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Application of the Adomian decomposition method for solving the heat equation in the cast-mould heterogeneous domain

R. Grzymkowski, M. Pleszczyński, D. Słota*
Institute of Mathematics, Silesian University of Technology, Kaszubska 23, 44-100 Gliwice, Poland
*Corresponding author. E-mail address: damian.slota@polsl.pl

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Abstract

The paper is focused on a method for solving the heat equation in a cast-mould heterogeneous domain. The discussed method makes use of the Adomian decomposition method. The derived calculations prove the effectiveness of the method for solving such types of problems.

Keywords: Application of information technology to the foundry industry, Heat conduction, Adomian decomposition method

1. Introduction

The Adomian decomposition method was called after its creator: George Adomian [1-4]. The method is useful for solving a variety of problems. A review of the application of the Adomian decomposition method for solving differential and integral equations was discussed in [2, 3]. It was also used for solving the linear and non-linear heat transfer equation in [5-9]; whereas its use for solving the wave equation was tested in [10-12]. In [13, 14] the Adomian decomposition method was utilized for solving the inverse problems of differential equations. The method may also be employed in mathematical models describing different technical problems as discussed in [15-17].

In [18, 19] the authors combined this method with optimization to obtain an approximate solution of the direct and inverse Stefan problem; whereas in [20, 21] it was used for solving the one-phase Stefan problem. The Stefan problem was first approximated by a system of ordinary differential equations, and next the system was solved with the use of the Adomian decomposition method. Such approach made it possible to obtain an approximate solution of the Stefan problem without constructing a functional and seeking its minimum.

The scope of this paper is the presentation of the application of the Adomian decomposition method for solving the heat transfer equation in the cast-mould heterogeneous domain, assuming an ideal contact at the cast-mould contact point.

2. Adomian decomposition method

The following operational equation is given:

$$F(u) = f,$$

where F denotes non-linear operator, f - given element, u - sought element. Operator F may be noted as:

$$F(u) = L(u) + R(u) + N(u),$$

where L denotes invertible linear operator, R - linear operator, which was derived by separating operator L from operator F ,

N denotes non-linear operator. By the two-sided use of inverse operator L^{-1} in the above operational equation we obtain:

$$L^{-1}L(u) = -L^{-1}R(u) - L^{-1}N(u) + L^{-1}(f).$$

The left side of this equation has the following form:

$$L^{-1}L(u) = u - g^*,$$

where g^* is the function related to the initial and boundary conditions (in the case of differential equations). On such grounds we obtain:

$$u = g^* - L^{-1}R(u) - L^{-1}N(u) + L^{-1}(f).$$

By inserting the following notation:

$$g_0 = g^* + L^{-1}(f),$$

the above equation may be written in the following form:

$$u = g_0 - L^{-1}R(u) - L^{-1}N(u). \qquad (1)$$

In the next step, function u is presented as a series:

$$u = \sum_{i=0}^{\infty} g_i,$$

the terms of which must be designated. In practice, it is sufficient to find an approximate solution:

$$u_n = \sum_{i=0}^{n} g_i, \qquad (2)$$

the accuracy of which is increased with a bigger number of the considered terms. Likewise, the nonlinear operator is also expanded in a series:

$$N(u) = \sum_{i=0}^{\infty} A_i, \qquad (3)$$

where A_i denote the polynomials introduced by the author of the method, referred to as: "Adomian polynomials". These polynomials may be derived from the following recurrent equations:

$$A_0 = N(g_0),$$
$$A_n = \frac{1}{n!}\left[\frac{d^n}{d\lambda^n} N\left(\sum_{i=0}^{n} \lambda^i g_i\right)\right]_{\lambda=0}, \qquad n = 1,2,\ldots,$$

where λ is a parameter. Relations (2) and (3) are substituted to equation (1) and, after some transformations, the following recurrent formula is derived:

$$g_0 = g^* + L^{-1}(f),$$
$$g_n = -L^{-1}R(g_{n-1}) - L^{-1}(A_{n-1}), \qquad n = 1,2,\ldots,$$

enabling the designation of the successive terms of the solution.

