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ON A METHOD OF THE RAPID APPROXIMATION OF A CUBIC BÉZIER CURVE BY QUADRATIC BÉZIER CURVES

Summary. In this paper the approximation of a cubic Bèzier curve by quadratic Bèzier curves is presented. For some method of the choose of quadratic Bèzier curves, the error of approximation, measured with the Frèchet distance is especially simple to calculate. Moreover, this error is easy to predict also for parts of a given cubic Bèzier curve. Both these features give us the opportunity, with some caution, of rapid approximation, where required precision is granted.

O METODZIE SZYBKIEJ APROKSYMACJI KRZYWEJ BEZIERA TRZECIEGO STOPNIA KRZYWYMI BEZIERA DRUGIEGO STOPNIA

Streszczenie. W artykule zaprezentowano aproksymację krzywej Bèziera trzeciego stopnia za pomocą krzywych Bèziera drugiego stopnia. Dla pewnej metody wyboru krzywych Bèziera drugiego stopnia błąd aproksymacji, mierzony odległością Frècheta, jest szczególnie łatwy do policzenia. Co więcej, błąd ten łatwo policzyć także dla fragmentów wyjściowej krzywej. Obie te właściwości, z pewnymi zastrzeżeniami, umożliwiają szybką aproksymację z wymaganą dokładnością.

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1. Preliminaries

1.1. The definitions of quadratic and cubic Bèzier curves

We will call the quadratic Bèzier curve [2] a curve defined on the plane $0XY$ by parametric equations of the t variable

$$\boldsymbol{B_2} = \begin{bmatrix} x(t) \\ y(t) \end{bmatrix} = \boldsymbol{C_2 T} = \begin{bmatrix} c_{0x} & c_{1x} & c_{2x} \\ c_{0y} & c_{1y} & c_{2y} \end{bmatrix} \begin{bmatrix} 1 \\ t \\ t^2 \end{bmatrix}$$

for $t \in \langle\, 0, 1 \,\rangle$. Similarly a cubic Bèzier curve [2] is defined as

$$\boldsymbol{B_3} = \begin{bmatrix} x(t) \\ y(t) \end{bmatrix} = \boldsymbol{C_3 T} = \begin{bmatrix} c_{0x} & c_{1x} & c_{2x} & c_{3x} \\ c_{0y} & c_{1y} & c_{2y} & c_{3y} \end{bmatrix} \begin{bmatrix} 1 \\ t \\ t^2 \\ t^3 \end{bmatrix}.$$

where $c_{0x}, c_{1x}, c_{2x}, c_{3x}, c_{0y}, c_{1y}, c_{2y}, c_{3y} \in \boldsymbol{R}$.

1.2. Equivalent form of Bèzier curves

An equivalent form of definition, a bit more intuitive, uses points on a plane.

The quadratic Bèzier curve is defined by 3 points: P_0, P_1 and P_2. The transformation matrix $\boldsymbol{A_2}$ guarantee the coordinates of these points and the coefficient matrix $\boldsymbol{C_2}$ are mutually interchangeable

$$\boldsymbol{C_2} = \boldsymbol{P_2 A_2},$$

$$\begin{bmatrix} c_{0x} & c_{1x} & c_{2x} \\ c_{0y} & c_{1y} & c_{2y} \end{bmatrix} = \begin{bmatrix} x_{P_0} & x_{P_1} & x_{P_2} \\ y_{P_0} & y_{P_1} & y_{P_2} \end{bmatrix} \begin{bmatrix} 1 & -2 & 1 \\ 0 & 2 & -2 \\ 0 & 0 & 1 \end{bmatrix}.$$

The cubic Bèzier curve is defined by 4 points: P_0, P_1, P_2 and P_3. The transformation matrix $\boldsymbol{A_3}$ guarantee the coordinates of these points and the coefficient matrix $\boldsymbol{C_3}$ are mutually interchangeable

$$\boldsymbol{C_3} = \boldsymbol{P_3 A_3}.$$

The form of matrix $\boldsymbol{A_3}$ will be presented later, see the equation (3).

Both matrices $\boldsymbol{A_2}$ and $\boldsymbol{A_3}$ are non singular, therefore they are invertible. Thus the coordinates of the points $\boldsymbol{P_2}$ can be computed from the coefficient matrix $\boldsymbol{C_2}$, and similarly $\boldsymbol{P_3}$ from $\boldsymbol{C_3}$.

