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Ryszard GESSING
Silesian Technical University

STOCHASTIC HIERARCHICAL CONTROL WITH PERIODIC COORDINATION FOR RESOURCE DISTRIBUTION[x]

Abstract. In the paper the two-level, hierarchical control structure for optimal resource distribution in a large-scale dynamic system in the case of incomplete information has been considered. It is assumed that the information of the decision-makers of the higher and lower levels differs from each other and that the decision-making frequency is less in the higher level than in the lower level. The method of determination of the optimal control strategies for all decision-makers has been presented. Namely, it has been shown that in the considered problem a decomposition of the calculations and realization of the control is possible.

Keywords. Large-scale systems; hierarchical systems; stochastic control; optimal control; dynamic programming; discrete time systems.

INTRODUCTION

A lot work has been done on the problems of hierarchical optimization and the methods of decomposition and coordination. There exist many papers and books where these problems are described in detail, at the same time deterministic [3, 8] and probabilistic [2, 4, 7] methods are described.

It seems that in the control of the large-scale system an important part play the methods which make it possible to decentralize the control decision making. One from this method has been described

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in [2] , where two-level approach is used in which the higher level collects measurements and control-decisions from the local controllers periodically with a less frequency in comparison to that of the lower level and in retern transmits coordinating variables.

In [4] the another method of decomposition and coordination for resource distribution in a large-scale, dynamic system has been described. It is assumed there, that in the two-level control system the decision-makers of the higher and lower levels have at disposal the information wich differs from each other. Each decision-maker of the lower level has at disposal the information which is essential for the particular subsystem, and the decision-maker of the higher level - the information which is essential for the whole system.

The presented paper differs from [4] by two additional assumptions: first that the decision-making frequency is less in the higher level than in the lower level, and second that the losses in the future time intervals are neglected in calculations of the lower level control strategies. The problem statement make it possible to obtain a more essential decomposition of the calculations and a real decentralized control.

DESCRIPTION OF THE PRIMARY PROBLEM

Let us consider the large-scale system composed of M subsystems, each of which is described by the difference equation

$$x^i_{n+1} = f^i_n(x^i_n, u^i_n, w^i_n). \tag{1}$$

$i = 1,2,\dots,M$, where x^i_n, u^i_n, w^i_n are the state, control and disturbance of the i-th subsystem, respectively, f^i_n are definite functions of their arguments, $n = 0,1,\dots,N$ denotes the basic discrete time and N- the stopping time.

The primary performance index which defines the losses in the whole system which we would like to minimize has the form

$$I = \sum_{n=0}^{N} \sum_{i=1}^{N} L^i_n(x^i_n, u^i_n, z^i_n) \tag{2}$$

wheres L^i_n are scalar, definite functions of their arguments, z^i_n - random variables upon which the losses in the i-th subsystem are dependent, they can e.g. define demands for the resources at the instant n in the i-th subsystem.

We assume that the controls u^i_n do not influence the random variables w^j_m, z^j_m, for $m \leq n$, $n = 0,1,\dots,N$, $i,j = 1,2,\dots,M$.

Further on, we assume that the resources of the control variables u_n^i are limited. Namely, we assume that there exist K magazines which accumulate the resources, at the same time the resource inflows d_n^i to the particular magazines in the particular instant of time are random variables. The control u_n^i denotes the decision about the resource ration for the i-th subsystem, taken at the instant n from the one definite magazine.

Let h_n^j be the resource level at the instant n in the j-th magazine. It would be desirable to fulfill the following constraints

$$h_{min}^j \leqslant h_n^j \leqslant h_{mx}^j \tag{3}$$

where h_{min}^j and h_{mx}^j denote the lower and upper level limitations, respectively.

