

Calculation of Polish power system angular stability factors based on instantaneous power swings

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Abstract

The paper presents results of the accuracy analysis of calculating the defined stability factors of the Polish Power System (PPS) on a basis of the PPS state matrix eigenvalues associated with electro-mechanical phenomena. The eigenvalues were calculated by analysis of the disturbance waveforms of the instantaneous power swings when taking into account introduction of a disturbance to different generating units working in PPS. There were analysed the power swing waveforms occurring after introducing the disturbance in the form of a rectangular impulse of appropriately selected width and height to the voltage regulation system of generators in one of the PPS generating units. The hybrid optimisation algorithm consisting of the genetic and gradient algorithms was used for computations. In order to increase the computation accuracy, computations were repeated many times for each analysed instantaneous power swing waveform. The computation results of real and imaginary parts of particular eigenvalues were averaged.

1. Introduction

Maintaining the angular stability of a power system (PS) is one of the most important aspects of its work. The PS angular stability can be assessed with use of stability factors calculated on a basis of the PS state matrix eigenvalues [1]. The eigenvalues can be calculated from the PS state equations but the calculation results then depend on the values of the PS state matrix elements. They also depend indirectly on the assumed PS models and their uncertain parameters [2]. The eigenvalues can also be calculated with good accuracy from analysis of the actual disturbance waveforms occurring in the PS after various disturbances.

The goal of the paper is to determine the stability factors of the Polish Power System (PPS) with use of the eigenvalues calculated on a basis of the analysis of instantaneous power swing waveforms in PPS generating units occurring after introducing a disturbance to one of the PPS generating units.

2. The linearised model of a power system

The PS model linearised around the steady working point is described by the state equation and the output equation [3]:

$$\Delta \dot{\mathbf{X}} = \mathbf{A}\Delta \mathbf{X} + \mathbf{B}\Delta \mathbf{U} \quad (1)$$

$$\Delta \mathbf{Y} = \mathbf{C}\Delta \mathbf{X} + \mathbf{D}\Delta \mathbf{U} \quad (2)$$

where: $\Delta \mathbf{X}$, $\Delta \mathbf{U}$, $\Delta \mathbf{Y}$ - deviations of the vectors of: state variables, input and output variables, respectively. The waveforms of input quantities of the linearised PS model can be calculated directly by solving the state equation, or by using the eigenvalues and eigenvectors of the state matrix [3].

The waveform of the given output value is a superposition of the modal components depending on the eigenvalues and eigenvectors of the state matrix. For example, in case of a disturbance in the form of a Dirac pulse of the j -th input value:

$$\Delta U_j(t) = \Delta U \delta(t) \quad (3)$$

the i -th output value (at $\mathbf{D} = \mathbf{0}$ and assuming only single eigenvalues of the state matrix) is [4]:

$$\Delta Y_i = \sum_{h=1}^n F_{ih} e^{\lambda_h t} \quad (4)$$

where:

$$F_{ih} = \mathbf{C}_i \mathbf{V}_h \mathbf{W}_h^T \mathbf{B}_j \Delta U \quad (5)$$

where: $\lambda_h = \alpha_h + j \nu_h$ - h -th eigenvalue of the state matrix, F_{ih} - participation factor of the h -th eigenvalue in the i -th output waveform, \mathbf{C}_i - i -th row of \mathbf{C} matrix, \mathbf{V}_h , \mathbf{W}_h - h -th right-side and left-side eigenvector of the state matrix, \mathbf{B}_j - j -th column of \mathbf{B} matrix, n - dimension of the state matrix \mathbf{A} . The eigenvalues λ_h and their participation factors F_{ih} can be real or complex.

In case of the waveforms of instantaneous power swings in PS, the eigenvalues associated with motion of generating units rotors, called *electromechanical*

eigenvalues in the paper, are of decisive significance. They are complex conjugate eigenvalues with imaginary parts corresponding to the oscillations frequency range (0.1-2 Hz), hence their imaginary parts fall into the range (0.63-12.6 rad/s). The electromechanical eigenvalues influence the instantaneous power waveforms of particular generating units in different ways, which is related to the different values of the participation factors of these eigenvalues.

3. Exemplary calculations

Calculations were performed for the Polish Power System model shown in Fig. 1. In the model there were taken into account 49 selected generating units working in high and highest voltage networks as well as 8 equivalent generating units representing influence of power systems of neighbouring countries.

