Adaptive Filtering of Engine Vibration in a Vehicle Measurement System

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Abstract

The paper presents modified LMS based filtering algorithm of non-stationary nonlinear engine-induced acceleration signal. Analysis is dedicated to vehicle measurement system which is a key element of vibration control system. Engine-induced vibrations, which can be modelled using multi-notch filter, are assumed to corrupt stationary wideband desired signal defined in a band of 0 – 100 [Hz]. Initially, it was stated, based on offline analysis, the optimal filter parameter exists for all experiment conditions. Adaptation constant of LMS algorithm as well as bandwidth parameter of multi-notch filter are made dependent on cross-correlation of filtered signal and its delayed version as well as on power estimation of input corrupted signal. Time-frequency domain analysis proved significant efficiency of the adaptive filtering algorithm.

Keywords: vehicle engine nonlinear vibrations, LMS based measurement filtering, multi-notch filter.

1. Introduction

Nowadays kinematic sensors based measurement systems used in vehicles require high accuracy and reliability, especially in applications dedicated to vibration control, road profile scanning and ride safety. However, such measurement data, especially if taken using accelerometers, can be very easily and strongly corrupted with vibration and acoustic noise generated by vehicle engine. Engine-induced nonlinear vibrations contribute to the resultant vibration measurements, apart from desired road and maneuver-induced vibration measurements.

Further analysis is dedicated to all-terrain vehicle which exhibits high level of engine-induced vibrations, such as in [1]. Frequency band, in which these disturbances are defined in most cases overlaps band of desired measurements so disturbances cannot be cancelled using straight-forward linear band-pass filtering method. Moreover, vehicle engine speed and frequency of engine ignitions are not constant which makes the disturbance non-stationary and requires adaptive filtering methods to be used [2].

Adaptive LMS (Least-Mean Square) based algorithms include reference model which is, in many cases, constituted as FIR filter [3]. However, in frequency tracking applications the reference model needs to be suited to the nonlinear multi-harmonics disturbance, which can be satisfied by narrowband filter. Parameters of all notch sub-filters are related which reduces the number of multi-harmonic parameters and leads to increase of algorithm accuracy. Usually kinds of narrowband filters are taken into account in tracking of single harmonic, *i.e.*: zeroing polynomial filter [4] and notch filter [5], which includes additional bandwidth parameter.

Frequency tracking methods dedicated to single harmonic signals lose its accuracy in case of nonlinear multi-harmonic signals. Separate narrowband filters can be used; however, all sub-filters may track the same dominant harmonic. Instead, comb filters [6] or cascade multi-harmonics filters [7] are recommended to accompany LMS algorithm. Modifications of multi-notch filter based adaptive algorithms additionally include a scheme of parameters adaptation such as [8] in which notch filter bandwidth is made dependent on LMS adaptation constant. Both LMS adaptation constant and multiple-notch filter bandwidth can be also made dependent on cross-correlation derived based on filter output and delayed output [9]; additionally, stability analysis was presented.

The article is organized as follows. Chapter 2 presents features of an experimental vehicle and its measurement system. In chapter 3 an offline solution for the problem is reported. Chapter 4 introduces modified multi-notch filter which is based on LMS algorithm. In chapter 5 efficiency of the algorithm is validated. Finally, chapter 6 concludes the results.

2. Experimental vehicle and measurement system characteristics

Research is dedicated to an ATV (all-terrain vehicle), strictly ATV Sweden CF-Moto 500 (Figure

1), which is the key element of a vehicular vibration control system [10]. Measurement part of the system consists of accelerometers which are located in the vehicle body and underbody parts. There are numerous components which can contribute to the resultant acceleration measurements. Some of them describe road-induced or maneuver-induced vehicle vibration as well as vibration generated by other vehicle elements; others have origin in vehicle engine vibrations, which are not desired.

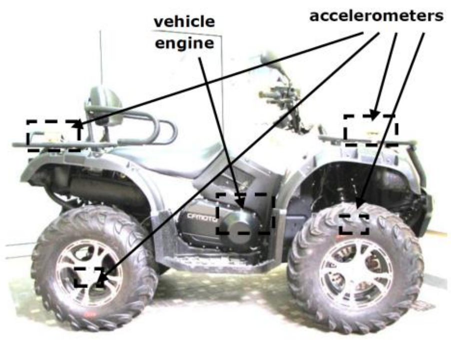

Figure 1. Experimental vehicle including accelerometers and engine.