2. Aproximation $\boldsymbol{B_3}$ curve by $\boldsymbol{B_2}$ curve

2.1. Method of approximation

The approximation $\boldsymbol{B_3}$ curve by $\boldsymbol{B_2}$ curve rely on indicating 3 points Q_0, Q_1 and Q_2 which specify the searched curve. Let's define the method $\overline{\boldsymbol{HH}}$ of approximation:

Definition 1. *Given is a cubic Bézier curve, specified by 4 points:* P_0,P_1,P_2 *and* P_3*. The approximating quadratic Bézier curve, specified by 3 points:* Q_0,Q_1 *and* Q_2*. The method* $\overline{\boldsymbol{HH}}$ *determines the coordinates of these points as follows*

$$\begin{aligned} Q_0 = &\quad P_0, \\ Q_1 = &\quad \frac{3(P_1+P_2)-(P_0+P_3)}{4}, \\ Q_2 = &\quad P_3. \end{aligned}$$

The illustration of this method is shown on the Figure 1. In matrix notation, the above method is as follows

$$\boldsymbol{Q} = \begin{bmatrix} x_{Q_0} & x_{Q_1} & x_{Q_2} \\ y_{Q_0} & y_{Q_1} & y_{Q_2} \end{bmatrix} = \boldsymbol{P}\ \boldsymbol{K_H} =$$

$$= \begin{bmatrix} x_{P_0} & x_{P_1} & x_{P_2} & x_{P_3} \\ y_{P_0} & y_{P_1} & y_{P_2} & y_{P_3} \end{bmatrix} \left(\frac{1}{4} \begin{bmatrix} 4 & -1 & 0 \\ 0 & 3 & 0 \\ 0 & 3 & 0 \\ 0 & -1 & 4 \end{bmatrix} \right). \quad (1)$$

As can be seen from the above equation, the method $\overline{\boldsymbol{HH}}$ has the following characteristics:

- both curves begins at the same point,
- both curves ends at the same point.

Feature worth mentioning is the third obligatory point common to both curves, parameter sets it to $t = \frac{1}{2}$.

To show the position of the point Q_1, we need two auxiliary points H_1 i H_2. Let's define them as follows:

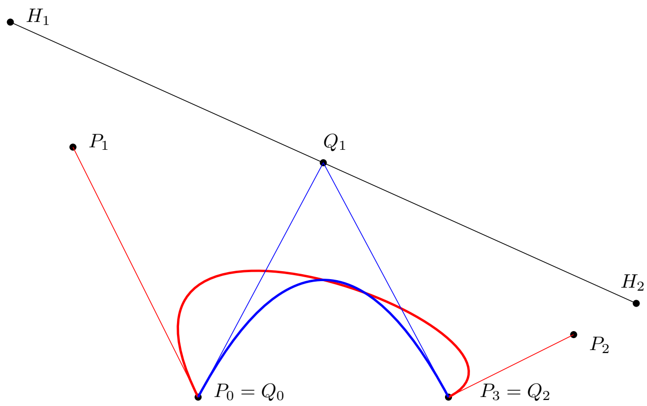

Fig. 1. Method $\overline{\boldsymbol{HH}}$ of approximation
Rys. 1. Metoda $\overline{\boldsymbol{HH}}$ aproksymacji

Definition 2. *Given is a cubic Bézier curve, specified by 4 points: P_0,P_1,P_2 i P_3. The auxiliary points H_1 i H_2 can be set as follows*

$$H_1 = \frac{3P_1 - P_0}{2},$$
$$H_2 = \frac{3P_2 - P_3}{2}.$$

With these points, the location Q_1 is easy to determine geometrically (Figure 1):

$$Q_1 = \frac{H_1 + H_2}{2}.$$

2.2. Approximation error

The Hausdorff distance d_H [3] can be used as a natural candidate to evaluate the conformity between the approximated and approximating curves. Unfortunately, the analytical calculation in the general case does not seem to be possible, moreover numerical methods are time consuming and error prone. For these reasons, the Fréchet distance d_F [3] were used to evaluate the error. Although it is less precise than the Hausdorff distance, because of the following estimate