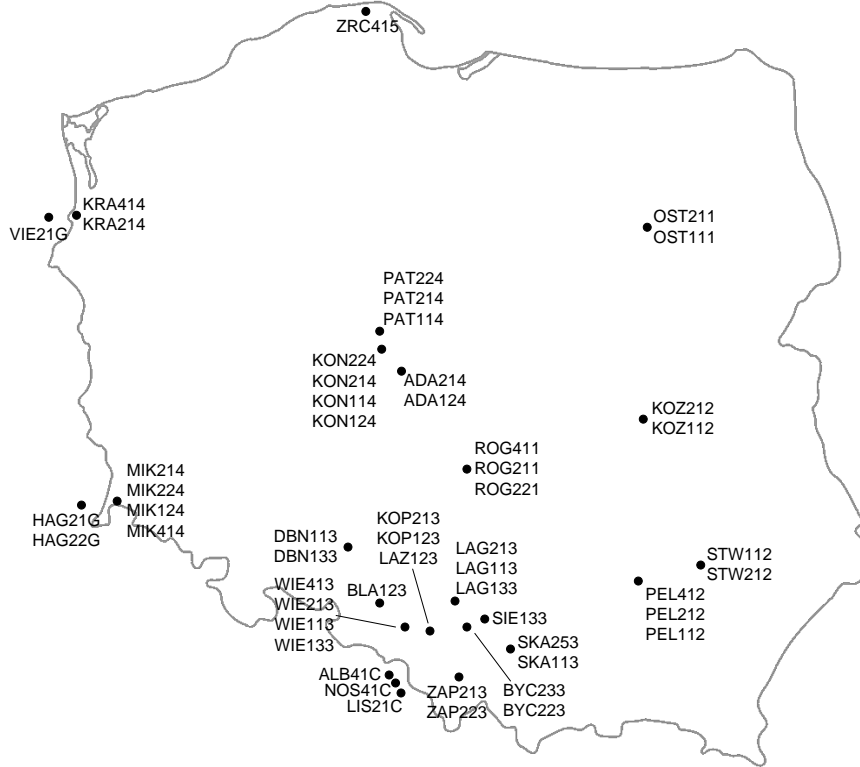


Fig.1. Generating units included in the PPS model.

The method for calculations of electromechanical eigenvalues used in investigations consists in approximation of instantaneous power swing waveforms in particular generating units with use of expression (4). The electromechanical eigenvalues and their participation factors are the unknown parameters of this approximation. In the approximation process, these parameters are iteratively selected to minimise the value of the objective function defined as a mean square error between the approximated and approximating waveform:

$$\varepsilon_w(\lambda, \mathbf{F}) = \sum_{i=1}^N (P_{i(m)} - P_{i(a)}(\lambda, \mathbf{F}))^2 \quad (6)$$

where: λ – vector of electromechanical eigenvalues, \mathbf{F} – vector of participation factors, N – number of waveform samples. The index m denotes the approximated waveform, while the index a denotes the approximating waveform of the instantaneous power P , calculated from the searched eigenvalues and participation factors.

The eigenvalues with small participation factor modules in particular waveform are neglected in calculations based on this waveform. The objective function (6) is minimised by a hybrid algorithm which is a serial combination of a genetic algorithm with a gradient algorithm. The results of the genetic algorithm are the starting point of the gradient algorithm. The genetic algorithm seeks the global minimum of the objective function in the given range of the determined parameters. The starting point is randomly selected from the search interval, so it is not necessary to define it precisely. However, the algorithm is slowly convergent. The gradient algorithm is more quickly convergent, but it seeks the local minimum of the objective function, due to which the initial parameter values must be carefully selected to obtain correct results. The serial combination of the genetic and gradient algorithms eliminates their basic disadvantages [4, 5].

For the purpose of calculations, the measured instantaneous power waveforms will be used as the input data (approximated during the calculation), but in order to verify the calculation method accuracy,

the instantaneous power waveforms obtained from simulations with use of the PPS model are employed. The eigenvalues and participation factors calculated from the assumed structure and parameters of the PPS model are assumed to be the reference point [4].