The experimental vehicle is equipped with four-stroke petrol engine including one cylinder; consequently, every second engine revolution an ignition of fuel-air mixture occurs. Engine-induced vehicle body vibrations exhibit high level of power and include multiple higher harmonics. It can be stated that first harmonic should be correlated with engine ignitions; second harmonic corresponds to engine speed. Further harmonics are derivatives of first two fundamental harmonics.

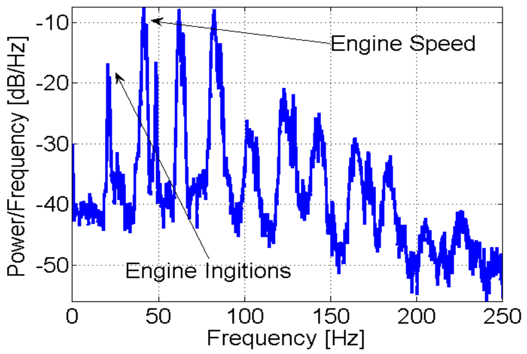

Figure 2. Power spectral density function for vehicle engine run at 42 [Hz] of engine speed.

Initially, engine-induced disturbance was analyzed during 50 [sec] long engine's run for manually set engine speed of 42 [Hz] with results presented as power spectral density function (Figure 2). It can be noticed that theoretical expectations about frequency of engine ignitions and engine speed as well as multi-harmonics nature of acceleration measurements are justified by experiment results. Frequency of the first and the second harmonics are equal to 21 [Hz] (engine ignitions) and 42 [Hz] (engine speed), respectively.

3. Offline filtering of measurements

Two classes of measurement filtering algorithms can be distinguished, *i.e.*: online and offline filtering methods. Online filters process data during real-time algorithm execution. Offline filters are more strictly introduced as smoothers due to their a posteriori data processing and utilized if an object is observed rather than controlled. Offline smoother output is available after all input data are acquired; however, offline algorithms can be more accurate, are non-causal and take advantage of future samples. Mean values of certain parameters and quantities can be, initially, estimated for measurement data and used again in the filtering method. The following offline analysis can mainly show that there exists an optimal solution to the problem of engine-induced signal's filtering.

3.1. Cascade connection of notch sub-filters

Nonlinear engine-induced acceleration signal can be modelled with high accuracy as multi-harmonic filter. Two classes of narrowband digital filters have been taken into account, *i.e.*: zeroing polynomial filter and notch filter. Notch filters, which have been selected for the task, exhibit non-constant group delay; however, bandwidth of zeroing polynomial filter cannot be tuned as in case of 2-order notch filters due to the poles' absence.

Center frequencies of all sub-filters included in the multi-notch filter have been made dependent on consecutive multiplications of one parameter denoted as fundamental frequency parameter. Consecutive harmonics of engine-induced disturbance can be filtered out using such cascade notch-based filter which is defined in z domain as follows:

$$\mathbf{H}_{Notch-M}(z, r_H, \omega_H) = \prod_{m=1}^{M} \frac{1-2\cos(m\omega_H)z^{-1}+z^{-2}}{1-2r_H\cos(m\omega_H)z^{-1}+r_H^2 z^{-2}} \quad (1)$$

where $M = 5$ is a number of notch sub-filters included in the cascade filter. Symbol r_H denotes bandwidth parameter of all notch sub-filters and ω_H denotes their center frequency parameter.

3.2. Nonlinear solution space of quality index

Problem of optimizing parameters of the multi-notch filter (1) can be assumed as modification of searching for optimal Wiener filter parameters. However, in this case a filter, which is being adjusted, is nonlinear and includes one parameter. According to Wiener filter theory, desired and estimated signals are defined. These signals are denoted as $x(n)$ and $\hat{x}_M(n)$ and correspond to corrupted engine-induced acceleration signal and the estimated signal which is the output of the filter (1), respectively. Estimated signal $\hat{x}_M(n)$ is derived based on $x(n)$ as follows:

$$\hat{x}_M(n) = [1 - h_{Notch-M}(n, r_H, \omega_H)] * x(n) \qquad (2)$$

where $h_{Notch\text{-}M}(n)$ is an impulse response of the multi-notch filter (1).

