Memetic Algorithm for identification of linear-bilinear time-series model.

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Abstract

There are multiple difficulties which have to be overtaken to achieve accurate estimation results of coefficients of time-series models containing bilinear substructures. This paper presents a Memetic Algorithm, designed for identification of a linear-bilinear time-series models with separated estimation approach.

1. Introduction

The time-series modelling and analysis is a discipline of mathematical science dominated by linear models. They are easy to identified and proved to be useful for prediction, analysis and classification of many phenomena. The less explored but more challenging subfield of time-series modelling is nonlinear modelling. The first scientists which commenced research in this area of research are, Granger and Andersen [1]. They have also introduced the theory of bilinear time-series models in 1978 [2] which are the addressed in this paper and also one of the simplest proposition of nonlinear time-series modelling.

The bilinear time-series models were next concerned by Subba Rao [3] in 1981 and Quinn in 1982. In 1985 and 1986, Pham proposed a Markovian representation of bilinear time-series models [5] and later, analysed its equivalent - an autoregressive linear model with varying coefficients [6]. Three years later, the Method of Lest Squares for estimation of diagonal variant of bilinear time-series model was proposed by Pham and Guegan. In meantime in 1987 Gooijger and Heuts [8] performed a statistical analysis of higher order moments of certain bilinear models which was a introduction to Method of Moments identification procedure [13]. In 1994, the stability condition for certain bilinear models, has been proposed by Lee and Mathews [9]. The same year Bielińska and Nabagło proposed a modification to Least Squares (LS) method [10], which was based on the concept of limiting estimates of prediction error. The purpose of this modification was stabilisation of an identification procedure.

A general bilinear time-series model (BARMA) is very difficult to indentify. Even the highly simplified variant, the elementary linear-bilinear time series model (LEB), requires many uncommon approaches in order to be identified with unbiased estimates of its coefficients. In 1995, Brunner and Hess presented experimental research concerning LEB model [11]. They showed that the cost function of maximum likelihood algorithm has a complex multimodal shape which cases the biased estimates of model coefficients. Later (2007), an extensive monograph concerning bilinear time-series models has been written by Bielińska [13]. She proposed several identification procedure for different variants of LEB. Finally in 2011 Maliński proposed a evolutionary algorithm [14] to overcame the problem of multimodality of the Mean Square Error (MSE) cost function and solution to indentify indivertible elementary bilinear time-series model [15] which, is the part of LEB model.

The current research presented in this paper are focused on usage of Memetic Algorithm (MA) designed for identification of LEB models. The main concept is inspired by work of Wang [12] who has proposed an identification procedure for complex BARMA model based on separated estimation of each component of this model. This way, in algorithm proposed in this paper a linear part of LEB model is estimated separately by a Least Squares (LS) algorithm which is suitable for identification of linear models. The bilinear part is identified by MA because of the multimodality of the cost function caused by it.

2. Theoretical background

The most general variant of bilinear time-series models is BARMA(dA,dC,dK,dL) model [13], defined below:

$$y(t)=\sum_{i=1}^{dA}a_i y(t-i)+\sum_{j=0}^{dC}c_j e(t-j)+ \\ +\sum_{k=1}^{dK}\sum_{l=1}^{dL}\beta_{kl}e(t-k)y(t-l) \quad (1)$$

where: $y(t)$ is a discrete output signal, t is a discrete time indicator, coefficients a_i and c_j determine linear part of the model and β_{kl} are coefficients of the bilinear part. Finally, dA, dC, dK

and dL are structure parameters defining the particular ranks of each model component. An innovation signal $e(t)$ is assumed to be independent, identically distributed random sequence.

The model, described above, is very complex and an analysis of its features is difficult. There are also numerous problems to be found in attempts of identification of such model. Therefore, many authors limit their considerations to analysis of simplified structures [4,10-13]. On following pages the elementary linear-bilinear LEB(m,k,l) model, defined in (2), will be considered.

$$y(t) = e(t) + \alpha y(i-m) + \beta e(t-k)y(t-l) \quad (2)$$

Assuming that, innovation signal $e(t)$ has following statistical properties:

$$\begin{aligned} &E\{e(t)\} = 0; \\ &E\{e(t)^2\} = \lambda^2; \\ &E\{e(t)e(t-1)\} = 0; \\ &E\{e(t)^3\} = 0; \end{aligned} \quad (3)$$

The stability condition of bilinear part of the LEB(m,k,l) model can be defined by means of model coefficient and innovation signal variance [13]:

$$\beta^2 \lambda^2 < 1 \quad (4)$$

where: λ^2 is a variance of the white noise $e(t)$.