The account of the resource inflows and outflows for the j-th magazine gives the equation

$$h_{n+1}^j = h_n^j + P^j v_n - R^j u_n + d_n^j \tag{4}$$

$j = 1,2,\ldots,K$, where $v_n = \left[v_n^1, v_n^2,\ldots,v_n^D\right]^T$, $u_n = \left[u_n^1,u_n^2,\ldots,u_n^M\right]^T$, v_n^l, $l = 1,2,\ldots,D$ – denotes the controlled resource flow between definite magazines, or the controlled resource outflow from a definite magazine beyond the system, D – the number of these flows, u_n^i – the controlled resource consumption of the i-th subsystem – all variables at instant n; P^j or R^j are the row-vectors defining the flows-structure between the j-th and other magazines, or between the j-th magazine and some definite subsystems, respectively. The successive i-th component of the vector P^j takes the values 1, or – 1, or 0 if the variable v_n^i determines inflow, or outflow, or – is not connected with the j-th magazine, respectively. The successive i-th component of the vector R^j takes the values 1 or 0 if the i-th subsystem takes, or not the resources from the j-th magazine, respectively. The equations (4) can also be written in the vector – form

$$h_{n+1} = h_n + Pv_n - Ru_n + d_n \tag{5}$$

where $h_n = \left[h_n^1, h_n^2,\ldots, h_n^K\right]^T$, $d_n = \left[d_n^1, d_n^2,\ldots, d_n^K\right]^T$,

P and R are K x D and K x M matrices, created from the rows P^j and R^j, respectively. The above mentioned variables should fulfill the constraints

$$v_n \in V, \quad u_{min}^i \leqslant u_n^i \leqslant u_{mx}^i \tag{6}$$

for $n = 0,1,\ldots,N$, $i = 1,2,\ldots M$, where V denotes some given set in R^D, u^i_{min} and u^i_{mx} are lower and upper limitations, respectively.

Since in the considered primary problem there exist a large number of random variables, then the value of the primary performance index is also a random variable. Then in the case of incomplete information it is impossible to find the solution of the primary problem, i.e. to define the controls u^i_n for which the primary performance index takes the minimal valus. The formulation of the secondary problem which will be able to be solved, will be dependent upon the available information, as well as, upon the structure of the control system which will be proposed.

CONTROL SYSTEM STRUCTURE AND AVAILABLE INFORMATION

We assume that the decisions about the resource distribution are made in a two-level control system. In the lower level the decision-makers (the number of which is equal to M) define the resource distribution policy for each subsystem, separately. In the higher level the decision-maker defines the resource distribution between particular subsystems.

The decisions of the lower and higher level with different frequencies are made. The decisions in the lower level are made with the frequency to be defined by occuring in (1) the basic discrete time $n = 0,1,\ldots,N$, and in the higher level - by the discrete time of the higher level defined by $k = W(n/L)$. Here $W(n/L)$ denotes the whole part of the fraction n/L , and L is the whole number defining the ratio of the decision-making frequencies of the lower and higher levels. Thus, the same instants of time are denoted by the integer $\dot{k}$ - in the higher level, and by the integer $n = L\dot{k}$ - in the lower level. Further on, if necessary, we will say that some instant of time is described by the higher or lower level time integer and - denote them by $\dot{k}$ or n, respectively. To more exactly differentiate both integers the higher level time integer will be used with a dot.

We assume that every decision-maker has information about the system which consists of two components. The first one results from the experience in the past and has the shape of appropriate probability distribution functions; the second one - results from a current measurements to be made in particular points of the system. Thus we assume that each decisionmaker of the i-th subsystem in the lower level has at disposal at the instant described by the lower level time integer n the vector of current information $\vec{y}^i_n$, components of which result from measurements. The decision-maker of the higher level has at disposal the vector $\vec{y}_{k.}$, where k. denotes the higher level time integer. The vector $\vec{y}_n^i$ contains more detailed information concerning the i-th subsystem, the vector $\vec{y}_{k.}$ contains the information which is essential for the whole system and not contains some components which occur in the vectors $\vec{y}_n^i$, for $n = kL$. We assume that the vectors $\vec{y}_{k.}$, $\vec{y}_n^i$ accumulate the information from the past i.e. all components which occur in the vectors $\vec{y}_{k.}$, $\vec{y}_n^i$ also occur in the vectors $\vec{y}_{k+1.}$, $\vec{y}_{n+1}^i$, respectively, and the last vectors contain additionally the results of new measurements[1].