Main goal of the optimization problem can be defined as searching for parameter ω_H which minimizes a mean square value of the estimation error $e_M(n)$. If such optimization condition is met, the filter output will be the MMSE (minimum mean-squares error) estimate of desired signal $x(n)$. Mean squared estimation error can be approximated by estimation quality index as follows:

$$\begin{aligned} QI_{eM} &= MSE[e_M(n)] = \\ &MSE[h_{Notch-M}(n, r_H, \omega_H) * x(n)] \end{aligned} \qquad (3)$$

where

$$e_M(n) = x(n) - \hat{x}_M(n) \qquad (4)$$

Values of the quality index (3) were derived in the offline manner based on measurements for engine speed equal to 58 [Hz] and presented in Figure 3. Two cases are analyzed which differ in value of r_H parameter of the filter (1), *i.e.*: 0.65 and 0.85. Due to strong nonlinearity of filter transform function (1) a solution space of quality index (3) includes numerous local minimum points and is not unambiguous. An optimal filter parameter ω_H can be obtained for the global minimum point of the quality index (3).

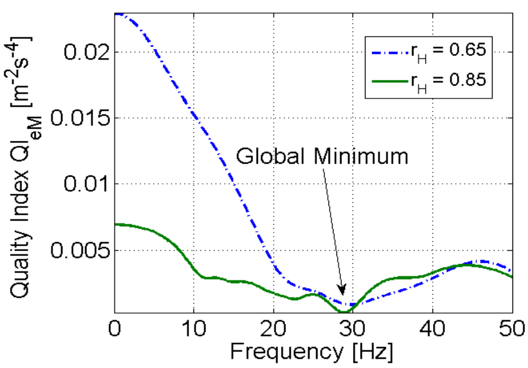

Figure 3. Solution space of QI_{eM} quality index for engine speed equal to 58 [Hz] and bandwidth parameter values 0.65 and 0.85.

Nonlinear shape of QI_{eM} solution space significantly increases difficulty of minimum searching algorithm which tends to converge to local minimum points. It is worth noticing that the lower parameter of multi-notch filter, the smoother the solution space and greater its slope (Figure 3). For lower r_H equal to 0.65 it is easier to reach the neighborhood of the global minimum; however, for such conditions the minimum searching algorithm is not accurate enough. After increasing r_H parameter, global minimum point (Figure 3) occurs at frequency equal to 29 [Hz] which is equivalent to the frequency of engine ignitions.

4. Online filtering of measurements

Real-time applications require a minimum point of the solution space of function (3) to be found iteratively and online. Gradient-based methods, such as LMS algorithm, can converge to the minimum point online, which makes them widely used in the field of vibrations cancellation and filtering.

4.1. Multi-notch filter based LMS algorithm

The LMS adaptive algorithm was selected to be used in measurement filtering, accompanied by multi-notch filter (1) as a reference model; it is defined including squared estimation error (4) as follows:

$$\omega_H(n+1) = \omega_H(n) - \frac{\mu}{2}\frac{\partial[e_M^2(n)]}{\partial \omega_H} \qquad (5)$$

where symbol μ denotes adaptation constant of the LMS algorithm. According to the stability analysis presented in [10], LMS algorithms remain stable if μ remains within range of $(0 \,;\, 1/\lambda_{max})$, where λ_{max} is the maximum eigenvalue of the auto-correlation matrix which is derived based on the desired input signal $x(n)$. The LMS correction element can be rewritten as follows:

$$\frac{\partial[e_M^2(n)]}{\partial \omega_H} = 2 \cdot e_M(n)\frac{\partial[e_M(n)]}{\partial \omega_H} \qquad (6)$$

Substituting (6) to (5) gives a final version of LMS algorithm as follows:

$$\omega_H(n+1) = \omega_H(n) - \mu \cdot e_M(n) \cdot \frac{\partial[e_M(n)]}{\partial \omega_H} \qquad (7)$$

