Vessel Detection Method Based on Eigenvalues of the Hessian Matrix and its Applicability to Airway Tree Segmentation

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Abstract

This paper presents a 3D image processing method that is based on the analysis of Hessian matrix eigenvalues combined with a multiscale image analysis approach. The method, originally developed for blood vessels detection in medical images, can also be used in other areas, where finding line-like structures in the image is required. Theoretical background, advantages and disadvantages of the method are described. Possible modifications required to allow the method to detect structures of different character (airway tree) are mentioned. An implementation of the method was tested on synthetic images containing airway-like structures as well as on real medical images from chest CT scan. Results show that the method in general can be used to airway detection in 3D medical images, however it requires improvements and some adaptation to this specific purpose.

1. Introduction

Airway tree segmentation is an important step of medical image analysis. It helps the radiologist to assess the state of patient's airways, find anomalies (like stenosis, nodule, foreign body) and perform surgery planning, when using the minimally invasive surgery approach. In the case of 2D images (e.g. chest radiogram) manual image analysis is usually sufficient in the terms of performing the diagnosis and coarse surgery planning. In the case of 3D volumetric data like CT, MR (Computed Tomography, Magnetic Resonance), manual approach is tedious and time consuming because many (from 50 to 200) images have to be analyzed.

For the medical analysis of patient's blood vessels the problem is quite similar. There were developed methods for automatic detection or enhancement of vessel-like structures in medical images, mainly addressed to blood vessel detection. Presented in this paper vessel detection method is based on the analysis of eigenvalues of the image Hessian matrix [2, 3, 4, 6] combined with a multiscale image analysis approach [1]. The advantage of this approach is that it can operate in 2D and/or 3D. Furthermore, basing on the eigenvalues not only vessel-like, but also sheet-like or blob-like structures can be detected [3, 6]. Combined with the multiscale image analysis approach [1] gives a versatile tool for blood vessel enhancement and detection. However, because the method was developed mainly for blood vessels detection, it cannot be directly used for airway tree detection/enhancement, which in general has the same tubular structure like blood vessels, but different intensity cross section profile. The paper considers the possibility to adapt the method to airway tree detection

The paper is organized as follows: Section 2 presents theoretical background of the method: properties of the eigenvalues of the Hessian matrix and the multiscale image analysis approach. Section 3 points out advantages of the method and drawbacks that need to be overcome if the general method is to be adopted to enhancement and/or segmentation of the airway tree. In Section 4 testing and results of an implementation of the method are presented. Section 5 indicates some ideas that should allow the method to be used for the airway tree segmentation. The paper ends with a short summary in Section 6.

Throughout the paper following notions are used: I – grayscale input image (volumetric data represented as a 3D array), x, y, z – coordinates of a voxel within I, H – Hessian matrix, λ_i – eigenvalues of the Hessian matrix, G_σ – gaussian kernel with standard deviation σ.

2. Theory of operation

2.1 Analysis of the eigenvalues of the Hessian matrix

For a given voxel of the input image a Hessian matrix is composed from the image 2nd order partial derivatives (1).

$$H = \begin{bmatrix} \frac{\partial^2 I}{\partial x^2} & \frac{\partial^2 I}{\partial x \partial y} & \frac{\partial^2 I}{\partial x \partial z} \\ \frac{\partial^2 I}{\partial y \partial x} & \frac{\partial^2 I}{\partial y^2} & \frac{\partial^2 I}{\partial y \partial z} \\ \frac{\partial^2 I}{\partial z \partial x} & \frac{\partial^2 I}{\partial z \partial y} & \frac{\partial^2 I}{\partial z^2} \end{bmatrix} \quad (1)$$

The partial derivatives are calculated as voxel intensity differences in the neighborhood of the voxel. The Hessian matrix describes the 2nd order local image intensity variations around the selected voxel [2]. For the obtained Hessian matrix its eigenvalues λ_i and eigenvectors are calculated. Eigenvector decomposition extracts an orthonormal coordinate system that is aligned with the second order structure of the image [3].

Having the eigenvalues and knowing the (assumed) model of the structure to be detected and the resulting theoretical behavior of the eigenvalues, the decision can be made if the analyzed voxel belongs to the structure being searched.