We also assume that mutual relations between the vectors $\vec{y}_{k.}$ and $\vec{y}_{kL}^i$, $i = 1,2,\ldots,M$, $k = 0,1,\ldots,F = W(N/L)$, are such that given the components of the vector $\vec{y}_{kL}^i$ one can define the all components of the vector $\vec{y}_{k.}$, which have the essential meaning for the control of the i-th subsystem in the time interval $kL \leq n \leq (k+1)L-1$, i.e. the components of $\vec{y}_{k.}$ which concern the i-th subsystem. Generally speaking the components of the vectors $\vec{y}_{k.}$ and $\vec{y}_n^i$ are chosen in this manner that it is possible to define the appropriate probability distribution functions, which are utilized at the time of performing the averaging operations to be defined later. Additionally, we assume that the decisions u_n^i do not influence the components of the vectors $\vec{y}_{n+1}^j$, $j \neq i$, $i,j = 1,2,\ldots,M$.

The vector $\vec{y}_{k.}$ can contain the information about the resource levels $h_{k.}^j$ at the instant defined by the higher level time integer k. in all magazines, which has an essential meaning for the fulfilment of the constraints (3) to the best ability.

The decision-makers of the lower level besides the results of measurements to be contained in the vectors $\vec{y}_n^i$ obtain additionally from the higher level the information about the resource quantity $p_{k.}^i$ to be assigned for the i-th subsystem by the higher level for utilization in the time interval $kL \leq n \leq (k+1)L-1$. Decisions about the quantities $p_{k.}^i$ are made in the higher level on the basis of the information contained in the vector $\vec{y}_{k.}$, and decisions about the controls u_n^i are made by the i-th decision-maker of the lower level on the basis of the information contained in the vector $\vec{y}_n^i$ and the information about $p_{k.}^i$, where $k = W(n/L)$. The quantity of $p_{k.}^i$ is the estimate of the sum $s_{kL}^i = \sum_{n=kL}^{(k+1)L-1} u_n^i$ to be

[1] In some cases this assumption can have no meaning.

determined on the basis of the information contained in $\vec{y}_{k.}$, in the form of the conditional expectation. Then we have

$$p^i_{k.} = E_{|\vec{y}_{k.}} s^i_{kL} = E_{|\vec{y}_{k.}} \sum_{n=kL}^{(k+1)L-1} u^i_n \qquad (7)$$

for $k = 0,1,\ldots,F$, $i = 1,2,\ldots,M$, where $E_{|\vec{y}_{k.}}$ denotes the operation of conditional averaging, given $\vec{y}_{k.}$. Additionally, we assume that the value s^i_{kL} is determined at the instant kL in the lower level on the basis of the information contained in the vector $\vec{y}^i_{kL}$, $k = 0,1,\ldots,F$, and this value of s^i_{kL} defines the sum of the resources, which may be used in the whole interval $kL \leqslant n \leqslant (k+1)L-1$ by the i-th subsystem.

The quantities $p^i_{k.}$ are the coordinating variables. The formula (7) leaves some freedom in choosing the controls u^i_n for the decision-makers of the lower level, owing to this the more detailed information contained in the vector $\vec{y}^i_n$ can be better utilized.

In the higher level the decisions about the flows $v^i_{k.}$, $k = 1,2,\ldots,F$, $i = 1,2,\ldots,M$, are also made. Here $v^i_{k.}$ denotes the i-th flow in the time interval $kL \leqslant n \leqslant (k+1)L-1$, and k. or n denote the higher or lower level time integer, respectively. We assume that v^i_n is the constant value defined by $v^i_n = v^i_{k.}/L$ in the time interval $kL \leqslant n \leqslant (k+1)L-1$. The decisions of the higher level should be made in this manner so as to fulfill the constraints (3) to the best ability.

SECONDARY PROBLEM FORMULATION

In the considered case of incomplete information one can neither determine the controls u^i_n for which the performance index takes the minimal value, nor ensure the fulfilment of the constraints (3). Then in the further considerations the constraints (3) will be replaced by modified constraints (3) in which in the place of the levels h^i_n the one (higher integer) step ahead estimates of these levels will occur in the shape of an appropriate conditional expectations to be determined on the basis of the information of the higher level.

