Utility Tests Of The New Reversibility Criterion For Elementary Bilinear Time-Series Model

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Abstract

The problem of the reversibility of the elementary bilinear time-series model is a major issue of its parametric identification due to ambiguity of the parameters estimates. The well known reversibility condition requires knowledge of model parameter which cannot be known before identification procedure so it cannot be relied upon that model has been identified properly. The purpose of this paper is to perform utility tests of the reversibility criterion proposed in [1]which is based on a model output sequence alone. The a'priori knowledge of the potential irreversibility of the elementary bilinear time-series model can be very useful in selection of the approach to a parametric identification.

1. Introduction

The paper presents results of utility tests performed on *a'priori* criterion of reversibility for elementary bilinear time-series model (EB) proposed in [1]. The concerned time series model is the most simple sub-model of general bilinear time series models and was introduced by Granger and Andersen [2], further analysed by Tong [3], Granger and Teräsvirta [4], Martins [5], and Berlin Wu [6]. and more recently by Bielińska [7] and Hili [8]. There is enough evidence that can be found in work of Brunner, Hess [9], Bielińska and Maliński [10,11] that identification of EB model is a difficult task due to several reasons. One of them is non-stationarity of the random process obtained from EB time-series model, which was shown in work of Maliński and Figwer [12].

This paper is focused on reversibility of the EB time-series model, because irreversible, but still stable and useful models, cannot be simply identified by minimisation of mean square value of prediction error without certain modifications in an algorithm. These modifications should not be applied to reversible models due to high probability of damaging estimation results. This causes the demand to estimate reversibility of the model before model identification. Actual knowledge about EB time-series models defines both stability and reversibility conditions by means of its coefficient value and a white noise variance, which are not known before an identification. In the sequel a definition and properties of EB model are presented along with the proposed output based reversibility criterion [1].

2. Problem formulation

The elementary bilinear time-series model definition is presented below:

$$y(i) = e(i) + \beta_{kl} e(i-k) y(i-l) \tag{1}$$

where *e(i)* is a white noise sequence with the zero mean value and the limited variance $m_e^{(2)}$, β_{kl} is the coefficient of EB time-series model.

The structure of the model can be distinguish by relation between *k* and *l* parameters into three possible types:

- Superdiagonal for *k* < *l*
- Diagonal for *k* = *l*
- Subdiagonal for *k* > *l*

The EB time-series model is stable if the following condition is satisfied:

$$\beta_{kl}^2 m_e^{(2)} < 1 \tag{2}$$

and the general reversibility condition for the EB time-series model is presented below:

$$\beta_{kl}^2 m_y^{(2)} < 1, \tag{3}$$

where $m_y^{(2)}$ is the variance of the model output sequence (time-series) *y(i)*.

For various types of model structures the output variance is a function of its coefficient and statistical properties of white noise *e(i)*. In this paper the normal distribution of the white noise sequence were assumed, which implies the following relations:

$$E\{e(i)\}=0; \quad E\{e(i)^2\}=\lambda^2; \quad E\{e(i)e(i-1)\}=0; \quad E\{e(i)^3\}=0; \quad E\{e(i)^4\}=3\lambda^4 \tag{4}$$

Considering (4) and (1), the variance $m_y^{(2)}$ of the output sequence can be represented for:

- superdiagonal structure as:

$$m_y^{(2)}=\frac{\lambda^2}{1-\beta_{kl}^2\lambda^2} \tag{5}$$

- diagonal structure as:

$$m_y^{(2)}=\beta_{kk}^2\frac{3\lambda^4}{1-\beta_{kl}^2\lambda^2}+\lambda^2 \tag{6}$$

- subdiagonal structure as:

$$m_y^{(2)}=\beta_{kl}^{2(k-l+1)}\lambda^{2(k-l+1)+2}\frac{3}{1-\beta_{kl}^2\lambda^2}+\sum_{i=0}^{k-l}\beta_{kl}^{2i}\lambda^{2i+2} \tag{7}$$

Finally, taking into account (2), (4) and (5—7) the stability and reversibility conditions can be simplified to formulas presented in the table below:

Tab.1.

Stability and reversibility conditions for bilinear time-series models.