In the literature several models were analyzed to find the relation between the eigenvalues and "vesselness" of a voxel (meaning the likelihood that the voxel belongs to a blood vessel) [4]. What all proposed models have in common is that the intensity within the vessel exhibits a gaussian distribution. For example a simple cylindrical model (2) is shown in Fig.1 center and Fig.1 top left.

$$I(x,y,z) = G_\sigma(x,y,z) = const \cdot e^{-\frac{x^2+y^2}{2\sigma^2}} \quad (2)$$

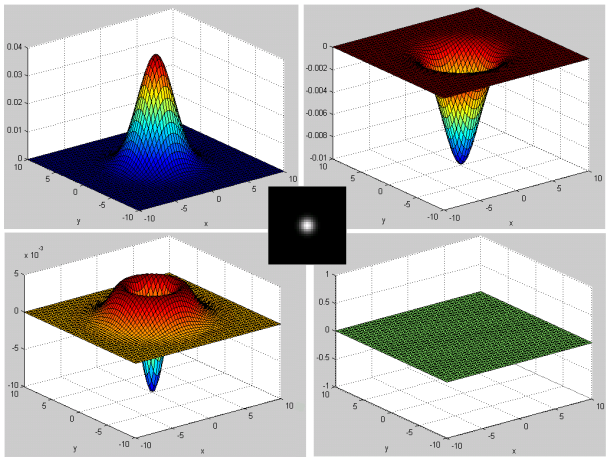

Fig.1. Cylindrical vessel model (center – vessel cross-section, top left – intensity distribution, top right, bottom left, bottom right – analytical eigenvalues).

For the proposed vessel model the analytical expressions of λ_i are calculated and analyzed how they behave in the center of the vessel model (Fig.1 top right, bottom). Krissian et al. [4] shown that the eigenvector corresponding to the eigenvalue of the smallest magnitude determines the direction along the vessel (direction of smallest intensity variations). However, at the vessel contours the method fails because two of the eigenvalues become zero [5]. Analytical expressions of the eigenvalues and eigenvectors for several vessel models can be found in [4]. Other presented models, despite their increasing complexity, also share the same intensity profile across the vessel. This is because of the appearance of blood vessels in medical images (CT, MR) as filled curvilinear cylinders.

Following [3] the eigenvalues are sorted so that $|\lambda_1| \leq |\lambda_2| \leq |\lambda_3|$. Tab.1 summarizes the relations between λ_i and orientation of a structure in the image.

Tab.1.

Eigenvalues of the Hessian matrix and image structure orientation (L low, H+ high positive, H- high negative)

λ_1	λ_2	λ_3	structure orientation
L	L	L	noise (no preferred structure)
L	L	H–	bright sheet-like structure
L	L	H+	dark sheet-like structure
L	H–	H–	bright tubular structure
L	H+	H+	dark tubular structure
H–	H–	H–	bright blob-like structure
H+	H+	H+	dark blob-like structure

Several formulas were proposed to calculate the "vesselness" of a voxel basing on the values of λ_i [2, 3, 4, 6]. For the aforementioned example one of the eigenvalues is always zero and two other exhibit a large negative value in the center of the vessel. This indicates the character of the structure within the image (see Tab.1) and is used as a criterion in the function that calculates voxel's "vesselness". Additionally the "vesselness" function should incorporate a term minimizing the influence of image noise [3]. The output image is created voxel-wise using the calculated "vesselness" values.

The method is usually combined with the multiscale image analysis theory [1] that allows using the same method for finding small and large objects provided that the object is similar in the terms of its model, but its size (length, diameter) varies.

2.2 Multiscale image analysis

The idea of multiscale image analysis is to add a new dimension to the analysis – image scale. The image is transformed into a set of derived images, each representing the original, but at a different scale. With increasing the scale the image gets less detailed. The obtained set is called the scale-space representation of the image.