The decision u^i_n of the i-th decision-maker of the lower level should be calculated in this manner so as to minimize the sum of estimates of the actual and future losses. But the estimate of the future losses can not be calculated in the lower level at the time instants $n = kL-1$, $k = 1,2,\ldots,F$, because the information of the particular decision-makers in this level is not sufficient for the calculation. Namely, on the basis of the information of the particular lower level decision-maker the estimate of the succesive of $p^i_{k+1.}$ which influences the estimate of the future losses can not be calculated. The estimate of $p^i_{k+1.}$ can be calculated on the basis of the information of the higher level, only. Thus, we assume that in the lower level the instant $(k+1)L-1$ defines the stopping time for the control in the time interval $kL \leqslant n \leqslant (k+1)L-1$, while the estimates of the future losses corresponding to the intervals $lL \leqslant n \leqslant (l+1)L-1$, $l = k+1, k+2, \ldots, F$, are taken into account in the higher level.

Let us introduce the vector $p_{k.} = \left[p^1_{k.}, p^2_{k.}, \ldots, p^M_{k.}\right]^T$. As the admissible control strategies of the i-th decision-maker, $i = 1,2,\ldots, M$, of the lower level and the decision-maker of the higher level we will understand the sets of the functions $u^i_n = a^i_n(\vec{y}^i_n, p^i_{k.})$, $n = 0,1,\ldots,N$, $k = W(n/L)$, and $p_{k.} = b_{k.}(\vec{y}_{k.})$, $v_{k.} = c_{k.}(\vec{y}_{k.})$, $k = 0,1,\ldots,F$, respectively, each of which maps the appropriate set of vectors $\vec{y}^i_n$, $p^i_{k.}$, or $\vec{y}_{k.}$ to the appropriate set of values u^i_n, or $p_{k.}$, $v_{k.}$, and for which the constraints (6), the modified constraints (3) and also the constraints resulting from (7) in the form

$$b^i_{k.}(\vec{y}_{k.}) = E_{|\vec{y}_{k.}} \sum_{n=kL}^{(k+1)L-1} a^i_n\left[\vec{y}^i_n, b^i_{k.}(\vec{y}_{k.})\right] \tag{8}$$

for $n = 0,1,\ldots,N$, $i = 1,2,\ldots,M$, $k = W(n/L)$, (where $b^i_{k.}(\vec{y}_{k.})$ denotes the i-th component of the vector function $b_{k.}(\vec{y}_{k.})$) are fulfilled. Additionally, we assume that for these functions the secondary performance index in the form

$$I(a,b,c) = E\sum_{n=0}^{N} \sum_{i=1}^{M} L^i_n\left[x^i_n, a^i_n\left[\vec{y}^i_n, b^i_{k.}(\vec{y}_{k.})\right], z^i_n\right] \tag{9}$$

(where $k = W(n/L)$) takes a definite value. The denotation $I(a,b,c)$ has been introduced to stress that the value of the secondary performance index depends upon the set of functions

$a = \left\{a^1_0, \ldots, a^1_N, \ldots, a^M_0, \ldots, a^M_N\right\}$, $b = \left\{b_{0.}, \ldots, b_{F.}\right\}$ and $c = \left\{a_{0.}, \ldots, a_{F.}\right\}$. The dependence of the secondary performance index on c results from taking into account the above mentioned constraints.

For a similarity reason of the calculations we additionally assume that the stopping time fulfills the relation

$$N = (F+1)L-1 \tag{10}$$

where F - some integer defining the number of control steps in the higher level.

Secondary Problem

Among admissible control strategies of the considered large-scale system the optimal strategy for the i-th, $i = 1,2,\ldots,M$, decision-maker of the lower level $u_n^i = a_n^{io}(\vec{y}_n^i, p_{k.}^i)$, $n = 0,1,\ldots,N$, $k = W(n/L)$, and the optimal strategy for the decision-maker of the higher level $p_{k.} = b_{k.}^o(\vec{y}_{k.})$, $v_{k.} = c_{k.}^o(\vec{y}_{k.})$, $k = 0,1,\ldots,F = W(N/L)$, are to be found for which the secondary performance index (9) takes the minimal value, or

$$I(a^o,b^o,c^o) = \operatorname*{Min}_{a,b,c} I(a,b,c) \tag{11}$$

SOLUTION OF THE SECONDARY PROBLEM

The similar considerations as in the papers [1, 5] (in which the lemma 1 [5, p. 450], or which is equivalent lemma 3.2 [1, p. 261] and the principle of optimality is utilized) make it possible to apply the stochastic dynamic programming method. Owing to the assumptions and the shape of the primary performance index the calculations may be performed on some components of this index, separately, which make it possible to decompose the calculation problem.

