GB-problem in the class of locally graded groups

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Abstract

A problem, we consider, is equivalent to the one posed in 1981 by G. Bergman: *Let G be a group and S a subsemigroup of G which generates G as a group. Must each identity satisfied in S be satisfied in G?* The first counterexample was found in 2005 by S. Ivanov and A. Storozhev. It gives a negative answer to the problem in general. However we show that the problem has an affirmative answer for locally residually finite groups and for locally graded groups containing no free noncyclic subsemigroups.

Keywords: Semigroup identities; Generating subsemigroup in a group.

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1. Introduction

An identity in a semigroup has a form $u(x_1, ..., x_n) = v(x_1, ..., x_n)$ where the words u and v are written without inverses of variables. Such an identity in a group is called a *semigroup identity.* It is clear that abelian groups and groups of finite exponent satisfy semigroup identities.

A subsemigroup S of a group G is called *a generating semigroup* if elements of S generate G as a group. The following question is equivalent to the one posed by G. Bergman in [1].

GB**-problem** *Let G be a group and S a generating semigroup of G. Must each identity satisfied in S be satisfied in G?*

Another formulation of this problem is whether every proper variety of semigroups is closed with respect to groups of fractions ([16], Question 11.1). The GB-problem can be approached from two sides. One can concentrate either on the identities or on the properties of the group G. We say that an identity $u = v$ is *transferable* if while satisfied in S, it is necessary satisfied in G. Thus the problem is whether every semigroup identity is transferable. It is clear that the abelian identity is transferable. In 1986 the problem was discussed in G. Bergman's "Problem Seminar" in Berkeley [2]. It was shown that the identity $x^n y^n = y^n x^n$ is transferable for $n = 2$. The proof of transferability for $n > 2$ was found in 1992 [9]. As to another approach, it is known ([3], Theorem D) that for soluble-by-finite groups the GB-problem has an affirmative answer.

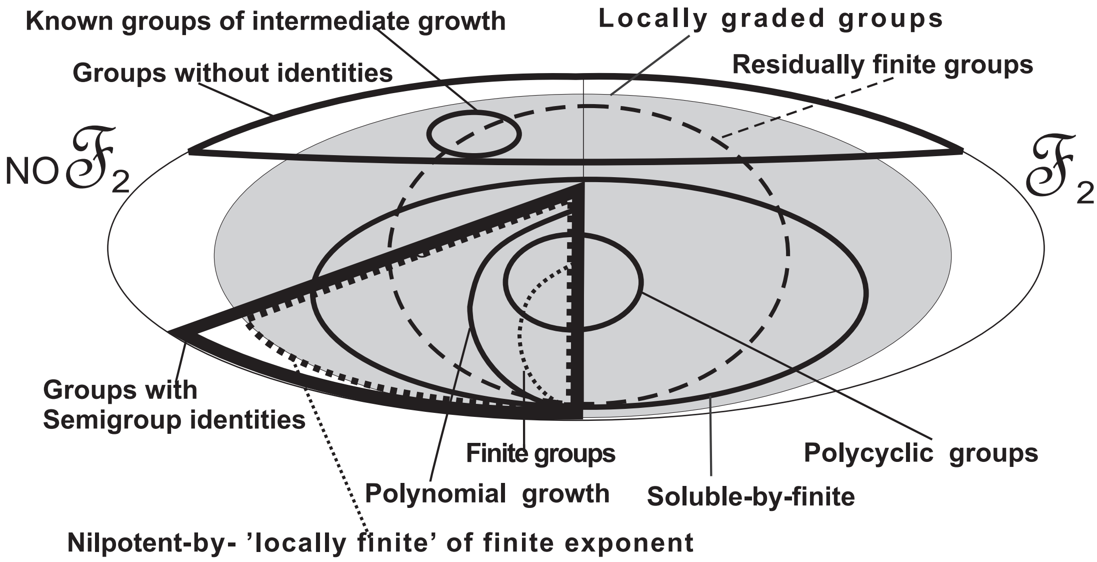
The first example of a non-transferable identity was given in 2005 by S. Ivanov and A. Storozhev [8]. They found a counterexample-group G (in fact a family of them) with a generating semigroup S such that the group G does not satisfy an identity holding in S (the identity is similar to that introduced by A. Yu. Ol'shanskii in [13]). So the GB-problem has a negative answer in general. The problem of describing the class of counterexample-groups is now open. We show (Theorem 1) that looking for the counterexample-groups it suffices to consider only the groups with relatively free generating semigroups and that each such a group (with not less then three generators) either contains a free noncyclic semigroup or satisfies a semigroup identity (Theorem 2). We show also (Theorem 3) that there is no counterexample-group in the large class of locally graded groups with no free noncyclic subsemigroups.