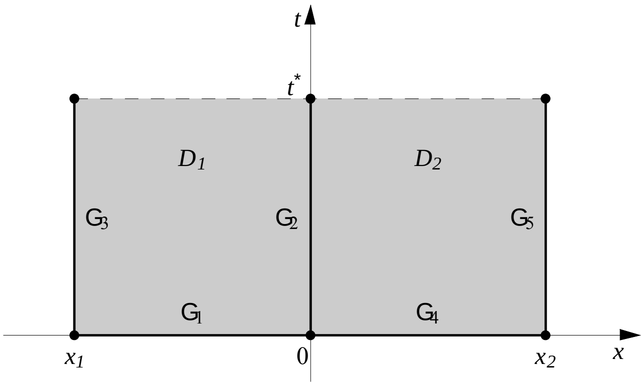

Fig. 1. Domain of the problem

3. Formulation of the problem

Let the domains D_1 and D_2 are given (Fig. 1):

$$D_1 = \{(x,t) : x \in [x_1,0], t \in [0,t^*]\},$$
$$D_2 = \{(x,t) : x \in [0,x_2], t \in [0,t^*]\}, \qquad (4)$$

and their boundaries:

$$\Gamma_1 = \{(x,0) : x \in (x_1,0)\},$$
$$\Gamma_2 = \{(0,t) : t \in [0,t^*]\},$$
$$\Gamma_3 = \{(x_1,t) : t \in [0,t^*]\}, \qquad (5)$$
$$\Gamma_4 = \{(x,0) : x \in (0,x_2)\},$$
$$\Gamma_5 = \{(x_2,t) : t \in [0,t^*]\}.$$

In these domains the sought functions are defined: $u(x,t)$ in D_1 and $v(x,t)$ in D_2. These functions satisfy the heat transfer equation inside the domains:

$$\frac{\partial u}{\partial t} = a_1 \frac{\partial^2 u}{\partial x^2} \quad \text{in} \quad D_1, \qquad (6)$$

$$\frac{\partial v}{\partial t} = a_2 \frac{\partial^2 v}{\partial x^2} \quad \text{in} \quad D_2, \qquad (7)$$

whereas, at the boundaries the above functions satisfy the initial and boundary conditions:

$$u(x,0)=\varphi_1(x) \quad \text{on} \quad \Gamma_1, \tag{8}$$

$$v(x,0)=\varphi_2(x) \quad \text{on} \quad \Gamma_4, \tag{9}$$

$$u(x_1,t)=\psi(t) \quad \text{on} \quad \Gamma_3, \tag{10}$$

$$\frac{\partial v(x_2,t)}{\partial x}=q(t) \quad \text{on} \quad \Gamma_5, \tag{11}$$

$$u(0,t)=v(0,t) \quad \text{on} \quad \Gamma_2, \tag{12}$$

$$\lambda_1\frac{\partial u(0,t)}{\partial x}=\lambda_2\frac{\partial v(0,t)}{\partial x} \quad \text{on} \quad \Gamma_2, \tag{13}$$

where a is the heat diffusion coefficient and λ is the thermal conductivity. We also assume that the initial and boundary conditions meet the consistency conditions.

4. Method of solution

In accordance with the decomposition method, let us replace the discussed equations with operational ones. For this purpose the following operators of partial derivatives are defined:

$$L_t(\cdot)=\frac{\partial(\cdot)}{\partial t} \quad \text{and} \quad L_x(\cdot)=\frac{\partial^2(\cdot)}{\partial x^2}$$

and the corresponding inverse operators:

$$L_t^{-1}(\cdot)=\int_0^t(\cdot)\,dt,$$

$$L_x^{-1}(u)=\int_x^0\int_x^0 u(x,t)\,dx\,dx \quad \text{in } D_1,$$

$$L_x^{-1}(v)=\int_0^x\int_0^x v(x,t)\,dx\,dx \quad \text{in } D_2.$$

As two equivalent operators were derived, equation (6) and (7) shall be considered separately with the main operator L_x alternating with L_t. Thus, the system of equations (6)-(7), after the introduction of additional functions denoted as $\tilde{u}(x,t)$, $\tilde{v}(x,t)$, $\hat{u}(x,t)$ and $\hat{v}(x,t)$ to the solutions with the main operators: L_t and L_x is reduced to the following form:

$$\begin{cases} L_t(\tilde{u})=a_1L_x(\tilde{u}),\\ L_x(\hat{u})=\dfrac{1}{a_1}L_t(\hat{u}),\\ L_t(\tilde{v})=a_2L_x(\tilde{v}),\\ L_x(\hat{v})=\dfrac{1}{a_2}L_t(\hat{v}).\end{cases} \tag{14}$$

In both sides of the above equations we use the corresponding inverse operators:

$$\begin{cases} L_t^{-1}L_t(\tilde{u})=a_1L_t^{-1}L_x(\tilde{u}),\\ L_x^{-1}L_x(\hat{u})=\dfrac{1}{a_1}L_x^{-1}L_t(\hat{u}),\\ L_t^{-1}L_t(\tilde{v})=a_2L_t^{-1}L_x(\tilde{v}),\\ L_x^{-1}L_x(\hat{v})=\dfrac{1}{a_2}L_x^{-1}L_t(\hat{v}).\end{cases} \tag{15}$$

Consequently, the left sides of the equations assume the following form:

$$L_t^{-1}L_t(\tilde{u})=\int_0^t\frac{\partial\tilde{u}(x,t)}{\partial t}dt=\tilde{u}(x,t)-\tilde{u}(x,0)=\tilde{u}(x,t)-c_1k_1(x),$$

$$L_x^{-1}L_x(\hat{u})=\int_0^x\int_0^x\frac{\partial^2\hat{u}(x,t)}{\partial x^2}\,dx\,dx=\int_0^x\left(\frac{\partial\hat{u}(x,t)}{\partial x}-\frac{\partial\hat{u}(0,t)}{\partial x}\right)dx=$$
$$=\int_0^x\left(\frac{\partial\hat{u}(x,t)}{\partial x}-c_2k_2(t)\right)dx=\hat{u}(x,t)-xc_2k_2(t)-\hat{u}(0,t)=$$
$$=\hat{u}(x,t)-xc_2k_2(t)-c_3k_3(t)$$

and, accordingly:

$$L_t^{-1}L_t(\tilde{v})=\int_0^t\frac{\partial\tilde{v}(x,t)}{\partial t}dt=\tilde{v}(x,t)-\tilde{v}(x,0)=\tilde{v}(x,t)-c_4k_4(x),$$

$$L_x^{-1}L_x(\hat{v})=\int_0^x\int_0^x\frac{\partial^2\hat{v}(x,t)}{\partial x^2}\,dx\,dx=\hat{v}(x,t)-xc_5k_5(t)-c_6k_6(t),$$

where: $c_i\,k_i$ for $i=1,2,\ldots,6$ are the unknown functions obtained as a result of the integration. After considering the obtained results, the system of equations (15) assumes the following form:

$$\begin{cases} \tilde{u}(x,t)=c_1k_1(x)+a_1L_t^{-1}L_x(\tilde{u}),\\ \hat{u}(x,t)=xc_2k_2(t)+c_3k_3(t)+\dfrac{1}{a_1}L_x^{-1}L_t(\hat{u}),\\ \tilde{v}(x,t)=c_4k_4(x)+a_2L_t^{-1}L_x(\tilde{v}),\\ \hat{v}(x,t)=xc_5k_5(t)+c_6k_6(t)+\dfrac{1}{a_2}L_x^{-1}L_t(\hat{v}),\end{cases} \tag{16}$$

where c_i, $i=1,2,\ldots,6$ are any constants, whereas k_i, $i=1,2,\ldots,6$, are any functions of variable t or x, respectively. The right sides of the system of equations (16) generate the following form of the terms $\tilde{g}_0^u$, $\hat{g}_0^u$, $\tilde{g}_0^v$, $\hat{g}_0^v$:

$$\begin{aligned} \tilde{g}_0^u&=c_1k_1(x),\\ \hat{g}_0^u&=xc_2k_2(t)+c_3k_3(t),\\ \tilde{g}_0^v&=c_4k_4(x),\\ \hat{g}_0^v&=xc_5k_5(t)+c_6k_6(t).\end{aligned} \tag{17}$$