Let us denote

$$s_n^i = \sum_{j=n}^{(k+1)L-1} u_j^i, \qquad k = W(n/L) \tag{12}$$

Beginning from the last instant of time we denote

$$S_N^i(\vec{y}_N^i, s_N^i) = E_{|\vec{y}_N^i} I_N^i(x_N^i, u_N^i, z_N^i) \tag{13}$$

where $u_n^i = s_n^i$, which results from (10) and (12). The functions S_N^i should be determined for the vectors $\vec{y}_N^i$ from a set with probability 1 and for s_N^i from the interval

$$u_{min}^i \leqslant s_N^i \leqslant u_{mx}^i \tag{14}$$

The last constraint results from (6) and from the dependence $s^i_N = u^i_N$.

Calculating recurrently let us assume that for some n from the interval $FL \leqslant n < N$ we know from previous calculations the function $S^i_{n+1}(\vec{y}^i_{n+1}, s^i_{n+1})$. Then utilizing the dynamic programming method in the stochastic version we obtain the Bellman's equation

$$S^i_n(\vec{y}^i_n, s^i_n) = \underset{u^i_n}{\mathrm{Min}}\, E_{|\vec{y}^i_n}\left[L^i_n(x^i_n, u^i_n, z^i_n) + S^i_{n+1}(\vec{y}^i_{n+1}, s^i_n - u^i_n)\right] \tag{15}$$

where

$$u^i_{min} \leqslant u^i_n \leqslant u^i_{mx} \tag{16}$$

The functions S^i_n should be determined for the vectors $\vec{y}^i_n$ from a set with probability 1 and for s^i_n from the interval

$$(N-n+1)u^i_{min} \leqslant s^i_n \leqslant (N-n+1)u^i_{mx} \tag{17}$$

wich results from (12) and (6). Performing the operation of minimization in (15) we determine the function $u^i_n = \bar{a}^{io}_n(\vec{y}^i_n, s^i_n)$.

From the equation (15) we determine finally the function $S^i_{FL}(\vec{y}^i_{FL}, s^i_{FL})$. The additional function of the lower level optimal strategy $s^i_{FL} = \bar{\bar{a}}^{io}_{FL}(\vec{y}^i_{FL}, p^i_{F.})$ we can determine solving the minimization problem in the espression

$$\bar{S}^i_{F.}(\vec{y}_{F.}, p^i_{F.}) = \underset{\bar{\bar{a}}^i_{FL}}{\mathrm{Min}}\, E_{|\vec{y}_{F.}}\, S^i_{FL}\left[\vec{y}^i_{FL}, \bar{\bar{a}}^i_{FL}(\vec{y}^i_{FL}, p^i_{F.})\right] \tag{18}$$

by constraints

$$E_{|\vec{y}_{F.}}\, \bar{\bar{a}}^i_{FL}(\vec{y}^i_{FL}, p^i_{F.}) = p^i_{F.} \tag{19}$$

$$Lu^i_{min} \leqslant \bar{\bar{a}}^i_{FL}(\vec{y}^i_{FL}, p^i_{F.}) \leqslant Lu^i_{mx} \tag{20}$$

The last inequality results from (12) and (6).

The functions $u^i_n = \bar{a}^{io}_n(\vec{y}^i_n, s^i_n)$, $FL \leqslant n \leqslant (F+1)L-1$, together with the equation

$$s^i_{n+1} = s^i_n - u^i_n \tag{21}$$

resulting from (12), with initial condition $s^i_{FL} = \bar{\bar{a}}^{io}_{FL}(\vec{y}^i_{FL}, p^i_{F.})$ define the optimal strategy functions $u^i_n = a^{io}_n(\vec{y}^i_n, p^i_{F.})$, $FL \leqslant n \leqslant (F+1)L-1$ to be sought in the secondary problem.