$$d_H \leqslant d_F.$$

But with usage of Fréchet distance we can solve the problem analytically. To present it, let's introduce the vector $\overrightarrow{h}$ (Figure 2):

$$\overrightarrow{h} = \overrightarrow{H_1 H_2}.$$

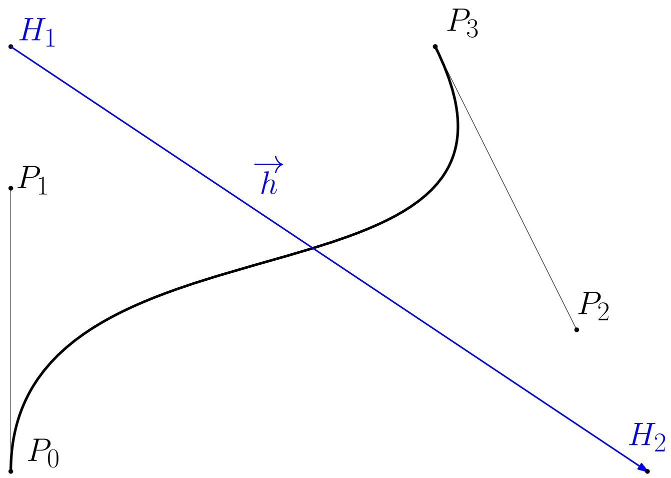

Fig. 2. Determination of $\vec{h}$ vector
Rys. 2. Określenie wektora $\vec{h}$

With this help, the error of this method is determined in the following theorem:

Theorem 3. *The cubic Bézier curve is approximated by quadratic Bèzier curve determined by $\overline{HH}$ method. If the Hausdorff distance d_H is an error of such approximation, its upper limit is equal to the Fréchet distance d_F:*

$$d_H \leqslant d_F = \frac{|\overrightarrow{h}|}{6\sqrt{3}}.$$

The proof of this theorem is given in [1].

3. The subdivision of Bèzier curve $\boldsymbol{B_3}$ into N smaller parts

Theorem 4. *Each section of cubic Bézier curve is also a cubic Bézier curve.*

The proof of this theorem is given [1].

Definition 5. *Every section of cubic Bézier curve is a member of K-family originated by this curve.*

Theorem 6. *Let the cubic Bézier curve B_3 have a vector $\overrightarrow{h}$. Let's create a K-family by dividing B_3 to N equal parts. Any such created part is also a cubic Bézier curve $B_{3,i}$ and has its own vector $\overrightarrow{h_i}$. Within such K-family, these vectors have the relationship*

$$\overrightarrow{h_i} = \frac{\overrightarrow{h}}{N^3}, \qquad i = 1, 2, \ldots, N. \tag{2}$$

An equal division should be understood as follows: when converting Bézier curve to an equivalent algebraic form, the length of the parameter Δt is equal for all N parts

$$\Delta t = t_{i+1} - t_i = \frac{1}{N},$$

where t_i indicates a dividing point. This is a special case of a more general theorem on invariant of K-family presented and proven in [1].

The above theorem is very useful in case of exceeding the acceptable error of approximation. After splitting the initial curve $\boldsymbol{B_3}$ into N parts, theorem (6) guarantees to reduce the estimation of error of the extent N^3.

4. Mathematical transformations

4.1. The coefficients of the polynomial

At the beginning, we set 4 points: P_0, P_1, P_2 and P_3. We transform them into a set of coefficients C, designed to present the Bézier curve in the algebraic form

$$\begin{cases} x(t) = c_{0x} + c_{1x}\,t + c_{2x}\,t^2 + c_{3x}\,t^3\,, \\ y(t) = c_{0y} + c_{1y}\,t + c_{2y}\,t^2 + c_{3y}\,t^3\,, \end{cases}$$

To do this, a transformation matrix $\boldsymbol{A}$ is needed

$$\boldsymbol{C_3} = \begin{bmatrix} c_{0x} & c_{1x} & c_{2x} & c_{3x} \\ c_{0y} & c_{1y} & c_{2y} & c_{3y} \end{bmatrix} =$$

$$= \boldsymbol{P_4}\boldsymbol{A_3} = \begin{bmatrix} x_{P_0} & x_{P_1} & x_{P_2} & x_{P_3} \\ y_{P_0} & y_{P_1} & y_{P_2} & y_{P_3} \end{bmatrix} \begin{bmatrix} 1 & -3 & 3 & -1 \\ 0 & 3 & -6 & 3 \\ 0 & 0 & 3 & -3 \\ 0 & 0 & 0 & 1 \end{bmatrix}. \tag{3}$$