The scale-space theory introduced by Lindeberg uses for the purpose of detail removal a convolution with a gaussian kernel [1]. For an n-D image I its scale-space representation $L(t)$ at scale t is the image I convolved with an n-D gaussian kernel G_σ where $t=\sigma^2$:

$$L(t) = I(x_1,...,x_n) * G_{\sigma=\sqrt{t}}(x_1,...,x_n) \quad (3)$$

Spatial derivatives of the scale-space image representation $L(t)$ can be calculated as a convolution of the image with the derivative of the gaussian kernel at scale t (4):

$$\frac{\partial}{\partial x_i} L(t) = \frac{\partial}{\partial x_i} G_{\sigma=\sqrt{t}}(x_1,...,x_n) * I(x_1,...,x_n) \quad (4)$$

In order to be able to compare the derivatives across multiple scales one can normalize the free variables: $\hat{x}_i = \frac{x_i}{\sigma}$. Then the derivatives of the gaussian become normalized by its standard deviation σ: $\frac{\partial}{\partial \hat{x}_i} G_\sigma = \frac{\partial}{\partial x_i} G_\sigma \frac{\partial x_i}{\partial \hat{x}_i} = \sigma \frac{\partial}{\partial x_i} G_\sigma$, leading to:

$$\frac{\partial}{\partial x_i} L(t) = \sigma \frac{\partial}{\partial x} G_{\sigma=\sqrt{t}}(x_1,...,x_n) * I(x_1,...,x_n) \quad (5)$$

what allows the responses across scales to compared. Similarly, the 2nd order normalized derivative of *L(t)* is calculated using:

$$\frac{\partial^2}{\partial x_i \partial x_j} L(t) = \sigma^2 \frac{\partial^2}{\partial x_i \partial x_j} G_{\sigma=\sqrt{t}}(...) * I(...) \quad (6)$$

Scale-space representation simplifies the contents of the image depending on the chosen scale. This allows to search for objects of similar dimensions as the chosen scale, or to analyze the image across wide range of scales to see if any object of unknown size but known model can be found.

2.3 Multiscale vessel detection

The multiscale vessel detection is performed for scales between t_{min} and t_{max} (corresponding to σ_{min} and σ_{max}). For each $\sigma \in \langle \sigma_{\min}; \sigma_{\max} \rangle$ the Hessian matrix entries are calculated using (6). Then the eigenvalue analysis is performed as described in Section 2.1 and the result for a given scale is obtained. The final result of the multiscale analysis is the voxel-wise maximum of obtained results over all analyzed scales.

3. Properties

In general, the method based on the Hessian eigenvalues analysis is capable of detecting not only tubular structures, but also blob-like and sheet-like structures within the image [3, 6]. This only requires finding proper formulas for "blobness" and "sheetness" as functions of λ_i. Also dark vessel detection is not a problem [3] – as it only requires change of conditions imposed on values of λ_i during calculation of voxel's "vesselness". However, the intensity distribution within the vessel still has to be more-or-less of gaussian shape to ensure maximal values of the two eigenvalues at the center of the vessel.

The main advantage of the method is that there is no discretization of the vessel orientation – by eigenvector decomposition the principal directions of the 2nd order image structure are found. This approach is less computationally expensive than performing multiple filtering in multiple discrete orientations [3]. Due to the fact that the analysis of the theoretical eigenvalues is performed at the center of the vessel, the method by itself extracts the centerlines of the vessels, what can be considered as another advantage. Also because the output is calculated for each voxel separately the method may successfully detect vessels with high degree of stenosis (or even disconnected segments), whereas methods based on voxel connectivity fail to segment the part of the vessel after the obstruction and require special detection of such cases.

However, the lack of connectivity and neighborhood analysis is also a drawback when the vessels exhibit other than gaussian intensity profile (although the multiscale approach partially takes care of this issue – see Fig.2 and Fig.3) and the behavior of the eigenvalues farther from the vessel center should also be considered. Additionally, the formulas used to calculate the "vesselness" provide good response to tubular objects and good noise and other structure (blobs, sheets) suppression, however at vessel bifurcations the method's response is weak. This is due to the fact that the vessel bifurcation does not exhibit a tubular structure, but rather is blob-like. Another issue is connected with small vessels (few voxels in diameter), where due to finite image resolution and partial volume effect, the vessel center is generally not in the center of a voxel. In such case the eigenvalues are not calculated at the vessel center, thus the response is weak [4].