The stability of the single coefficient, auto-regressive linear part of the LEB(m,k,l) model is well known [16]:

$$|\alpha| < 1 \quad (5)$$

As long as the linear part of the LEB(m,k,l) model can be easily identified using common Recursive Least Squares (RLS) algorithm, one of the main issues concerning bilinear part of the LEB(m,k,l) model identification is invertibility which, is described by condition (6):

$$\beta^2 m'^{(2)}_y < 1 \quad (6)$$

where, $m'^{(2)}_y$ is a variance of a output signal $y(t)$.

Assuming that, we generated time-series using LEB(m,k,l) model, then if original β value do not satisfy condition (6) the inverted model become unstable and the result of this coefficient estimation may be significantly biased. More information and solution (Saturated Mean Square Error - SMSE) to overcame this phenomenon is presented in [15].

Another, major problem of identification of the bilinear part of the LEB(m,k,l) model is multimodality of the cost function (MSE/SMSE). This issue has been addressed in [14] and solution proposed there was usage of Evolutionary Algorithms (EA).

In following sections the inspired by Wang [12] concept of identification of linear and bilinear part separately will be presented along with Memetic Algorithm design which is superior to EA.

3. Separated Identification

In 2004 Wang [12] has proposed the unique identification procedure for subdiagonal bilinear time-series model. The major feature of this algorithm was separated identification of each part of model using common Least Squares (LS) method. However, as long as common LS method is sufficient for identification of linear models it is not recommended for bilinear ones due to multimodality of the its cost function [17].

To solve this problem a Memetic Algorithm has been designed, which general principles are explained in Figure 1.

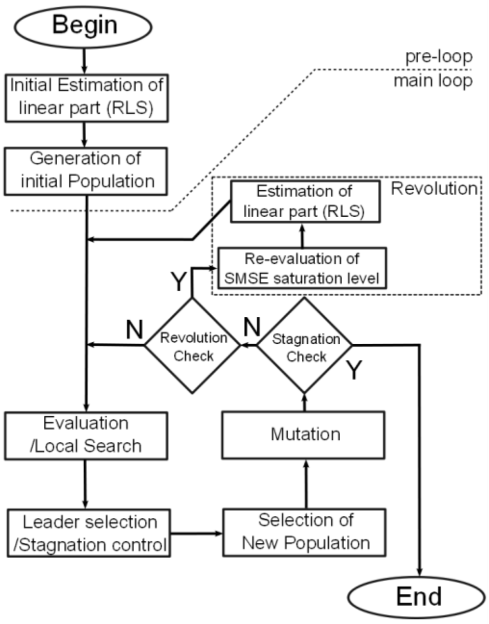

Fig. 1. The MA design.

The cost function used in this particular MA design is, mentioned before, SMSE function defined by equation (7) with support of (8) and (9) :

$$J(w, y, \hat{\beta}, \hat{\alpha}, k, l) = \frac{1}{N}\sum_{t=1}^{N} \hat{e}(t)^2 \quad (7)$$

$$\hat{e}(t) = \begin{cases} w & \text{for} \quad \varepsilon(t) \le w \\ \varepsilon(t) & \text{for} \quad -w < \varepsilon(t) < w \\ -w & \text{for} \quad \varepsilon(t) < -w \end{cases} \quad (8)$$

$$\varepsilon(t) = y(t) - \hat{\alpha} y(t-m) - \hat{\beta}\hat{e}(t-k)y(t-l) \quad (9)$$

A similar saturation function (8) is also applied during computation of one step prediction error in commonly known RLS algorithm [16] used in estimation of linear part of LEB(*m*,*k*,*l*) model.

The identification of the linear part is performed at the very beginning of the identification algorithm assuming $\hat{\beta} = 0$. Then repeated in every Revolution, for current best estimate for β coefficient found by MA. Re-evaluation of the saturation level for SMSE function is also performed during Revolution basing on principles presented in [18]. The identification of the bilinear part is done by MA in every single iteration of the algorithm for current best estimate of α coefficient.

What it is worth mentioning, this particular solution presents the optimisation problem with nonstationary (changing) solution space therefore, a lot of flexibility is required from MA algorithm.

4. Results of Experiment

In order to present the run of proposed identification procedure a single time-series realization obtained from LEB(1,1,1) model has been generated. The coefficients of original model were α = 0.2 and β = 0.3 and the innovation signal $e(t)$ used for stimulation model was white noise of Gaussian distribution with zero mean value and unary variance. The time-series contained N = 1000 samples but was generated from 1500 realizations of the white noise and first 500 samples were omitted and treated as the buffer to fade initial conditions.

During the identification run the model structure parameters (*m*,*k*,*l*) were assumed to be known. This way, in ideal situation, the estimates of the coefficients obtained should be convergent to the original ones. The run was performed using small population of L = 20 candidate solutions. The probability of mutation was chosen relatively high P_m = 0.5, in order to obtain high diversity of population. The probability of performing a local search on candidate solutions was chosen to be low (P_{LS} = 0.2), to reduce computational cost of the algorithm. The stagnation limit *SL* (number of interactions after which the algorithm will stop if superior solution cannot be found) was set to 5.