The unknown products $c_i k_i$, $i = 1,2,\ldots,6$ shall be determined from conditions (8)-(13), as they must be satisfied also by a single term approximation of the solution. The successive terms g_n of each equation from the system (16) shall be derived from condition $g_n = cL^{-1}L(g_{n-1})$, where c is the corresponding constant for a given equation. Hence, we obtain:

$$\begin{aligned}
\tilde{g}_1^u &= a_1 \int_0^t \frac{\partial^2 \tilde{g}_0^u}{\partial x^2}\, dt, \\
\hat{g}_1^u &= \frac{1}{a_1} \int_x^0 \int_x^0 \frac{\partial \hat{g}_0^u}{\partial t}\, dx dx, \\
\tilde{g}_1^v &= a_2 \int_0^t \frac{\partial^2 \tilde{g}_0^v}{\partial x^2}\, dt, \\
\hat{g}_1^v &= \frac{1}{a_2} \int_0^x \int_0^x \frac{\partial \hat{g}_0^v}{\partial t}\, dx dx.
\end{aligned} \tag{18}$$

It should be remembered that in the previously calculated functions $c_i k_i$ constant c_i is correspondent only for g_0, so, in system (18) constants c_i shall appear as unknowns, which, as in the case of a single-term solution, may be determined in consideration of conditions (8)-(13) for a two-term solution. Following such procedure, we obtain any n-term solution for $\tilde{u}(x,t)$, $\hat{u}(x,t)$, $\tilde{v}(x,t)$, $\hat{v}(x,t)$. The final solution is obtained by averaging the results:

$$\begin{cases}
u(x,t) = \frac{1}{2}(\tilde{u} + \hat{u}), \\
v(x,t) = \frac{1}{2}(\tilde{v} + \hat{v}).
\end{cases}$$

4. Example

Let us examine the example where: $x_1 = -1$, $x_2 = 1$, $a_1 = \frac{1}{4}$, $a_2 = 1$, $\lambda_1 = 1$, $\lambda_2 = 2$, $\varphi_1(x) = e^{2x}$, $\varphi_2(x) = e^x$, $\psi(t) = e^{t-2}$ and $q(t) = e^{t+1}$. Following the discussed method we obtain (considering the initial and boundary conditions) a system of equations containing unknowns $c_i k_i$:

$$\begin{cases}
-c_2 k_2(t) + c_3 k_3(t) = e^{t-2}, \\
c_5 k_5(t) = e^{t+1}, \\
c_1 k_1(x) = e^{2x}, \\
c_4 k_4(x) = e^x, \\
c_3 k_3(t) = c_6 k_6(t), \\
c_2 k_2(t) = 2c_5 k_5(t),
\end{cases} \tag{19}$$

from which we derive:

$$\begin{cases}
c_1 k_1(x) = e^{2x}, \\
c_2 k_2(t) = 2e^{t+1}, \\
c_3 k_3(t) = 2e^{t+1} + e^{t-2}, \\
c_4 k_4(x) = e^x, \\
c_5 k_5(t) = e^{t+1}, \\
c_6 k_6(t) = 2e^{t+1} + e^{t-2}.
\end{cases} \tag{20}$$

Substituting the derived relations to system (17) we obtain:

$$\begin{cases}
\tilde{g}_0^u = \tilde{u}_0(x,t) = e^{2x}, \\
\hat{g}_0^u = \hat{u}_0(x,t) = 2(x+1)e^{t+1} + e^{t-2}, \\
\tilde{g}_0^v = \tilde{v}_0(x,t) = e^x, \\
\hat{g}_0^u = \hat{v}_0(x,t) = (x+2)e^{t+1} + e^{t-2}.
\end{cases} \tag{21}$$