For those reasons the method is usually used as a preprocessing step or as a support in a more complex vessel segmentation methods [4, 5].

4. Testing and results

Because the airways exhibit a tubular structure the method was tested to assess its applicability to airway detection from CT data.

Airways' cross-section profiles are similar (but not sufficiently) to the "dark vessel" cylindrical model. Fig.2 shows three exemplary cross-section intensity profiles from real CT dataset: top – trachea, middle – main bronchus, bottom – bronchiole. As can be seen the trachea (Fig.2 top) is similar to a dark vessel, because tissues of higher density surround it. Although, the profile has a flat bottom comparing to the cylindrical vessel model, the multiscale approach allows that model to be used (Fig.3). The farther from the trachea the airway is, the more its shape becomes a pipe – a hollow tube with thin walls surrounded by tissues of almost the same density as the inside of the tube (bronchiole, Fig.2 bottom). However the intensity profile is then more-less of gaussian shape.

Fig.3 shows the first two cross-sections from Fig.2, but convolved with gaussian kernel with σ similar to the radius of the airway. This illustrates what happens during the multiscale image analysis. It

can be seen that although the trachea originally does not have a gaussian profile (and the all eigenvalues at the center would be close to zero), at a larger scale the profile becomes of gaussian shape, thus the Hessian eigenvalue analysis can give correct results.

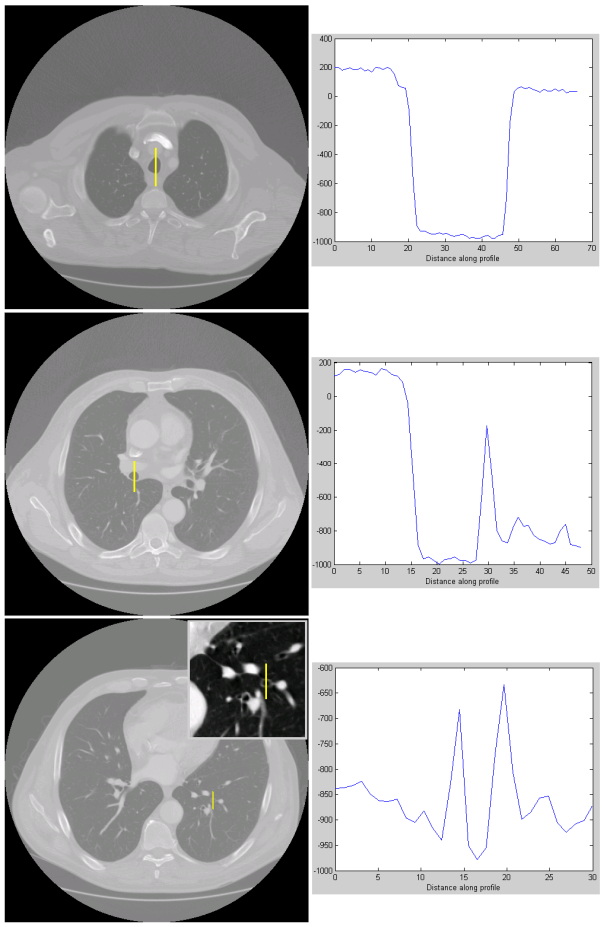

Fig.2. Airway cross-section profiles: top – trachea, middle – main bronchus, bottom – bronchiole. Vertical scale in the profiles is in Hounsfield Units