The functions $p_{F.} = b^o_{F.}(\vec{y}_{F.})$ and $v_{F.} = c^o_{F.}(\vec{y}_{F.})$ of the higher level optimal strategy we can determine solving the minimization problem in the expression

$$\bar{\bar{S}}_{F.}(\vec{y}_{F.}) = \operatorname*{Min}_{p_{F.}, v_{F.}} \sum_{i=1}^{M} \bar{S}^{i}_{F.}(\vec{y}_{F.}, p^{i}_{F.}) \tag{21}$$

by constraints

$$v_{F.} \in \bar{V}, \quad Lu^{i}_{min} \leqslant p^{i}_{F.} \leqslant Lu^{i}_{mx}, \qquad i = 1,2,\ldots,M, \tag{22}$$

$$h^{j}_{min} \leqslant \hat{h}^{j}_{F.} + P^{j}v_{F.} - R^{j}p_{F.} + \hat{d}^{j}_{F.} \leqslant h^{j}_{mx}, \quad j = 1,2,\ldots,K, \tag{23}$$

where

$$\bar{V} = \left\{ v_{F.} : v_{F.} = v_{FL}L,\ v_{FL} \in V \right\} \tag{24}$$

The inequality contraints (22) result from (17), (10) and (19). The constraints (23) result from the modified constraints (3), and $\hat{h}^{j}_{F.} = E_{|\vec{y}_{F.}}\, h^{j}_{F.}$, $\hat{d}^{j}_{F.} = E_{|\vec{y}_{F.}}\, d^{j}_{F.}$.
Of course, if the vector $\vec{y}_{F.}$ contains the information about $h^{j}_{F.}$ then we have $\hat{h}^{j}_{F.} = h^{j}_{F.}$.

Applying the recurrent method let us assume that we now know the function $\bar{\bar{S}}_{k+1.}(\vec{y}_{k+1.})$ determined from previous calculations. Let us consider the time interval $kL \leqslant n \leqslant (k+1)L-1$ and let us determine the control strategy functions for particular decision-makers of the lower level. In accordance with one of the assumptions in the mentioned time interval the lower level optimal strategy functions may be calculated assuming that the instant $n = (k+1)L-1$ defines the last instant of control. Therefore the calculations of these functions may be performed utilizing the formulae (13) - (21) in which the instants N and F. should be replaced by the instants $(k+1)L-1$ and k., respectively. As the result of the calculations the functions $\bar{S}^{i}_{k.}(\vec{y}_{k.}, p^{i}_{k.})$, $i = 1,2,\ldots,M$, are also obtained from the formula like to (18).

The functions $p_{k.} = b^{o}_{k.}(\vec{y}_{k.})$ and $v_{k.} = c^{o}_{k.}(\vec{y}_{k.})$ of the higher level optimal strategy we can determine solving the minimization problem in the expression

$$\bar{\bar{S}}_{k.}(\vec{y}_{k.}) = \operatorname*{Min}_{p_{k.}\ v_{k.}} \left[\sum_{i=1}^{M} \bar{S}^{i}_{k.}(\vec{y}_{k.}, p^{i}_{k.}) + \bar{\bar{S}}_{k+1.}(\vec{y}_{k+1.}) \right] \tag{25}$$

by constraints

$$v_{k.} \in \bar{V}, \quad Lu^{i}_{min} \leqslant p^{i}_{k.} \leqslant Lu^{i}_{mx}, \qquad i = 1,2,\ldots,M, \tag{26}$$

$$h^j_{min} \leqslant \hat{h}^j_{k.} + P^j v_{k.} - R^j p_{k.} + \hat{d}^j_{k.} \leqslant h^j_{mx}, \quad j = 1,2,\dots,K \qquad (27)$$

in which the denotations utilized have the same meaning as in the constraints (22), (23).