	Superdiagonal	Diagonal	Subdiagonal
Stability	$\beta_{kl}^2\lambda^2<1$		
Revers-ibility	$\beta_{kl}^2\lambda^2<0.5$	$\beta_{kl}^2\lambda^2<0.36$	$\beta_{kl}^2\lambda^2<\sqrt[k-l-2]{0.5}$

As was stated in the introduction, these reversibility conditions require knowledge about both the variance λ^2 of white noise *e(i)* and the model coefficient β_{kl} value. These cannot be known before model identification so there is no indication if a time-series model is reversible and has been properly identified. The proposed solution is the criterion (8) based on statistical moments of an output sequence. It utilizes the different sensitivity of the second and the forth order statistical moments to occurrence of explosions in an output sequence of a random process enquired from EB model [1].

$$K=\frac{(\hat{m}'^{(2)}_y)^2}{\hat{m}'^{(4)}_y} \tag{8}$$

where: $\hat{m}'^{(2)}_y$ is the estimated variance of *y(i)*, $\hat{m}'^{(4)}_y$ is the estimated fourth central moment of *y(i)*:

$$\hat{M}'^{(r)}_y=\frac{1}{N-1}\sum_{i=1}^{N}[y(i)-\bar{y}]^r \tag{9}$$

In order to determine irreversibility of EB model for the particular real time-series using the index *K*, following steps have been proposed:

1) An identification of the structure type and structure parameters (*k* and *l*) using the third central moment [7].

2) A Computation of the empirical S_y^2 and $\hat{M}'^{(4)}_y$ values.

3) A Computation of the *K* index value and comparison with the proper critical value C_N from Tables 1-2:

a) K ≤ C_N – the model should be consider irreversible.

b) K > C_N – the model should be considered reversible

The critical values C_N are presented in Tables 2 and 3.

Tab. 2.

Irreversibility threshold critical values of index K for superdiagonal and diagonal EB models.

N (no. of samples)	Superdiagonal	Diagonal
100	0,30	0,25
1000	0,21	0,15
10000	0,17	0,12

Tab. 3.

Irreversibility threshold critical values of index K for chosen structres of subdiagonal EB model.

N (no. of samples)	EB(2,1)	EB(3,2)	EB(3,1)
100	0,14	0,21	0,15
1000	0,04	0,09	0,14
10000	0,02	0,03	0,01

In the next chapter of this paper the utility tests of this criterion are described along with results.

3. Utility testes and results

In order to test utility of the reversibility criterion for EB time-series model a large number of simulations of the random processes obtained from the EB model have been performed for superdiagonal and diagonal structures. For the subdiagonal structure type a strong dependency of K - criterion critical values upon *k* and *l* EB time-series model structure parameters was noticed and therefore the utility tests were omitted.

For each model structure three different values of white noise *e(i)* variance λ^2 have been taken into consideration to find if this parameter has any impact on usefulness of the criterion. Basing on the original reversibility conditions (Table 1) three sets of the test EB model coefficient β_{kl} values have been chosen for each white noise variance λ^2 value. The first value of each set was selected from reversibility range of the model coefficient β_{kl} values, second was selected near the reversibility threshold and last was selected from the irreversibility range of β_{kl} and all of them are presented in Tables 4 and 5.

Tab. 4.

Test EB model coefficient values for superdiagonal model.

Variance λ^2 of the white noise $e(i)$	β_{kl} (for reversible model)	β_{kl} (near irreversibility threshold)	β_{kl} (from irreversibility range)
1	0.200	0.707	0.800
2	0.141	0.500	0.565
0.5	0.283	1.000	1.131

Tab. 5.

Test EB model coefficient values for diagonal model.

Variance λ^2 of the white noise $e(i)$	β_{kl} (for reversible model)	β_{kl} (near irreversibility threshold)	β_{kl} (from irreversibility range)
1	0.200	0.600	0.800
2	0.141	0.424	0.565
0.5	0.283	0.849	1.131

In the next step for each pair of the white noise variance λ^2 and the model coefficient value β_{kl}, R = *10000* simulations of the *N*-sample time-series were performed and for each these simulated time-series the index *K* (8) was computed and the tested criterion was applied to classify the reversibility of the model. Finally, all irreversible classifications were counted. These tests were repeated for different *N* values with an assumption of Gaussian distribution of the white noise *e(i)* to assure a maximum compatibility with the proposed criterion. The final results are shown in Table 6. and they represents the percentage of positive irreversibility classifications.