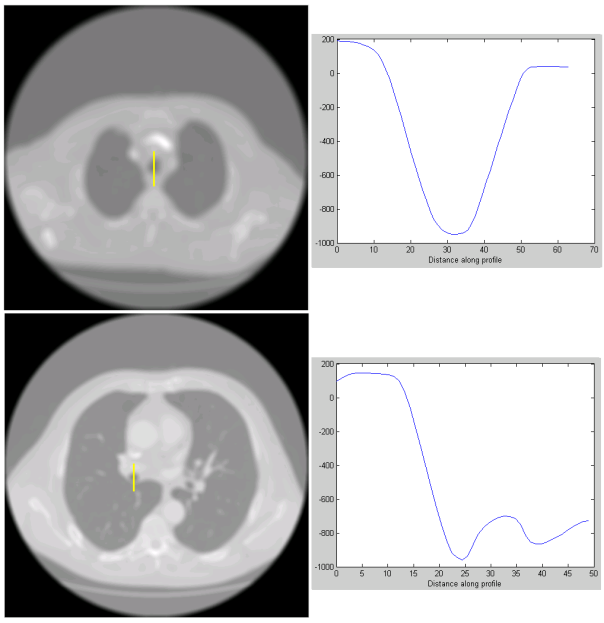

Fig.3. Airway cross-section profiles after convolution with a gaussian kernel: top – trachea, σ=10; bottom – main bronchus, σ=5. Vertical scale in the profiles is in Hounsfield Units.

An implementation of the Frangi's "vesselness filter" [3] was tested to assess its usefulness to detection of the airways in 3D medical CT images. The testing was performed on synthetic 3D image data (cross section shown in Fig.4) containing pipes with bright walls and dark vessels with gaussian intensity distribution and radii from 1 to 8 pixels. The performance of the method in the presence of noise was also tested (Fig.5).

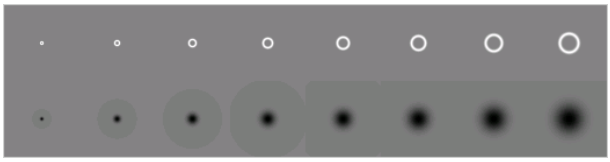

Fig.4. Synthetic image containing airways and dark vessels cross sections (radii from 1 to 8 pixels).

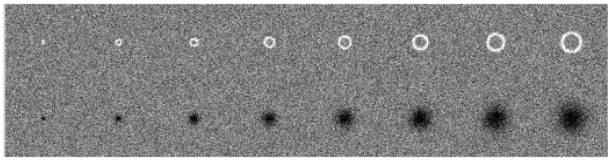

Fig.5. Synthetic image containing airways and dark vessels cross sections with superimposed gaussian noise.

The multiscale image analysis was performed for σ from 1 to 8 that approximately correspond to the diameters of the objects in the input images.

The results can be seen in Fig.6 and Fig.7 respectively. From the results one can notice that although the filter response is much higher to the dark vessels, the method is still able to detect the airways correctly.

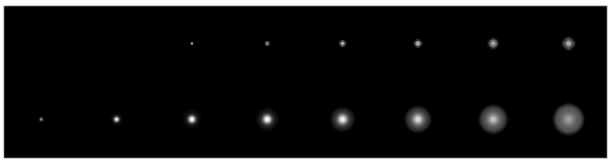

Fig.6. Testing results of the Frangi's method on noiseless image.

Fig.7. Testing results of the Frangi's method on noisy image.

Finally, the implemented method was used to detect airways from real medical images. The original CT data was preprocessed in order to extract only the lungs volume. Obtained lung volume was subject to further analysis.

Fig.8 presents volume rendered result (without any postprocessing) of the airway detection from two exemplary CT datasets. As can be seen the trachea and two main bronchi were found. The bifurcations following the main bronchi are missing and higher order bronchioles are not detected. Additionally some other anatomical structures are present in the results.

