works in a random setting of any specific test points of the solution space given at the beginning of the sample interval. GA method is thus able to simulate the work of the entire study area solutions.

Operation of the GA is based on the criteria posed in the simulation algorithm. Searching the solution in the space of the test portion of the model is to find the best point of the search criterion for a population of randomly selected values of the radius and angle. Space searching algorithm returns the first encountered point which is satisfying the criterion. In the case of a moving ball the energy criterion is reduced to determine the height of the swing of the beam. In Figure 9 and Figure 10 are presented the values of these characteristics that set the GA simulation for the points shown in Figure 7 and Figure 8.

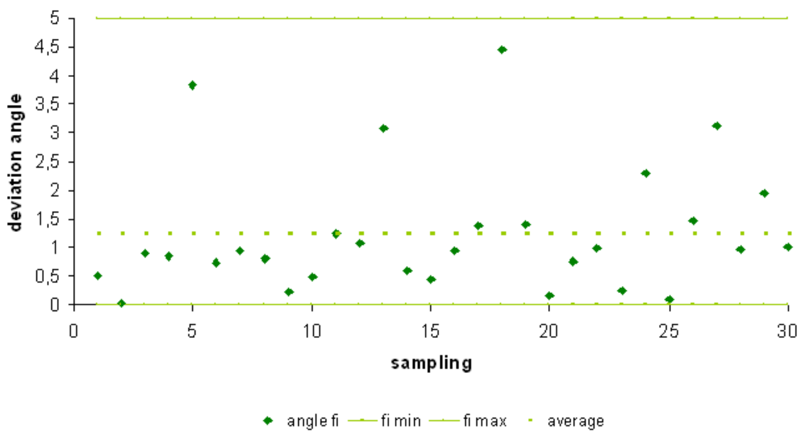

Fig. 8. The results of the GA simulation of the radius φ distance for individuals from Tables 1 and 2

Rys. 8. Wyniki symulacji GA dla kąta φ dla osobników z Tabel 1 i 2

Designated criterion values are within the specified range for a particular test that the illustrations show in the solid lines. Not everywhere has been shown upper or lower limits since the GA algorithm marked the correct values in a rather small dispersion. Which means that the algorithm is correct and accurate simulation gives the opportunity to test the system. Force values applied to the beam to keep the balance has been designated by the simulation algorithm on both sides of the axis. This means that the algorithm is able to correctly determine the values of force operating on the system regardless of whether it will be thrust from the top of the beam or supporting the beam from below. Algorithm based on the selected value is also able to calculate other characteristics describing the system. In the case of ball it is the speed of movement as shown in Figure 11. GA algorithm also marked the values of potential energy in the system shown in Figure 12.

Analysis of the results of the GA simulation algorithm has shown that it is good method for obtaining correct results. At the same time, this method allows to simulate the system at any point in state space solutions. By applying the method of GA we can simulate not only the correct operation of the system, but also it's failure. The algorithm allows the determination of the characteristics that lead to damage to the system. On the way of a computer simulation, we can accurately describe the state space by setting the proper operation of the system characteristics, as well as leading to damage or total depravity.

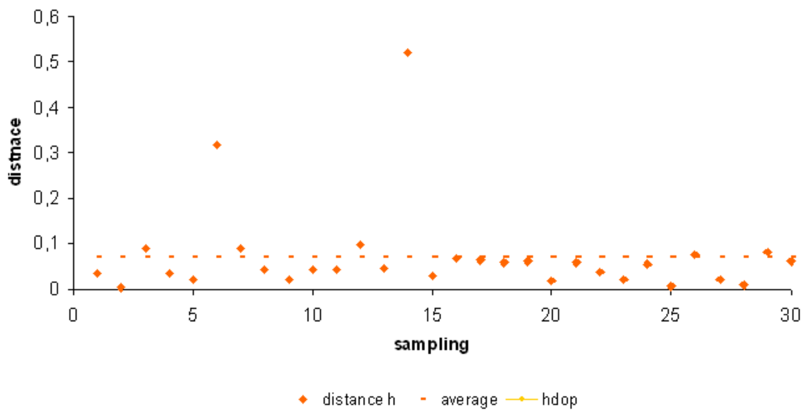

Fig. 9. The results of GA simulations of beam deflection from the equilibrium for individuals from Tables 1 and 2

Rys. 9. Wyniki symulacji GA dla wychylenia z punktu równowagi dla osobników z Tabel 1 i 2

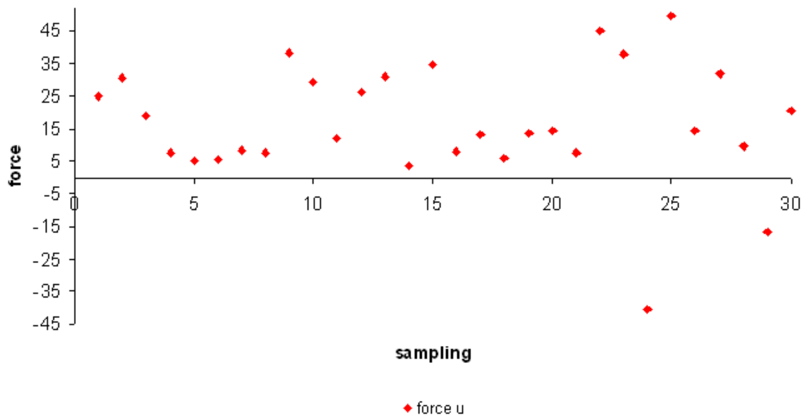

Fig. 10. The results of GA simulations of force characteristics that must be applied to the beam to remain the balance for individuals from Tables 1 and 2