FINAL CONCLUSIONS

The essence of the presented method are the assumptions that different decision-makers of higher and lower levels have at disposal a different information and that the decisions in the higher level are made less frequently than in the lower level. Owing to these assumptions the decision-makers of the lower level obtaining from the higher level the values of $p^i_{k.}$ have yet some freedom in choosing decisions u^i_n and they can better utilize the more detailed information. Owing to this, also, the higher level can have at disposal the information which is essential for the whole system. The freedom of the decisions of the lower level is expressed by the constraint (12) and the unconventional constraint (19) . To solve the problem of minimization in (18) with the constraint (19) one can use the Lagrange multipliers method.

The problem statement make it possible to diminish the amount of information transmitted to and processed by the higher level. Of course the assumptions about the structure, available information, coordinating variables and frequency of the decision-making, as well as, the neglect in the lower level the losses corresponding to the future time intervals create some constraints for the problem and give some increase of the losses but simultaneously they make it possible to decompose the calculations and realization of the optimal control strategy.

It is wise to apply the presented coordination method in the case when the dynamics of the subsystems is significantly "faster" than the dynamics resulting from the capacity of the magazines, so that the neglect of the future intervals losses in the lower level can be accepted. In the opposite case a different coordination method can be proposed in which the coordinating variables in k-th interval, besides $p^i_{k.}$ also contain the prognoses of the resources $p^i_{k+1.}, p^i_{k+2.}, \dots, p^i_{k+1.}$ for a few next time intervals. The last method for the case of equal decision-making frequencies of the lower and higher levels has been described in [6].

In the paper it is assumed that every subsystem takes the resources from the one definite magazine, only. But without any difficulties the presented method can be generalized for the case when a subsystem takes the resources from a few magazines. If e.g. a subsystem takes the resources from two magazines simultaneously, then the higher level should determine for it two values of coordinating variables each one for each magazine, respectively.

With the large dimentions of the vectors $\vec{y}_k$ and $\vec{y}_n^i$ there is connected the necessity of application of a large memory capacity in the calculations. Then, to realize the calculations either a loose computational grid should be used and no further past of measurements should be taken into account, or appropriate recurrent formulae should be elaborated (of the type of the Kalman filter equations) which accumulate the information resulting from the past measurements.

It can be easy seen that in the case of linear system equation, quadratic performance index, when the inequality constraint (6) is not obligatory the optimal control strategies of the lower level can be solved analitically, utilizing the Certainty Equivalence Principle [1] and (to take into account the constraint (19)) the Lagrange multipliers method. But the higher level optimal strategy must be calculated by numerical way, which is caused by the existence of the magazine resource level constraints.

The presented method has been elaborated for the water resource distribution system and then - the controls u_n^i are scalars. But it is easy to see that all considerations remain also valid for the case when the controls are vectors.

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STEROWANIE HIERARCHICZNE ROZDZIAŁEM ZASOBÓW PRZY NIEPEWNOŚCI Z PERIODYCZNIE POWTARZANĄ KOORDYNACJĄ

Streszczenie. W pracy rozpatrywane jest dwupoziomowa struktura hierarchiczna sterowania rozdziałem zasobów w dużym systemie dynamicznym w przypadku niepełnej informacji. Zakłada się, że poszczególne punkty decyzyjne wyższego i niższego poziomu dysponują różną informacją oraz że decyzje na wyższym poziomie są podejmowane z mniejszą częstotliwością niż na poziomie niższym. Przedstawia się metodę wyznaczenia strategii optymalnych dla wszystkich punktów decyzyjnych. W szczególności pokazuje się, że w rozpatrywanym problemie możliwa jest dekompozycja obliczeń i realizacji sterowania.

ИЕРАРХИЧЕСКИЕ УПРАВЛЕНИЕ РАСПРЕДЕЛЕНИЕМ РЕСУРСОВ ПРИ НЕПОЛНОЙ ИНФОРМАЦИИ С ПЕРИОДИЧЕСКОЙ КООРДИНАЦИЕЙ

Резюме. В работе рассматривается двух уровенную иерархическую структуру управления распределением ресурсов в большой динамической системе, в случае неполной информации. Предполагается что отдельные решители имеют различную информацию и решения на высшим уровне принимаются с меншей частотой чем на низшим уровне. Показывается метод определения оптимальных управляющих решений для всех решителей. Особенно, показывается что в рассматриваемой проблеме существует возможность декомпозиции вычисления и реализации управления.