Finally, after 10 iterations of the MA and 2 revolutions (linear part estimation and saturation level evaluation) the following results have been obtained: $\hat{\alpha} = 0.18729$; $\hat{\beta} = 0.28514$; The estimated variance of the innovation signal $e(t)$ was: $\hat{\lambda}^2 = 0.96586$, and the history of the coefficients ($\hat{\alpha}$, $\hat{\beta}$) changes during algorithm run is presented in Figures 2 and 3 (dashed lines indicate original values). The history of changes in $\hat{\lambda}^2$ and saturation level evaluation is shown in Figure 4.

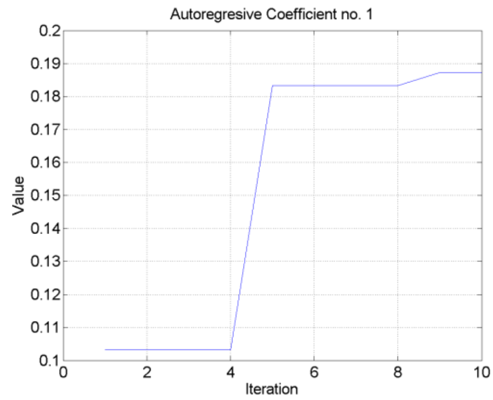

Fig. 2. The history of α coefficient.

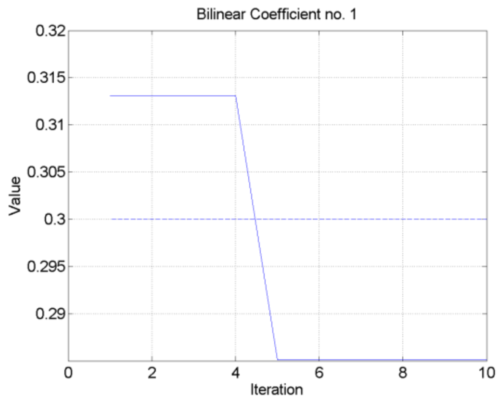

Fig. 3. The history of β coefficient.

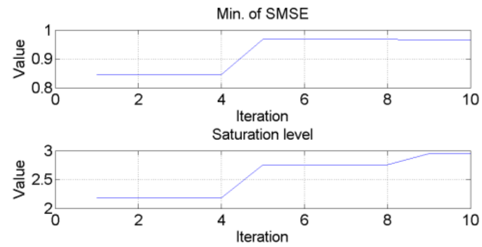

Fig. 4. The history of $\hat{\lambda}^2$ and saturation level evaluation

Final three Figures (5-7) shows the changes in shape of the cost function (SMSE) observed after initial identification of linear part of the model and during revolutions after 5th and 9th iterations:

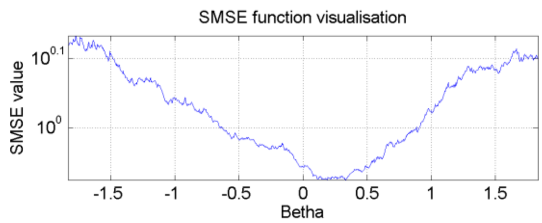

Fig. 6. The visualisation SMSE after initial identification of linear part.

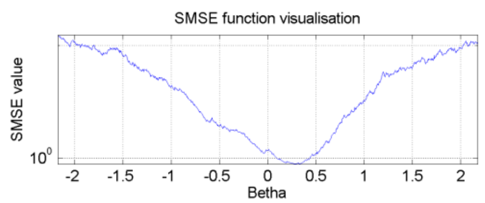

Fig. 6. The visualisation SMSE after revolution in 5th iteration.

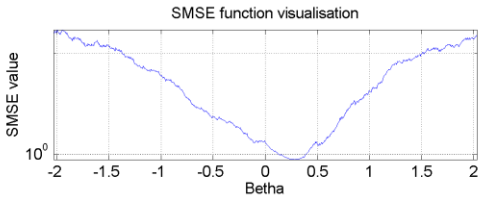

Fig. 7. The visualisation SMSE after revolution in 9th iteration.

5. Summary

As it can be observed in figures (previous section) the results looks quite optimistic. The estimates of the coefficients are slightly biased but it is expected. If an original values are low, the randomness originated from white noise is dominative and result with slight randomly biased estimates. Also differences between shape of the SMSE function of initial identification of linear part and after 5th and 9th iterations seems to be very subtle.

The results obtained, clearly encourage us to extend the research in this area to find computational optimisation of the algorithm itself as long as limitations and possible modifications for more complex and more demanding structures. Also, some statistical tests of algorithm efficiency should be performed.

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