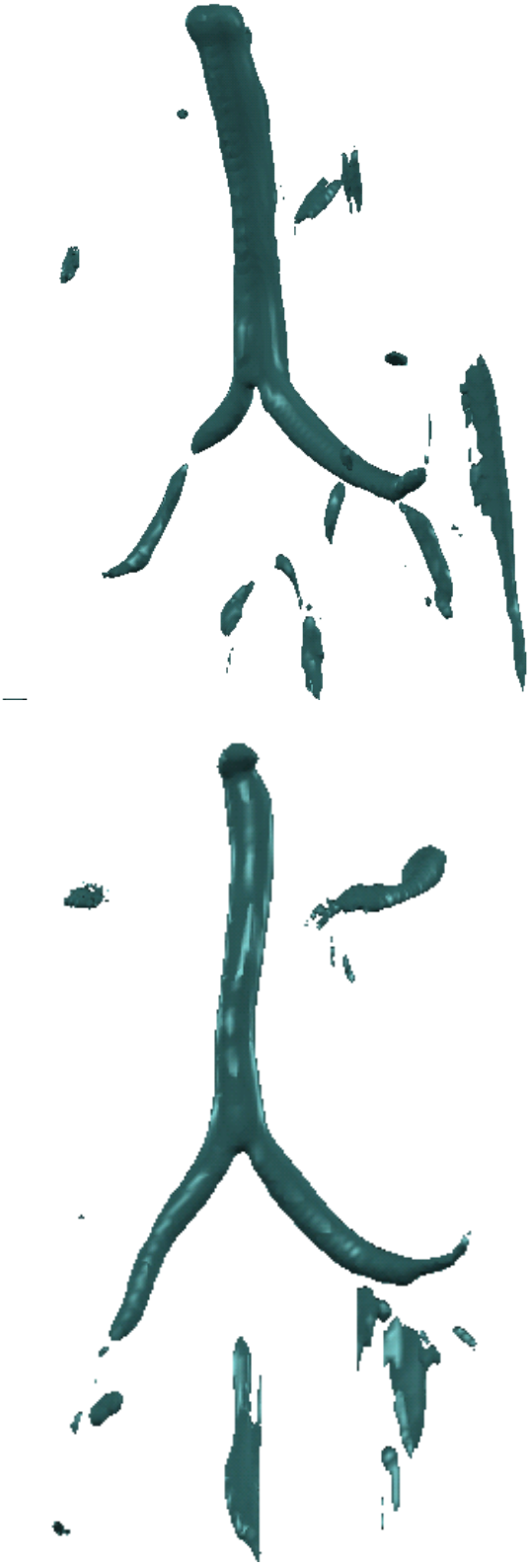

Fig.8. Detected airway trees (no post-processing applied).

For a comparison, Fig.9 shows the result of blood vessels detection inside the lung volume using the original Frangi's "vesselness filter".

Fig.9. Detected blood vessels inside lung volume (no post-processing applied).

5. Possible improvements – future work

First issue to be considered is the poor performance of the method in detecting small airways. This could be seen in the results of synthetic image and real image processing.

An airway cross section can be modeled by a Laplacean of a Gaussian (7) function (simpler to analyze, but not exactly fitting into the real airway cross section) or a "gaussian ring" with a relatively large radius *R* comparing to σ, for example by (8):

$$I(x,y,z) = const \cdot \left(\frac{\partial^2}{\partial x^2} e^{-\frac{x^2+y^2}{2\sigma^2}} + \frac{\partial^2}{\partial y^2} e^{-\frac{x^2+y^2}{2\sigma^2}} \right) \quad (7)$$

$$I(x,y,z) = const \cdot e^{-\frac{\left(R-\sqrt{x^2+y^2}\right)^2}{2\sigma^2}} \quad (8)$$

For those models the eigenvalue analysis has to be done not only at the center of the vessel (especially the in the second case where due to flat profile all λ_i are close to zero), but also in the closest neighborhood, where the "ring" is present. Thus the conditions for "airwayness" should include similar conditions as for "vesselness" and additionally those imposed by the presence of the "ring" around the center of the vessel. In this way the drawback concerning the lack of neighborhood analysis could be fixed.

Another possibility is to combine the vessel detection with wall detection. Airway walls are bright and of planar nature. By detecting planar objects around the vessels, the airways could be detected. However, there is risk of too high false positive detections as in real CT data there exist many

anatomical structures whose boundaries could be detected.

Mentioned above ideas are to be tested during further research on the presented methodology.

6. Summary

The general idea behind the method based on the Hessian eigenvalues analysis seems promising, but according to several authors [4, 5] by itself may be insufficient as a standalone vessel segmentation tool. However, obtained results show that the method is able to perform the airway tree detection, but it requires further development and parameter tuning to be fully adapted to this specific purpose. Thus the research is still being performed on the adaptation of the method to airway tree segmentation from real life medical images.

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