The adaptive measurements filtering problem can be formulated as frequency tracking problem which is dedicated to multi-harmonics signals. A novel approach to frequency tracking method was presented in [10]. Authors have included the multi-notch filter (1) in LMS algorithm and derived a recursive formula for the gradient of the estimation error (4) as follows:

$$\begin{aligned} &G_m(n) = G_{m-1}(n) - 2\cos[m\omega_H(n)]G_{m-1}(n-1) + \\ &2m\sin[m\omega_H(n)] \cdot e_{m-1}(n-1) + G_{m-1}(n-2) + \\ &+ 2r_H\cos[m\omega_H(n)]G_M(n-1) - r_H^2 G_m(n-2) \\ &- 2r_H m\sin[m\omega_H(n)]e_m(n-1) \end{aligned} \qquad (8)$$

for $m = 1,2,\ldots,M$, where

$$G_m(n) = \frac{\partial[e_m(n)]}{\partial \omega_H} \qquad (9)$$

Initial values of G_m and e_m used in the recursive expression (8) are constituted as follows: $e_0(n) = x(n)$, $G_0(n) = 0$, $G_0(n\text{-}1) = 0$, $G_0(n\text{-}2) = 0$. Authors [10] have proposed choosing the initial value of bandwidth parameter r_H in the neighborhood of quality index (3) minimum point to prevent the algorithm from converging to the local minimum point.

4.2. Tuning of adaptation constant μ and bandwidth parameter r_H

Due to the non-trivial shape of solution space function (Figure 3), authors [10] have made improvement to the presented algorithm (7) – (9). Consequently, parameter r_H can be adjusted adaptively and set to high values if the algorithm has already converged to the global minimum. In other cases r_H needs to be set to low values to increase slope of the solution space function, increase the algorithm's convergence rate and assure the algorithm will converge to the global minimum, not the local one. Instantaneous quality of adaptation result can be evaluated using cross-correlation estimation of the estimation error signal $e_M(n)$ and its delayed version as follows:

$$C_M(n) = e_M(n) \cdot e_M(n-1) \tag{10}$$

High frequency component of cross-correlation (10) is filtered out using low-pass 1-order filter including parameter λ_C as follows:

$$C_{M-LP}(n) = \lambda_C \cdot C_{M-LP}(n-1) + (1-\lambda_C) \cdot C_M(n) \tag{11}$$

Adaptation constant μ is also made dependent on cross-correlation estimation (11); that is, while algorithm is reaching the global minimum the parameter μ should be significantly greater than zero until the algorithm reaches the optimal solution and μ is set close to 0 to avoid algorithm's divergence. Dynamics of r_H and μ is defined in [10] using exponential functions as follows:

$$r_H(n) = r_{H\min} + e^{-\alpha|C_{M-LP}(n)|} \cdot (r_{H\max} - r_{H\min}) \tag{12}$$

$$\mu(n) = \mu_{\min} + (1 - e^{-\alpha|C_{M-LP}(n)|}) \cdot (\mu_{H\max} - \mu_{H\min}) \tag{13}$$

where r_H is limited to the chosen range of (r_{Hmin} ; r_{Hmax}) and μ is within (μ_{min} ; μ_{max}). Symbol α denotes saturation rate of Expressions (12) and (13); it can be also interpreted as a scaling factor of cross-correlation estimation $C_{M\text{-}LP}$. Additionally, an initial coarse estimation of frequency, which is to be tracked, was proposed; however, it is inapplicable for filtering problem presented in the current paper.

4.3. Estimation of engine-induced acceleration power

Experimental results showed that there exists dependence of adaptation constant μ and saturation rate α on power of the corrupted input signal $x(n)$. Power estimation problem is solved as follows:

$$V_{x-LP}(n) = \lambda_V \cdot V_{x-LP}(n-1) + (1-\lambda_V) \cdot x^2(n) \tag{14}$$

where $V_{x\text{-}LP}$ is corrupted signal power estimation after excluding high frequency components using filter parameter λ_V. Corrected adaptation parameter μ_V is inversely proportional to the power estimation (14) and linearly dependent on the original μ parameter as follows:

$$\mu_V(n) = \mu(n) \cdot V_{x-LP}^{-1}(n) \tag{15}$$

Moreover, it was stated based on measurements, that due to the ATV engine construction, the higher engine speed, the higher estimated power (14). Relation between estimated power $V_{x\text{-}LP}$ and saturation rate α was empirically approximated as:

$$\alpha(n) = \alpha_V \cdot \log[V_{x-LP}^{-1}(n)] \tag{16}$$

where α_V denotes the scaling parameter.