The class of locally graded groups was introduced in 1970 by Černikov to avoid groups such as the infinite Burnside groups or Ol'shanskii-Tarski monsters. A group G is called *locally graded* if every nontrivial finitely generated subgroup of G has a proper subgroup of finite index. We note that all locally or residually soluble groups and all locally or residually finite groups are locally graded. The class of locally graded groups is closed for taking subgroups, extensions and cartesian products.

On the map of Groupland (below) the region of locally graded groups is marked as the inner grey ellipse. The *left half* of Groupland denoted by symbol "NO $\mathcal{F}_2$" contains groups *with no* free noncyclic subsemigroups. In the *right half* there are groups *with* free noncyclic subsemigroups. The *north part* contains groups which satisfy *no identities.* The *south-west sector* contains groups which satisfy *semigroup identities.* The *dashed ellipse* contains *residually finite* groups. There are regions of groups of different types of growth, polycyclic groups and others.

It is visible that the class of locally graded groups satisfying semigroup identities consists of groups which are extensions of nilpotent groups by locally finite groups of finite exponent [3], and that not all groups of intermediate growth are residually finite [6] as it was conjectured. More information on Groupland can be found in [12] and on the Internet.

GROUPLAND



We prove that for locally graded groups with no free noncyclic subsemigroups (the left half of the inner grey ellipse) and for residually finite groups (the region inside the dashed ellipse) the answer to the GB-problem is affirmative.

2. Groups with generating semigroups satisfying identities

Let S be a generating semigroup of a group G. The statements of the following proposition hold if S has no free noncyclic subsemigroup, however we need it in a weaker form.

Proposition 1 *If a generating semigroup S of a group G satisfies a nontrivial identity, then (i) for every s, $t \in S$ there exist s', $t' \in S$ such that $ss' = tt'$ (left Ore condition), (ii) $G = SS^{-1}$, (iii) for every g, $h \in G$ there are $s, t, r \in S$ such that $g = sr^{-1}$, $h = tr^{-1}$.*

Proof A nontrivial n-variable identity in S implies a nontrivial 2-variable identity if replace the i-th variable by xy^i. By the cancellation property, it may be assumed as $xu(x,y) = yv(x,y)$. For $x{=}s,\ y{=}t,\ s'{:=}\ u(s,t),\ t'{:=}v(s,t)$, the statement (i) follows. Now since each $g \in G$ is a product of elements in $S{\cup}S^{-1}$, and by (i) for every $s,t \in S$, $s^{-1}t = s't'^{-1}$, the statement (ii) follows. Since $G = SS^{-1}$, for every $g,\ h \in G$ there exist $a,\ b,\ c,\ d \in S$ such that $g = ab^{-1}$, $h = cd^{-1}$. By (i) there exist $b',\ d' \in S$ such that $bb' = dd'$. We denote $r := bb' = dd'$ and $s := ab'$, $t := cd'$. Then $g = ab^{-1} = ab'b'^{-1}b^{-1} = sr^{-1}$ and $h = cd^{-1} = cd'd'^{-1}d^{-1} = tr^{-1}$, which gives (iii). □

Similarly, by using the last letters of the identity we can also prove that $G = S^{-1}S$. Then by Theorem 1.25 in [4] we obtain the following

Corollary 1 *A group G with a generating semigroup S where S satisfies a nontrivial identity is uniquely defined (up to an isomorphism) by the semigroup S.*