Tab. 6.

Utility tests result for Gaussian distribution of white noise

λ^2 =1	β_{kl} (Superdiagonal)			β_{kl} (Diagonal)		
N	0,200	0,707	0,800	0,200	0,600	0,800
100	14,96	54,62	70,77	3,34	40,35	79,84
1000	0,00	52,73	87,35	0,00	47,22	96,70
10000	0,00	54,48	99,73	0,00	47,97	100,00

λ^2 =2	β_{kl} (Superdiagonal)			β_{kl} (Diagonal)		
N	0,141	0,500	0,565	0,141	0,424	0,565
100	14,65	55,60	69,60	3,33	51,29	80,70
1000	0,00	52,99	87,68	0,00	46,50	96,50
10000	0,00	54,53	99,67	0,00	47,61	100,00

λ^2 = 0,5	β_{kl} (Superdiagonal)			β_{kl} (Diagonal)		
N	0,283	1,000	1,131	0,283	0,849	1,131
100	15,24	54,28	70,13	3,29	51,53	80,89
1000	0,00	53,26	87,81	0,00	47,27	96,89
10000	0,00	54,73	99,77	0,00	48,29	100,00

In order to check the robustness of the proposed criterion the second set of tests were performed for a uniform distribution of the white noise *e(i)*. Parameters of the uniform white noise generator were chosen to match the exact variance values from previous tests. The results obtained are presented in Table 7.

Tab. 7.

Utility tests result for uniform distribution of white noise

λ^2 =1	β_{kl} (Superdiagonal)			β_{kl} (Diagonal)		
N	0,200	0,707	0,800	0,200	0,600	0,800
100	0,00	21,04	44,35	0,00	3,58	39,35
1000	0,00	3,41	38,49	0,00	4,00	35,35
10000	0,00	5,00	28,50	0,00	0,00	36,45

λ^2 =2	β_{kl} (Superdiagonal)			β_{kl} (Diagonal)		
N	0,200	0,707	0,800	0,200	0,600	0,800
100	0,00	21,04	44,35	0,00	3,58	39,35
1000	0,00	3,41	38,49	0,00	4,00	35,35
10000	0,00	5,00	28,50	0,00	0,00	36,45

λ^2 = 0,5	β_{kl} (Superdiagonal)			β_{kl} (Diagonal)		
N	0,283	1,000	1,131	0,283	0,849	1,131
100	0,00	21,38	43,93	0,00	3,53	38,85
1000	0,00	3,23	38,80	0,00	7,00	36,02
10000	0,00	10,00	28,49	0,00	0,00	35,48

4. Summary

The final analysis of the utility tests results shown that the proposed in [1] criterion is quite effective for the time-series with the Gaussian distribution of white noise *e(i)* especially for a large number of time samples (*N*). Only for a short sequences reversible models were incorrectly classified in about 15% of tested cases. Moreover, for both superdiagonal and diagonal structure types results seems to be very similar.

The problem occurs when a distribution type of white noise is different from Gaussian and in that cases only clearly reversible examples were classified properly. This seem quite obvious, because the proposed criterion critical values were obtained with assumption of the Gaussian distribution of *e(i)*.

The most interesting observation is that regardless of model reversibility, structure type or distribution of *e(i)* the variance of white noise λ^2 value has no significant impact on criterion effectives, which in that case suggest strong robustness to changes of that parameter.

The final conclusion is that proposed criterion is not yet ready for universal usage and requires some improvements to compensate the problem of the *e(i)* distribution. Still for some cases it provides with valuable estimation of reversibility of the possible model which is yet to be identified. The another important remark is that the proposed criterion is based on empirical statistical moments of the time-series which in theory (estimated value computations) should be time independent. The results presented in [1] and in this paper clearly shows different situation which unfortunately counts against use of statistical moments for bilinear models.

5. References

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