Thus, by (18) we designate:

$$\begin{aligned}
\tilde{g}_1^u &= \frac{1}{4}\int_0^t \frac{\partial^2 \tilde{g}_0^u}{\partial x^2}\, dt = -c_1 t e^{2x}, \\
\tilde{g}_2^u &= \frac{1}{4}\int_0^t \frac{\partial^2 \tilde{g}_1^u}{\partial x^2}\, dt = c_1 \frac{t^2}{2} e^{2x}, \\
\tilde{g}_3^u &= \frac{1}{4}\int_0^t \frac{\partial^2 \tilde{g}_2^u}{\partial x^2}\, dt = c_1 \frac{t^3}{6} e^{2x}, \\
&\vdots \\
\tilde{g}_n^u &= \frac{1}{4}\int_0^t \frac{\partial^2 \tilde{g}_{n-1}^u}{\partial x^2}\, dt = c_1 \frac{t^n}{n!} e^{2x}.
\end{aligned}$$

Summing $\tilde{g}_i^u$ we arrive at $\tilde{u}(x,t)$:

$$\tilde{u} = \sum_{i=0}^{\infty} \tilde{g}_i^u = c_1 e^{2x} \sum_{i=0}^{\infty} \frac{t^i}{i!} = c_1 e^{2x+t}. \tag{22}$$

Likewise, we obtain $\hat{u}(x,t)$:

$$\begin{aligned}
\hat{g}_1^u &= 4\int_0^x\int_0^x \frac{\partial \hat{g}_0^u}{\partial t}\, dx\, dx = 2c_2 4 \frac{x^3}{6} e^{t+1} + c_3 4(2e^{t+1} + e^{t-2})\frac{x^2}{2}, \\
\hat{g}_2^u &= 4\int_0^x\int_0^x \frac{\partial \hat{g}_1^u}{\partial t}\, dx\, dx = 2c_2 4^2 \frac{x^5}{5!} e^{t+1} + c_3 4^2(2e^{t+1} + e^{t-2})\frac{x^4}{4!}, \\
\hat{g}_3^u &= 4\int_0^x\int_0^x \frac{\partial \hat{g}_2^u}{\partial t}\, dx\, dx = 2c_2 4^3 \frac{x^7}{7!} e^{t+1} + c_3 4^3(2e^{t+1} + e^{t-2})\frac{x^6}{6!}, \\
&\vdots \\
\hat{g}_n^u &= 4\int_0^x\int_0^x \frac{\partial \hat{g}_{n-1}^u}{\partial t}\, dx\, dx = \\
&= 2c_2 4^n \frac{x^{2n+1}}{(2n+1)!} e^{t+1} + c_3 4^n (2e^{t+1} + e^{t-2})\frac{x^{2n}}{(2n)!},
\end{aligned}$$

thus:

$$\hat{u}(x,t)=\sum_{i=0}^{\infty}\hat{g}_i^u=2c_2e^{t+1}\sum_{i=0}^{\infty}\frac{4^i x^{2i+1}}{(2i+1)!}+c_3(2e^{t+1}+e^{t-2})\sum_{i=0}^{\infty}\frac{4^i x^{2i}}{(2i)!},$$

and, on the grounds of the sums of the known power series:

$$\hat{u}(x,t)=2c_2e^{t+1}\sinh(2x)+c_3(2e^{t+1}+e^{t-2})\cosh(2x). \tag{23}$$

Finally, we get:

$$u(x,t)=\frac{1}{2}\Big(c_1e^{2x+t}+2c_2e^{t+1}\sinh(2x)+ +c_3(2e^{t+1}+e^{t-2})\cosh(2x)\Big). \tag{24}$$