4.2. Splitting of a Bézier curve

The next step is to split the cubic Bézier curve to N parts

$$\boldsymbol{D_i} = \boldsymbol{C_3 K_i}\,, \qquad i = 0, 1, \ldots, (N-1), \tag{4}$$

where $\boldsymbol{K_i}$ depends on $\tau = \tau(i, N)$ and $k = k(N)$:

$$\boldsymbol{K_i} = \boldsymbol{K}(\tau, k) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ \tau & k & 0 & 0 \\ \tau^2 & 2\tau k & k^2 & 0 \\ \tau^3 & 3\tau^2 k & 3\tau k^2 & k^3 \end{bmatrix},$$

where: N – number of parts, $k = \dfrac{1}{N}$, $\tau = i\,k$, $i = 0, 1, \ldots, (N-1)$.

4.3. Approximation of the parts of Béziera curve

A transition to the points of quadratic Bézier curve, which approximate these parts by the method $\overline{HH}$, is obtained using the transition matrix $\boldsymbol{K_2}$:

$$\boldsymbol{Q_i} = \begin{bmatrix} x_{Q_0} & x_{Q_1} & x_{Q_2} \\ y_{Q_0} & y_{Q_1} & y_{Q_2} \end{bmatrix} = \boldsymbol{D_i K_2} =$$

$$= \begin{bmatrix} d_{0x} & d_{1x} & d_{2x} & d_{3x} \\ d_{0y} & d_{1y} & d_{2y} & d_{3y} \end{bmatrix} \left(\frac{1}{4} \begin{bmatrix} 4 & 4 & 4 \\ 0 & 2 & 4 \\ 0 & 0 & 4 \\ 0 & -1 & 4 \end{bmatrix} \right).$$

Because of the aforementioned reasons, calculating the coordinates of the point Q_2 is unnecessary, therefore a shortened version of the transition matrix $\tilde{\boldsymbol{K_2}}$ is used

$$\tilde{\boldsymbol{Q_i}} = \begin{bmatrix} x_{Q_0} & x_{Q_1} \\ y_{Q_0} & y_{Q_1} \end{bmatrix} = \boldsymbol{D_i}\tilde{\boldsymbol{K_2}} =$$

$$= \begin{bmatrix} d_{0x} & d_{1x} & d_{2x} & d_{3x} \\ d_{0y} & d_{1y} & d_{2y} & d_{3y} \end{bmatrix} \left(\frac{1}{4} \begin{bmatrix} 4 & 4 \\ 0 & 2 \\ 0 & 0 \\ 0 & -1 \end{bmatrix} \right). \tag{5}$$

4.4. Consolidation

All the above transformations (3,4,5), covered collectively form a series of calculations

$$\tilde{\boldsymbol{Q_i}} = \boldsymbol{D_i}\tilde{\boldsymbol{K_2}} = \boldsymbol{C_3 K_i}\tilde{\boldsymbol{K_2}} = \boldsymbol{P_4 A_3 K_i}\tilde{\boldsymbol{K_2}}. \tag{6}$$

Since $\boldsymbol{A}$, $\boldsymbol{K_i}$ and $\tilde{\boldsymbol{K_2}}$ are the sparse matrices, using the above formula would be waste of computing time. After eliminating zero elements, the condensed form of the above formula is expressed as follows

$$
k = \frac{1}{N},
$$
$$
\left\{
\begin{array}{lll}
\text{for } i & = N-1,\ldots ,1,0 & \\
\tau & = \dfrac{i}{N}, & \\
\xi & = 1-\tau, & \\
Q_{0,i} & = & P_0\ \xi^3 \\
& & +3P_1\ \tau\xi^2 \\
& & +3P_2\ \tau^2\xi \\
& & +\ P_3\ \tau^3, \\
Q_{1,i} & = Q_{0,i} + \dfrac{k}{4}\ [& P_0(\ -6 + 12\tau - 6\tau^2 + k^2) \\
& & +P_1(\ 6 - 24\tau + 9\tau^2 - 3k^2) \\
& & +P_2(\ 12\tau - 18\tau^2 + 3k^2) \\
& & +P_3(\ 6\tau^2 - k^2)\ ].
\end{array}
\right.
\tag{7}
$$