For a group G with a generating semigroup S satisfying a nontrivial identity we shall obtain the natural presentations $S \cong \mathcal{F}/\rho$ and $G \cong F/N_\rho$ where F is a free group of appropriate rank, freely generated by a set $X{=}\{x_i,\ i \in I\}$, and $\mathcal{F}$ – a free semigroup, generated by the same set X. As in ([5] 12.8) for a congruence ρ on $\mathcal{F}$ we denote by $\mathcal{A}_\rho$ the following set

$$\mathcal{A}_\rho := \{\ ab^{-1} \mid (a,b) \in \rho\ \} \subseteq \mathcal{F}\mathcal{F}^{-1}.$$

By ngp $\mathcal{A}_\rho$ we denote the normal closure of the set $\mathcal{A}_\rho$ in F. In further text var(S) and var(G) denote the variety of semigroups and the variety of groups generated by S and G respectively.

Proposition 2 (cf [5] Construction 12.3 for a semigroup S)
Let G be a group with a generating semigroup S. If S satisfies a nontrivial identity then there exists a congruence ρ on $\mathcal{F}$ such that $S \cong \mathcal{F}/\rho$, and for the normal subgroup $N_\rho := \operatorname{ngp} \mathcal{A}_\rho$ in F, $G \cong F/N_\rho$ and $N_\rho \cap \mathcal{F}\mathcal{F}^{-1} = \mathcal{A}_\rho$.

Proof Let the semigroup S have a generating set $\{s_i,\, i \in I\}$ and let the set X have the same cardinality. In the following diagram the maps $X \to \mathcal{F} \to F$ are the embeddings and β sends $x_i \to s_i$.

$$
\begin{array}{ccccc}
 & & X & & \\
 & & \downarrow & & \\
\mathcal{F}/\rho \cong S & \overset{\beta}{\longleftarrow} & \mathcal{F} & \longrightarrow & F \\
 & {}_{\gamma}\searrow & & \swarrow_{\rho_1^{\#}} & \\
 & & F/N_\rho & &
\end{array}
\tag{1}
$$

The map β defines a congruence ρ on $\mathcal{F}$, such that $S \cong \mathcal{F}/\rho$. Namely, $(u(x_i), v(x_i)) \in \rho$ if and only if $u(s_i)$ and $v(s_i)$ are equal in S. The generators s_i correspond to the congruence classes of x_i under ρ.

The congruence ρ on $\mathcal{F}$ generates a congruence ρ_1 on F with the equivalence class of the unity equal to the normal subgroup $N_\rho := \operatorname{ngp} \mathcal{A}_\rho$. The cosets of N_ρ are the equivalence classes of ρ_1 in F. Hence the congruence ρ_1 defines the natural homomorphism $\rho_1^{\#} : F \to F/N_\rho$. It is shown in ([5], p. 291), that the map γ which sends each s_i to the coset $x_i N_\rho$, defines a homomorphism $\gamma : S \to F/N_\rho$ and S^γ generates F/N_ρ as a group. By Theorem 12.4 of [5] γ is the embedding, hence S is isomorphic to S^γ which is the generating semigroup of F/N_ρ. Now by Corollary 1 we have $G \cong F/N_\rho$.

By Corollary 12.8 in [5] for $a, b \in \mathcal{F}$, $ab^{-1} \in \operatorname{ngp} \mathcal{A}_\rho$ implies $ab^{-1} \in \mathcal{A}_\rho$

which can be written as $\mathcal{F}\mathcal{F}^{-1} \cap \operatorname{ngp} \mathcal{A}_\rho \subseteq \mathcal{A}_\rho$. The opposite inclusion is clear which finishes the proof. $\square$

We say that a subset of elements in F is $\operatorname{End}\mathcal{F}$-invariant if it is invariant to all mappings $X \to \mathcal{F}$.

Corollary 2 *If M is an $\operatorname{End}\mathcal{F}$-invariant normal subgroup of F, then the set $\mathcal{F}\mathcal{F}^{-1} \cap M$ defines a fully invariant congruence μ on $\mathcal{F}$ such that $\mathcal{A}_\mu = \mathcal{F}\mathcal{F}^{-1} \cap M$.* $\square$

The following theorem shows the importance of the relatively free generating semigroups.