Similarly, $\tilde{g}_i^v$ and $\hat{g}_i^v$ are obtained:

$$\tilde{g}_1^v=\int_0^t\frac{\partial^2\tilde{g}_0^v}{\partial x^2}dt=c_4te^x,$$

$$\tilde{g}_2^v=\int_0^t\frac{\partial^2\tilde{g}_1^v}{\partial x^2}dt=c_4\frac{t^2}{2}e^x,$$

$$\tilde{g}_3^v=\int_0^t\frac{\partial^2\tilde{g}_2^v}{\partial x^2}dt=c_4\frac{t^3}{6}e^x,$$

$$\vdots$$

$$\tilde{g}_n^v=\int_0^t\frac{\partial^2\tilde{g}_{n-1}^v}{\partial x^2}dt=c_4\frac{t^n}{n!}e^x$$

which means that:

$$\tilde{v}(x,t)=\sum_{i=0}^{\infty}\tilde{g}_i^v=c_4e^x\sum_{i=0}^{\infty}\frac{t^i}{i!}=c_4e^{x+t} \tag{25}$$

and

$$\hat{g}_1^v=\int_0^x\int_0^x\frac{\partial\hat{g}_0^v}{\partial t}dx\,dx=c_5\frac{x^3}{6}e^{t+1}+c_6(2e^{t+1}+e^{t-2})\frac{x^2}{2},$$

$$\hat{g}_2^v=\int_0^x\int_0^x\frac{\partial\hat{g}_1^v}{\partial t}dx\,dx=c_5\frac{x^5}{5!}e^{t+1}+c_6(2e^{t+1}+e^{t-2})\frac{x^4}{4!},$$

$$\hat{g}_3^v=\int_0^x\int_0^x\frac{\partial\hat{g}_2^v}{\partial t}dx\,dx=c_5\frac{x^7}{7!}e^{t+1}+c_64^3(2e^{t+1}+e^{t-2})\frac{x^6}{6!},$$

$$\vdots$$

$$\hat{g}_n^v=\int_0^x\int_0^x\frac{\partial\hat{g}_{n-1}^v}{\partial t}dx\,dx=c_5\frac{x^{2n+1}}{(2n+1)!}e^{t+1}+c_6(2e^{t+1}+e^{t-2})\frac{x^{2n}}{(2n)!}.$$

As in the case of $\hat{u}(x,t)$ we have:

$$\hat{v}(x,t)=\sum_{i=0}^{\infty}\hat{g}_i^v=c_5e^{t+1}\sum_{i=0}^{\infty}\frac{x^{2i+1}}{(2i+1)!}+c_6(2e^{t+1}+e^{t-2})\sum_{i=0}^{\infty}\frac{x^{2i}}{(2i)!}$$

and, once again, on the grounds of the known sums of power series:

$$\hat{v}(x,t)=c_5e^{t+1}\sinh(x)+c_6(2e^{t+1}+e^{t-2})\cosh(x). \tag{26}$$

Analogically to $\hat{u}(x,t)$ we derive:

$$v(x,t)=\frac{1}{2}\Big(c_4e^{x+t}+c_5e^{t+1}\sinh(x)+ +c_6(2e^{t+1}+e^{t-2})\cosh(x)\Big). \tag{27}$$

To find unknown constants c_i, $i=1,2,\ldots,6$ we use conditions (8)-(13). Accordingly, we obtain the system of equations, which, after the solution, renders the values of the sought constants c_i, $i=1,2,\ldots,6$:

$$c_1=c_4=2,\quad c_2=c_3=c_5=c_6=0.$$

Thus, we obtain the final solution of the initial differential equations system:

$$\begin{cases}u(x,t)=e^{2x+t},\\ v(x,t)=e^{x+t}.\end{cases}$$

It is easy to check that the solution satisfies all initial and boundary conditions, as well as the differential equations as such. So, our solution is not only approximate but, more importantly, exact.

5. Conclusion

The discussed method of an approximated solution of the heat transfer equation in the cast-mould heterogeneous domain is based on the Adomian decomposition method. The method provides a continuous function describing the sought temperature distribution $u(x,t)$ and $v(x,t)$. A computational example proves the usability of the method. The solution of the problem is provided with the assumption of an ideal contact between the cast and the mould. In further research the discussed method shall be employed to solve problems involving the presence of thermal resistance at the cast-mould contact.

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