Due to the existence of numerous local minima of the objective function in which the optimisation algorithm may freeze, the calculations of the eigenvalues were repeated many times for each analysed instantaneous power swing waveform. When the objective function values were higher than a certain assumed limit, the results were rejected. The assumed final result of the calculations of real and imaginary parts of the particular eigenvalues were the arithmetic means from the real and imaginary parts, respectively, of the eigenvalues obtained from the results not rejected in the repeated calculations.

The analysed PPS model was worked out in Matlab-Simulink environment. It consists of 57 models of generating units as well as the model of the network and loads.

In the calculations presented in this paper there were taken into account the models of: a synchronous generator (model GENROU) [4], a static or electromachine excitation system [4] operating in the PPS, a steam turbine IEEEG1 [4] or a water turbine HYGOV and, optionally, a power system stabilizer PSS3B [4]. For the equivalent generating units representing influence of power systems of the neighbouring countries there was used only the simplified model of a synchronous generator.

The assumed disturbance is a rectangular pulse of the voltage regulator reference voltage in one of PPS generating units. The system response to an input in the form of a short square rectangular pulse with a suitably selected height and length is close to the response to a Dirac pulse.

The appropriate selection of the height and length of the rectangular pulse of the voltage regulator reference voltage is an important factor which determines the accuracy of calculations. The amplitude of instantaneous power swings must be sufficiently high to allow separating these swings from the recorded waveforms of phase currents and voltages in particular system nodes. The amplitude is approximately proportional to the surface of the pulse in the voltage regulator reference voltage. The pulse height, however, must be limited to avoid a significant influence of PS nonlinearities on the instantaneous power waveforms. The rectangular pulse duration t_{imp} must also be limited, since its significant lengthening results in increasing differences in the system responses to the rectangular and Dirac pulse, which can decrease the accuracy of determining electromechanical eigenvalues [4].

Since there are only several modal components of significant amplitude in the instantaneous power waveform of a single generating unit, it is necessary

to analyse the instantaneous power waveforms of different generating units occurring after introducing a disturbance at various places.

Tab. 1 presents selected electromechanical eigenvalues λ calculated directly by a Matlab-Simulink program on a basis of the PPS model (called *original eigenvalues* in the paper) and absolute errors $\Delta\lambda$ of calculating these eigenvalues on a basis of the instantaneous power waveforms. Also the relative errors of calculating real $\delta\alpha$ and imaginary parts $\delta\nu$ of these eigenvalues are included.

Tab.1.

Selected original eigenvalues and errors of calculations of these eigenvalues

λ_1	$-1.3099 \pm j11.1792$	$\Delta\lambda_1$	$-0.0377 \mp j0.3307$
$\delta\alpha_1$	2.881 %	$\delta\nu_1$	-2.958 %
λ_7	$-1.1669 \pm j10.1882$	$\Delta\lambda_7$	$0.0215 \mp j0.1457$
$\delta\alpha_7$	-1.843 %	$\delta\nu_7$	-1.430 %
λ_{10}	$-1.0867 \pm j10.9129$	$\Delta\lambda_{10}$	$0.0022 \mp j0.2474$
$\delta\alpha_{10}$	-0.200 %	$\delta\nu_{10}$	-2.267 %
λ_{15}	$-1.0477 \pm j10.0241$	$\Delta\lambda_{15}$	$-0.0061 \pm j0.0214$
$\delta\alpha_{15}$	0.580 %	$\delta\nu_{15}$	0.213 %
λ_{19}	$-0.9956 \pm j9.7503$	$\Delta\lambda_{19}$	$-0.0035 \mp j0.1107$
$\delta\alpha_{19}$	0.352 %	$\delta\nu_{19}$	-1.136 %
λ_{23}	$-0.9891 \pm j10.3132$	$\Delta\lambda_{23}$	$0.0120 \pm j0.0465$
$\delta\alpha_{23}$	-1.211 %	$\delta\nu_{23}$	0.451 %
λ_{29}	$-0.8716 \pm j9.5518$	$\Delta\lambda_{29}$	$-0.0111 \pm j0.0301$
$\delta\alpha_{29}$	1.277 %	$\delta\nu_{29}$	0.315 %
λ_{36}	$-0.7765 \pm j9.1363$	$\Delta\lambda_{36}$	$-0.0086 \pm j0.0195$
$\delta\alpha_{36}$	1.102 %	$\delta\nu_{36}$	0.213 %
λ_{39}	$-0.7368 \pm j9.6011$	$\Delta\lambda_{39}$	$-0.0383 \pm j0.2604$
$\delta\alpha_{39}$	5.197 %	$\delta\nu_{39}$	2.712 %
λ_{45}	$-0.4955 \pm j7.3005$	$\Delta\lambda_{45}$	$0.0293 \mp j0.2115$
$\delta\alpha_{45}$	-5.914 %	$\delta\nu_{45}$	-2.897 %
λ_{49}	$-0.1710 \pm j4.9780$	$\Delta\lambda_{49}$	$-0.0064 \pm j0.1107$
$\delta\alpha_{49}$	3.717 %	$\delta\nu_{49}$	2.223 %