Theorem 1 *Let G have a generating semigroup S. There exists a group G_0 with a generating semigroup S_0 such that S_0 is free in var(S), and var$(G_0) =$ var(G).*

To prove this theorem we need the following

Lemma 1 *Let M, N be subgroups and A a subset of F. If $M \supseteq N$ then*

$$AN \cap M = (A \cap M)N. \tag{2}$$

Proof The inclusion "$\supseteq$" is clear. Let $t \in AN \cap M$, then $t = ar$ for some $a \in A$, $r \in N$. Since $t, r \in M$ we have $a = tr^{-1} \in A \cap M$. So $t = ar \in (A \cap M)N$, which finishes the proof. $\square$

Proof of Theorem 1 Let S be a generating semigroup of a group G, and var(S) be the variety of all semigroups. A free group in var(G) generating var(G) satisfies no semigroup identity and has a free generating semigroup which is also free in var(S). This proves the existence of G_0.

Let now var(S) be a proper variety of semigroups. By Proposition 2 there exists a congruence ρ on $\mathcal{F}$ such that $S \cong \mathcal{F}/\rho$, and $G \cong F/N_\rho$

for N_ρ :=ngp $\mathcal{A}_\rho$. We denote by ρ_0 the biggest fully invariant congruence on $\mathcal{F}$ contained in ρ. Then $\mathcal{A}_{\rho_0}$ is the biggest End $\mathcal{F}$-invariant subset in $\mathcal{A}_\rho$. The semigroup $S_0 := \mathcal{F}/\rho_0$ is free in var(S) with a free generating set $\{\bar{s}_i,\ i \in I\}$, where $\bar{s}_i$ are the congruence classes of x_i under ρ_0.

If repeat the construction (1) for the semigroup S_0 with the map $\bar{\beta} : x_i \to \bar{s}_i$ we obtain the above congruence ρ_0, the normal subgroup N_0 :=ngp $\mathcal{A}_{\rho_0}$ such that $N_0 \cap \mathcal{F}\mathcal{F}^{-1} = \mathcal{A}_{\rho_0}$ and the group $G_0 := F/N_0$ with the relatively free generating semigroup S_0.

Since $\rho_0 \subseteq \rho$ implies $N_0 \subseteq N_\rho$, there is a natural homomorphism $G_0 \to G$ which provides var$(G_0) \supseteq$ var(G) for varieties of groups.

To obtain the opposite inclusion it suffices to show that each identity satisfied in G must be satisfied in G_0. Let $w = 1$ be an identity satisfied in the group $G \cong F/N_\rho$. Then N_ρ contains a verbal subgroup V generated by the word w. Since V is fully invariant in F, we conclude that VN_0 is an *End* $\mathcal{F}$-invariant normal subgroup contained in N_ρ. By Corollary 2, VN_0 defines a fully invariant congruence $\mu \subseteq \rho$, hence $\mu \subseteq \rho_0$ and we have $\mathcal{A}_\mu \subseteq \mathcal{A}_{\rho_0}$, that is

$$\mathcal{F}\mathcal{F}^{-1} \cap VN_0 \subseteq \mathcal{F}\mathcal{F}^{-1} \cap N_0. \tag{3}$$

By (ii) of Proposition 1, $G_0 = S_0 S_0^{-1}$ implies $F = \mathcal{F}\mathcal{F}^{-1} N_0$ and then

$$VN_0 = F \cap VN_0 = \mathcal{F}\mathcal{F}^{-1} N_0 \cap VN_0 \overset{(2)}{=} (\mathcal{F}\mathcal{F}^{-1} \cap VN_0) N_0 \overset{(3)}{=} (\mathcal{F}\mathcal{F}^{-1} \cap N_0) N_0 = N_0.$$

So N_0 contains V and hence the identity $w = 1$ is satisfied in G_0. □

The following theorem shows that if a group with a relatively free generating semigroup lies in the left half of Groupland then it must be in the south-west sector. Unfortunately our proof requires at least three free generators in the generating semigroup.