In addition, the coordinates of the third point are obtained from the known coordinates of the first point

$$
Q_{2,i} = \begin{cases} P_3 & \text{if } i = N-1, \\ Q_{0,i+1} & \text{if } i < N-1. \end{cases} \tag{8}
$$

5. Example

5.1. Drawing constraints

The author's aim was to present an example that requires division into $N = 3$ parts. Unfortunately, such an illustration would be unreadable. This is due to the following calculations:

Suppose that all the points of cubic Bézier curve have to fit in a certain drawing space, limited by a circle of radius R. Then the largest Fréchet distance would be

$$
d_F = \frac{3R}{6\sqrt{3}} \approx 0,29R.
$$

When divided into $N = 3$ parts, approximation error would decrease at least N^3 times

$$
d_{F(1/3)} = d_F/N^3 = d_F/27 \approx 0,01R.
$$

Since the Hausdorff distance, which is the real measure of the distance between curves, would be even smaller, therefore approximating curves would overshadow the approximated curve. Thus, we present an example to illustrate a method for the $N = 2$, where the approximation is not as good, and then briefly show analogous calculations for the $N = 3$.

5.2. Computational limitations

Typically, the input data are selected in such a way that the calculations were as simple as possible. Therefore the following points forming the Bézier curve were selected

$$P_0[0,0],\ P_1[0,2],\ P_2[4,1],\ P_3[3,3].$$

Unfortunately, in the course of the calculations it is necessary to divide the coordinates of these points by 4, and by N^3 for $N = 2$ and $N = 3$. The postulate of avoiding fractions and decimal expansions resulted in need to multiply the initial data by the scale which is the product following factors

$$m = 4 \cdot 2^3 \cdot 3^3 = 864$$

which led to very large coordinates of points

$$P_0[0,0],\ P_1[0,1728],\ P_2[3456,864],\ P_3[2592,2592].$$

5.3. Formulation of the task

For 4 points in the plane

$$P_0[0,0],\ P_1[0,1728],\ P_2[3456,864],\ P_3[2592,2592]$$

depicting cubic Bézier curve and for a given accuracy of approximation

$$d_H = 86,4 \qquad (= m/10)$$

in the first variant, and

$$d_H = 17,28 \qquad (= m/50)$$

in the second variant, determine:

- N – count of quadratic Bézier curves that guarantee an approximation of specified accuracy d_H;
- list $K_1, K_2, \ \ldots\ , K_N$ of these curves.

5.4. Solution for $N = 2$

5.4.1. Calculation of the number of curves

We calculate the auxiliary vector $\overrightarrow{h}$ (Figure 2):

$$\begin{aligned}\overrightarrow{h} &= [h_x, h_y] \\ &= [3(P_{1x} + P_{2x}) - (P_{0x} + P_{3x})\ ,\ 3(P_{1y} + P_{2y}) - (P_{0y} + P_{3y})].\end{aligned}$$

In the presented task

$$\overrightarrow{h} = [3888, -2592].$$

Then we determine the length of L_h:

$$L_h = |\overrightarrow{h}| = \sqrt{3888^2 + 2592^2} \approx 4673.$$

It is used to estimate the size of the N, with the additional simplifying assumption $d_H = d_F = 86,4$:

$$N \geqslant \sqrt[3]{\frac{L_h}{6\sqrt{3}\ d_F}} \approx 1.73$$

thus

$$N = 2.$$

5.4.2. List of $N = 2$ curves

We find searched quadratic Bézier curves by giving the coordinates of three defining points: Q_0, Q_1 i Q_2. Next curves, marked with i, will refer with points marked as $Q_{0,i}$, $Q_{1,i}$ and $Q_{2,i}$.