For instance, Fig. 2 shows the instantaneous power swing waveforms of the generating unit ROG411 (Belchatów Power Plant) when introducing a disturbance to this unit.

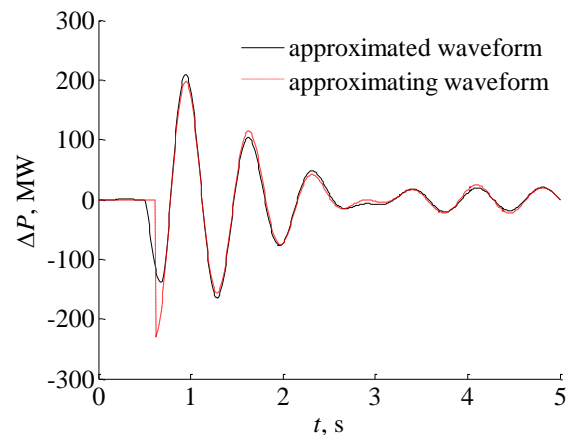


Fig.2. Instantaneous power swing waveforms of the generating unit ROG411.

From Fig. 2 it follows that in this case the quality of approximating the instantaneous power waveforms with the hybrid algorithm is worse in the time range of about 0.1 s after occurrence of the disturbance, which is caused by the influence of strongly damped modal components not associated with the electromechanical eigenvalues. In order to eliminate the influence of these modal components, the analysis of the waveform is started after 0.5 s after the disturbance occurrence [6].

The following stability factors determined on a basis of electromechanical eigenvalues were used for assessing the PPS angular stability [1]:

$$W_1 = \max(\alpha_h)$$

$$W_2 = \max(\xi_h) = \max\left(\frac{\alpha_h}{\sqrt{\alpha_h^2 + v_h^2}}\right) \quad (8)$$

$$W_3 = \min(\eta_h) = \min\left(\ln\left(2\pi \frac{-\alpha_h}{v_h}\right)\right)$$

The values of stability factors (8) determined on a basis of original and calculated by means of the hybrid algorithm eigenvalues are compared in Tab. 2. There are given the relative errors of calculating the stability factors by means of the hybrid algorithm.

Tab.2.

Calculation results of the PPS stability factors

	Calculated based on original eigenvalues	Calculated based on calculated eigenvalues	Error, %
W_1	-0.1710	-0.1773	3.717
W_2	-0.0343	-0.0348	1.460
W_3	-1.5335	-1.5189	-0.946

4. Summary

The investigations performed allow drawing the following conclusions:

- It is possible to determine the PPS state matrix electromechanical eigenvalues on a basis of the analysis of the instantaneous power waveforms in disturbance states. Based on them the PPS stability factors can be calculated. The higher participation of the modal components associated with the calculated eigenvalues in the instantaneous power waveform is, the more accurate calculations of eigenvalues based on that waveform are.
- The relative errors of calculating the PPS electromechanical eigenvalue real parts are higher for eigenvalues that have small modules of their real parts because they are related to these real parts. No relationship was found between the real parts of the eigenvalues and the absolute errors of these real parts.
- From the investigations performed it follows that the eigenvalue λ_{49} is of decisive significance for the

PPS angular stability. That eigenvalue has the largest real part (the smallest absolute value of the real part) from among the eigenvalues interfering in a significant way in the instantaneous power swing waveforms of generating units working in PPS.

- The calculation error of the stability factor W_3 appeared to be the smallest one (considering its module). A little greater is the calculation error of the stability factor W_2 . The greatest is the calculation error of the stability factor W_1 . The calculation errors of these stability factors were determined by calculation errors of the real and imaginary parts of the eigenvalue λ_{49} critical for the PPS stability.

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