Theorem 2 *Let G have a relatively free generating semigroup of rank greater then two. If G has no free noncyclic subsemigroup then G satisfies a semigroup identity.*

Proof Let S be a relatively free generating semigroup of G with a set of at least three free generators $s_1, s_2, s_3, \dots$. Since by assumption G has no free noncyclic subsemigroup, the elements $s_1 s_3^{-1}$, $s_2 s_3^{-1}$ generate a non-free subsemigroup. Then there exist nontrivial words $a(x,y)$, $b(x,y)$ such that

$$a(s_1 s_3^{-1},\, s_2 s_3^{-1}) = b(s_1 s_3^{-1},\, s_2 s_3^{-1}). \tag{4}$$

By (iii) of Proposition 1 for every $g,\, h \in G$ there are $s, t, r \in S$ such that $g = sr^{-1}$, $h = tr^{-1}$. Every map of the free generators s_i to elements of S defines an endomorphism of S, so we denote by ε the endomorphism which maps $s_1 \to s,\ s_2 \to t,\ s_3 \to r$ and fixes other free generators in S. Now by applying ε to (4) we obtain $a(g,h) = b(g,h)$ which means that the group G satisfies the semigroup identity $a(x,y) = b(x,y)$. □

3. Semigroup respecting groups (S-R groups)

Definition We call a group G *semigroup respecting* (S-R group) if all of the identities holding in each generating semigroup of G hold in G.

In case when none generating semigroup of G satisfies nontrivial identities G is vacuous an S-R group. The problem arises: which groups with generating semigroups satisfying nontrivial identities are S-R?

The following observation is immediate.

Corollary 3 *Torsion groups are S-R.*

Proof If the group G is torsion, then for each $s \in S$ there is some natural n such that $s^{-1} = s^{n-1} \in S$, so $G = S$. Hence G is S-R. □

We can see that the condition for a group to be semigroup respecting is a "local condition" in the sense of Mal'cev.

Proposition 3 *If every finitely generated subgroup of a group G is S-R, then so is G.*

Proof Suppose that for some generating semigroup S of G a nontrivial identity $u(x_1, ..., x_n) = v(x_1, ..., x_n)$ is satisfied in S and fails in G for elements $g_1, ..., g_n$, say. By (ii) of Proposition 1, each element g_i may be written as $s_i t_i^{-1}$, $s_i, t_i \in S$. We denote by S_1 the subsemigroup $sgp(s_1, ..., s_n, t_1, ..., t_n)$. Since $S_1 \subseteq S$, the identity $u = v$ is satisfied in the finitely generated subsemigroup S_1 but not in the subgroup $gp(S_1)$, since it contains $g_1, ..., g_n$. This contradicts the assumption. □

Proposition 4 *If a group G is residually S-R then it is S-R.*

Proof The group G being residually S-R is equivalent to G being a subcartesian product of some family $(G_i |\, i \in I)$ of S-R groups G_i. Each generating semigroup S of G will then be a subcartesian product of a family $(S_i |\, i \in I)$ of generating semigroups S_i of the G_i. If an identity holds in S but fails in G it would have to hold in each S_i but fail in at least one G_i, contradicting the assumption that the G_i are all S-R. □

By Corollary 3 finite groups are S-R, so by Propositions 4 residually finite groups are S-R. Now by Propositions 3 we get the following

Proposition 5 *Locally 'residually finite' groups are S-R.* □

Since by ([11], Prop. 7.11) every finitely generated linear group over a field is residually finite it follows

Corollary 4 *Every linear group over a field is S-R.*

Our main result is based on the following Lemma, which resembles the known fact for finitely generated groups: if G/N is finite, then N is a finitely generated subgroup of G.