For $i = 0, 1, \ldots, N-1$ we specify an auxiliary variable $\tau_i = \dfrac{i}{N}$. Then the starting points $Q_{0,i}$ of searched curves K_i we find as follows

$$Q_{0,i} = \begin{bmatrix} 1 & \tau_i & \tau_i^2 & \tau_i^3 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ -3 & 3 & 0 & 0 \\ 3 & -6 & 3 & 0 \\ -1 & 3 & -3 & 1 \end{bmatrix} \begin{bmatrix} P_0 \\ P_1 \\ P_2 \\ P_3 \end{bmatrix}.$$

With introduction of an additional variable $k = \dfrac{1}{N}$, the second point $Q_{1,i}$ which specifies search curves K_i is calculated as follows

$$Q_{1,i} = Q_{0,i} + \frac{k}{4} \begin{bmatrix} 1 & \tau_i & \tau_i^2 & k^2 \end{bmatrix} \begin{bmatrix} -6 & 6 & 0 & 0 \\ 12 & -24 & 12 & 0 \\ -6 & 18 & -18 & 6 \\ 1 & -3 & 3 & -1 \end{bmatrix} \begin{bmatrix} P_0 \\ P_1 \\ P_2 \\ P_3 \end{bmatrix}.$$

Since the ends of curves are also the beginnings of the next ones, so there is no need of specific computation. To avoid unnecessary calculations Q_2 can be determined as follows

$$Q_{2,i} = \begin{cases} P_3 & \text{for } i = N-1, \\ Q_{0,i+1} & \text{for } i < N-1. \end{cases}$$

Aforementioned observations force the order of evaluation by decreasing index i. Therefore, the calculation starts from $i = N - 1 = 1$:

$$\begin{aligned}
k = \frac{1}{N} \quad & = \frac{1}{2}, \\
\tau_2 = \frac{i}{N} \quad & = \frac{1}{2}, \\
Q_{0,1} \quad & = [\ 1620\ ,\ 1296\ ], \\
Q_{1,1} \quad & = [\ 2997\ ,\ 1458\ ], \\
Q_{2,1} = P_3 \quad & = [\ 2592\ ,\ 2592\ ].
\end{aligned}$$

For the second and the last curve $i = 0$:

$$\begin{aligned}
& \tau_0 = 0, \\
& Q_{0,0} \quad = [\ 0\ ,\ 0\ ], \\
& Q_{1,0} \quad = [\ 243\ ,\ 1134\ ], \\
& Q_{2,0} = Q_{0,1} \quad = [\ 1620\ ,\ 1296\ ].
\end{aligned}$$

The Figure 3 shows the result of approximation.

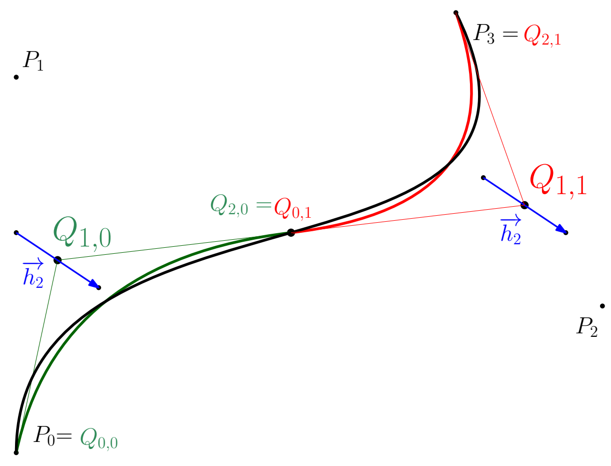

Fig. 3. Approximation for $N = 2$
Rys. 3. Aproksymacja dla $N = 2$

5.5. Solution for $N = 3$

5.5.1. Calculation of the number of curves

Knowing vector $\overrightarrow{h}$ (Figure 4):

$$\overrightarrow{h} = [3888, -2592].$$

and his length L_h:

$$L_h = |\overrightarrow{h}| \approx 4673.$$

we estimate N, with the additional simplifying assumption $d_H = d_F = 17,28$:

$$N \geqslant \sqrt[3]{\frac{L_h}{6\sqrt{3}\ d_F}} \approx 2.96$$

hence we get N:

$$N = 3.$$

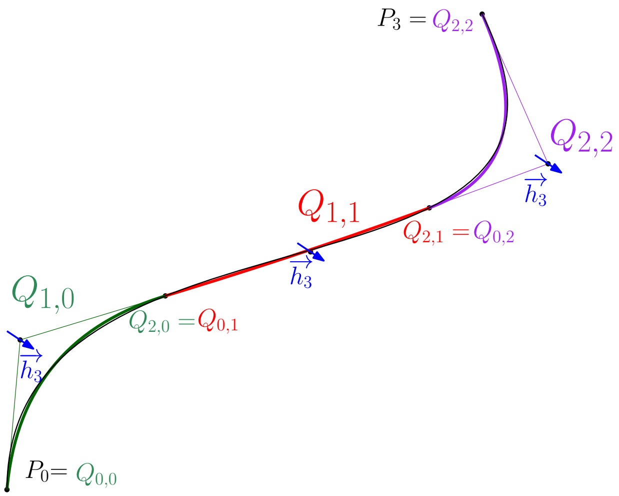

Fig. 4. Approximation for $N = 3$
Rys. 4. Aproksymacja dla $N = 3$

5.5.2. List of $N = 3$ curves

Analogous calculations start from $i = N - 1 = 2$:

$$\begin{aligned}
k &= \frac{1}{N} &&= \frac{1}{3},\\
\tau_2 &= \frac{i}{N} &&= \frac{2}{3},\\
Q_{0,2} & &&= [\ 2304\ ,\ 1536\ ],\\
Q_{1,2} & &&= [\ 2952\ ,\ 1776\ ],\\
Q_{2,2} &= P_3 &&= [\ 2592\ ,\ 2592\ ].
\end{aligned}$$

For the second curve $i = 1$, thus

$$\begin{aligned}
\tau_1 &= \frac{1}{3},\\
Q_{0,1} & &&= [\ 864\ ,\ 1056\ ],\\
Q_{1,1} & &&= [\ 1656\ ,\ 1296\ ],\\
Q_{2,1} &= Q_{0,2} &&= [\ 2304\ ,\ 1536\ ].
\end{aligned}$$

Fot the third and the last curve $i = 0$:

$$\begin{aligned}
\tau_0 &= 0,\\
Q_{0,0} & &&= [\ 0\ ,\ 0\ ],\\
Q_{1,0} & &&= [\ 72\ ,\ 816\ ],\\
Q_{2,0} &= Q_{0,1} &&= [\ 864\ ,\ 1056\ ].
\end{aligned}$$

The results are shown in the Figure 4. As expected, the approximation is so good that it is difficult to distinguish approximating curves from the approximated one.

5.6. Cusp case

Let us consider a case where there is a cusp curve, wherein the curvature is $k = \infty$ (Figure 5, case A).

It is known that non degenerate quadratic Bézier curve do not have such a cusp. To improve the assessment of the situation, approximating curves have been slightly shifted vertically. As shown, case B, where division point falls on a cusp, is difficult to accept. If it happens, it is required to increase the number of divisions by 1, which ensures that the jump over an obstacle. Emerging situation is similar to the variant C. If the results are not satisfactory again, one can reduce the acceptable error, by increasing the number of approximating curves. Of course, it is necessary to check whether the increased N does not repeat the situation B.

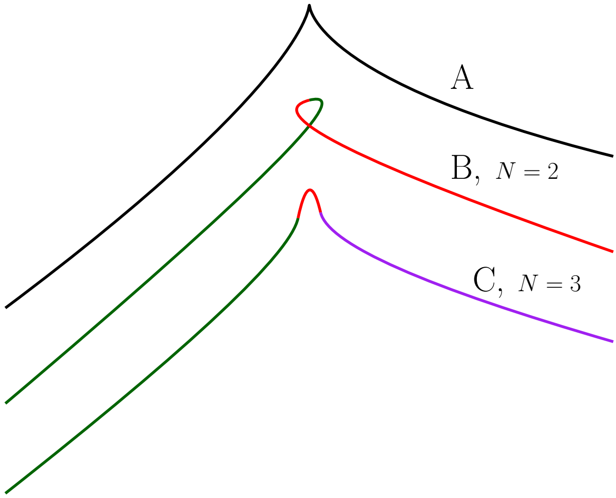

Fig. 5. Cusp case
Rys. 5. Przypadek ze szpicem

The same objections may arise during the approximation of Bèzier curves which contains a loop. Here, too, it is recommended to decrease the error of approximation. Fortunately, in practice such cases does not occur, so their in-depth analysis is not a pressing problem.

6. Concluding remarks

The presented method can be used in some environments to create graphics. If the environment does not have the ability to create cubic Bézier curves, and can create quadratic Bézier curves, then the method can be useful. Such environments are, for example, Adobe Flash, which uses SWF file format [4], currently very popular in the Internet.

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