Rys. 10. Wyniki symulacji GA dla charakterystyki siły, która musi być przyłożona aby utrzymać balans belki dla osobników z Tabel 1 i 2

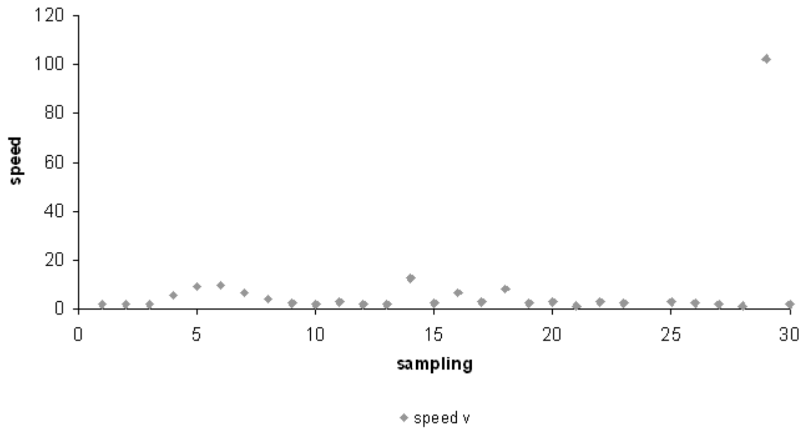

Fig. 11. The results of the GA simulation of ball movement speed for individuals from Tables 1 and 2

Rys. 11. Wyniki symulacji GA dla charakterystyki prędkości kulki dla osobników z Tabel 1 i 2

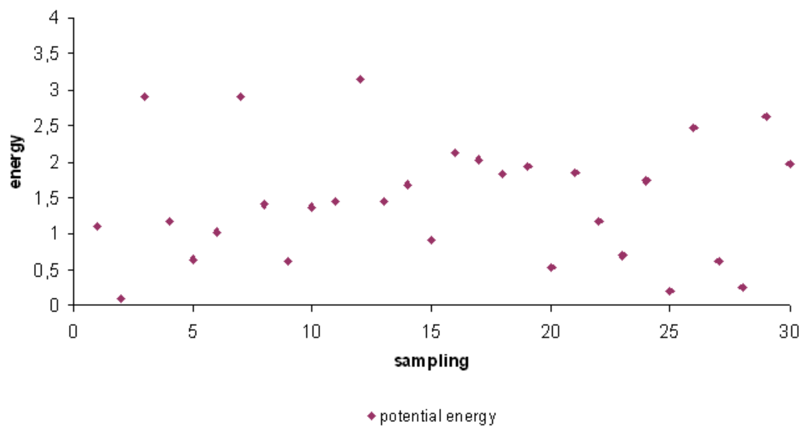

Fig. 12. The results of GA simulation of potential energy in the system for individuals from Tables 1 and 2

Rys. 12. Wyniki symulacji GA dla charakterystyki energii potencjalnej kulki dla osobników z Tabel 1 i 2

3. Conclusions

Actions that are described in the work show the possibility of using GA for the simulation of free moving ball on the beam. GA methods have great potential to simplify many operations by defining the relevant criterion function. Selection of criterion seems to be the most important thing. The quality of simulation results depends on the selection criterion. The presented GA algorithm can cope better with the simulation with large values of criterion. This effect is caused by the fact that the greater is acceptable criterion for the randomly selected population, the easier it is to find the individuals that are proper for the criterion. This behavior is clearly visible in Figures 3–4. Studies show that by adjusting the interval at which the simulation is made we can easily describe the characteristics of the test system at any point in state space solutions. This means that the GA method is suitable to simulate not only the proper operation of systems but also the values that lead to damage or even corruption. Application of GA methods in simulation systems can significantly reduce the costs in the course of research and shorten the entire process of data collection.

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Omówienie

W artykule przedstawiono metodę symulowania wartości z charakterystyk modeli dynamicznych dla systemów wspomagania decyzji opartych o zastosowanie technik inteligencji obliczeniowej.

Opisane zostało przykładowe zastosowanie algorytmów genetycznych do wyznaczenia poszczególnych stanów badanego obiektu dynamicznego. Metoda została przetestowana dla różnej liczby osobników w populacji początkowej, a następnie otrzymane wyniki przeanalizowano.

Wyniki opracowania statystycznego pozwoliły określić szybkość omówionej metody oraz jej efektywność dla symulacji stanów obiektów dynamicznych.