Lemma 2 *Let G be a finitely generated group with no free noncyclic subsemigroup. If G/N is nilpotent-by-finite, then N is a finitely generated subgroup of G.*

Proof By assumption G/N contains a nilpotent normal subgroup H/N of finite index, that is $|G/N : H/N| = |G : H| < \infty$. Since G is finitely generated, then H and H/N are finitely generated. Since H/N is nilpotent and finitely generated, it is supersoluble ([14] 5.4.6), and then there exists a finite normal series with, say, m cyclic factors: $H = N_0 \rhd N_1 \rhd ... \rhd N_m = N$.

Since N_0 is finitely generated we assume inductively that N_i is generated by, say, n elements and show that N_{i+1} is also finitely generated. The factor N_i/N_{i+1} is cyclic generated by a coset gN_{i+1}. If $\langle g\rangle \cap N_{i+1} \neq \{e\}$, then N_i/N_{i+1} is finite and hence N_{i+1} is finitely generated. So let now $\langle g\rangle \cap N_{i+1} = \{e\}$. Each generator in N_i is in some power of the coset gN_{i+1} and hence there exist $t_1, ..., t_n \in N_{i+1}$, such that $N_i = \langle g, t_1, ..., t_n\rangle$, that is N_i is generated by two subgroups: $\langle g\rangle$ and $T \coloneqq \langle t_1, ..., t_n\rangle$. Let T^{N_i} denote the normal closure of T in N_i. Then $T^{N_i} \subseteq N_{i+1}$ and $\langle g\rangle \cap T^{N_i} = \langle g\rangle \cap N_{i+1} = \{e\}$. Since $\langle g\rangle \cdot T^{N_i} = N_i$ we shall obtain $N_{i+1} = T^{N_i}$:

$$N_{i+1} = N_i \cap N_{i+1} = \langle g\rangle T^{N_i} \cap N_{i+1} \overset{(2)}{=} (\langle g\rangle \cap N_{i+1})T^{N_i} = (\langle g\rangle \cap T^{N_i})T^{N_i} = T^{N_i}.$$

By commutator calculus $T^{N_i} = T\cdot[T,\, N_i] = T\cdot[T,\, \langle T, \langle g\rangle\rangle] = T^{\langle g\rangle}$ is generated by all conjugates $g^k t g^{-k},\ k\in\mathbb{Z},\ t\in T$. So since $N_{i+1} = T^{\langle g\rangle}$, N_{i+1} is generated by subgroups $\langle t_i\rangle^{\langle g\rangle},\ i\le n$. We use the assumption that G has no free noncyclic subsemigroup. It is shown in ([10] Lemma 1), that in such a group for any two elements a, b, the subgroup $\langle a\rangle^{\langle b\rangle}$ is finitely generated. So N_{i+1} is finitely generated, which completes the induction, and proves that N is finitely generated. □

We recall here the following well-known lemma.

Lemma 3 *Every finite-by-nilpotent group is nilpotent-by-finite.*

Proof Let N be a finite normal subgroup of a group G and let G/N be nilpotent. Then each element a in N has only finitely many conjugates in G, all contained in N. Hence the index of the centralizer of a in G is finite. Now the centralizer of N, $C := C_G(N)$ is the intersection of these finitely many centralizers, so it also has finite index in G.

Since G/N is nilpotent, $\gamma_c(G)\subseteq N$ for some natural c. Then $\gamma_{c+1}(C) = [\gamma_c(C), C]\subseteq [N, C] = \{e\}$, hence C is nilpotent. Since C is of finite index, G is nilpotent-by-finite. □

Our main result shows that locally graded groups without free noncyclic subsemigroups are S-R, which gives a positive answer to the GB-problem in that class of groups. The idea of the proof is to reduce the situation to the case considered in Proposition 5.

Theorem 3 *Let G be a locally graded group with a generating semigroup satisfying a nontrivial identity. If G has no free noncyclic subsemigroup then G is an S-R group.*

Proof In view of Proposition 3 we can assume that G is finitely generated. Let N be the intersection of all normal subgroups of finite index in G. Assume that $N \neq \{e\}$, then G/N is residually finite and also has a generating semigroup satisfying nontrivial identities. Then by Proposition 5, G/N must be S-R and hence satisfies some semigroup identity. The Theorem of Semple and Shalev ([15] p.44), ensures that a finitely generated residually finite group with a semigroup identity is nilpotent-by-finite.

Thus G/N is nilpotent-by-finite and by Lemma 2, N is finitely generated. Being a finitely generated subgroup in a locally graded group, N must contain a proper subgroup R of finite index in N. By ([11] p.196), R contains a subgroup K characteristic in N of a finite index in N. Thus $N \supsetneq R \supseteq K$, where K must be normal in G. Since N/K is finite and $(G/K)/(N/K) \cong G/N$ is nilpotent-by-finite, we have that G/K is finite-by-(nilpotent-by-finite) and by Lemma 3, G/K is nilpotent-by-finite. Then by [7], G/K is residually finite. So the intersection of all normal subgroups of finite index in G is in K. That is $N \subseteq K$, which contradicts the above inequality $N \supsetneq K$. Hence $N = \{e\}$, G is residually finite and by Proposition 5, G is S-R. □

The counterexample-group found by S. Ivanov and A. Storozhev does not satisfy any identity, because it contains a free noncyclic subgroup. The natural questions arises

Question Does there exist a counterexample-group satisfying a nontrivial identity (a nontrivial semigroup identity)?

Note that by a result of Mal'tsev the metabelian identity does not affect the generating semigroup in a free group, however we cannot impose the metabelian identity on a counterexample-group to have a new counterexample group, because by [7] the metabelian groups are locally residually finite and by Proposition 5, are S-R.

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References

[1] Bergman G, (1981) Hyperidentities of groups and semigroups. *Aequat. Math.* 23:55-65.

[2] Bergman G, (1986) Questions in algebra. Preprint. Berkeley, U.C.

[3] Burns R. G., Macedońska O. and Medvedev Y. (1997) Groups Satisfying Semigroup Laws, and Nilpotent-by-Burnside Varieties. *J. Algebra* 195:510-525.

[4] Clifford A. H. and Preston G. B. (Vol. II, 1964) The Algebraic Theory of Semigroups. *Math. Surveys, Amer. Math. Soc. Providence, R.I.*

[5] Clifford A. H. and Preston G. B. (Vol. II, 1967) The Algebraic Theory of Semigroups. *Math. Surveys, Amer. Math. Soc. Providence, R.I.*

[6] Erschler A. (2004) Not residually finite groups of intermediate growth, commensurability and non-geometricity. *J. Algebra* 272:154-172.

[7] Hall P. (1959) On the finiteness of certain soluble groups. *Proc. London Math. Soc.* 9:595-622.

[8] Ivanov S. V. and Storozhev A. M. (2005) On identities in groups of fractions of cancellative semigroups. *Proc. Amer. Math. Soc.* 133:1873-1879.

[9] Krempa J. and Macedońska O. (1992) On identities of cancellative semigroups. *Contemporary Mathematics* 131:125-133.

[10] Longobardi P., Maj M. and Rhemtulla A. H. (1995) Groups with no free subsemigroups. *Trans. Amer. Math. Soc.* 347:1419-1427.

[11] Lyndon R. C. and Schupp P. E. (1977) Combinatorial Group Theory. *Springer-Verlag Berlin, Heidelberg, New York*.

[12] Macedońska O. (2003) Groupland. *London Math. Soc. Lecture Note Ser.* 305:400-404.

[13] Ol'shanskii A. Yu. and Storozhev A. (1996) A group variety defined by a semigroup law. *J. Austral. Math. Soc. Series A,* 60:255-259.

[14] Robinson D. J. S. (1982) A Course in the Theory of Groups. *Springer-Verlag Berlin, Heidelberg, New York*.

[15] Semple J. F. and Shalev A. (1993) Combinatorial conditions in residually finite groups I. *J. Algebra* 157:43-50.

[16] Shevrin L. N. and Sukhanov E. V. (1989) Structural aspects of the theory of varieties of semigroups (in Russian). *Izv. Vyssh. Uchebn. Zaved. Mat.* 6:3-39.
English translation: (1989) *Soviet Math.* (Iz. VUZ 6), 33:1-34.

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