5. Experimental results

All experiments were performed using the experimental vehicle (Figure 1). Three classes of measurement data analysis are distinguished, *i.e.*: frequency and time domain analysis of stationary engine-induced acceleration signals as well as frequency/time analysis of non-stationary measurements. For the first two classes, measurements, which were taken for different values of engine speed, were used for tuning and validating parameters of the adaptive filtering algorithm. According to the third class, a non-stationary measurement signal was composed based on different stationary measurement signals and used in final time/frequency domain non-stationary validation.

5.1. Experiment conditions

Engine-induced vibrations mostly propagate into the vehicle body so the front right accelerometer is the sensor which captures such vibrations with the highest gain. All measurement data were acquired with sample rate of 500 [Hz]. Experiments were performed for four conditions which differ in value of manually set engine speed: 2500, 3000, 3500 and 4000 revolutions per minute (42, 50, 58 and 67 revolutions per second). Each measurement set was limited to 50 seconds of steady state and constant engine speed.

It is assumed that the vibration measurement system is dedicated to acceleration acquisition in the frequency band within the range of 0 – 100 [Hz]. Such frequency band overlaps the band of engine-induced acceleration signal. For the need of algorithm validation a desired stationary wideband signal has been generated, denoted as $s_{ref}(n)$ and defined in the band of 0 – 100 [Hz]. Power of the desired signal is adjusted to the certain engine-induced acceleration power and described by DSENR coefficient (desired signal to engine noise ratio).

The adaptive filtering algorithm was validated based on all measurement data sets including added wideband desired signal. Frequency parameter $\omega_H(n)$, which was estimated in the first stage, was used in the second stage in filtering task using constant bandwidth parameter r_F:

$$y_F(n) = h_{Notch-M}[n, r_F, \omega_H(n)] * x(n) \qquad (17)$$

Parameter r_F is set to 0.85 which corresponds to significantly narrowband notch filter and prevents multi-notch filter from cancellation of wideband desired signal components.

5.2. Frequency domain analysis of stationary engine-induced vibrations

In the following analysis engine-induced vibrations signal is assumed as stationary. Parameters of the algorithm were estimated for all measurement data sets (Table 1) and were used in further validation. It was stated that for all experiment conditions the algorithm converges to the correct value of parameter r_H. The power spectral density function is presented in Figure 4 for engine speed equal to 58 [Hz] and for DSENR of 1.0.

Tab.1.

Parameters of LMS based filtering algorithm

$M = 5$	$\lambda_C = 0.995$	$\lambda_V = 0.999$
$r_{Hmin} = 0.50$	$\mu_{min} = 0.01$	$\alpha_V = 900$
$r_{Hmax} = 0.85$	$\mu_{max} = 0.1$	$r_F = 0.85$

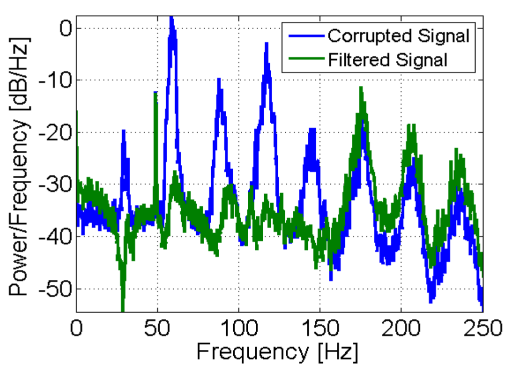

Figure 4. Power spectral density function of corrupted and filtered stationary engine-induced vibrations for engine speed of 58 [Hz] and desired signal to engine noise ratio equal to 1.0.

The figure shows an averaged PSD of corrupted and filtered signals estimated for 50 seconds. The bandwidth of harmonic peaks mainly comes from inaccuracy of setting engine speed; however, it is sufficient in the analyzed case. It can be noticed that fundamental frequency of measurement signal is being tracked properly and the final harmonic filter (17) cancels engine-induced harmonic peaks accurately not corrupting the desired wideband signal.

5.3. Time domain analysis of stationary engine-induced vibrations

In order to qualify performance of the filtering algorithm, a quality index has been defined:

$$QI_F = \frac{1}{N_F} \sum_{n=1}^{N_F} [s_{noisy}(n) - s_{ref}(n)]^2 \qquad (18)$$

where $s_{ref}(n)$ denotes desired wideband signal, $s_{noisy}(n)$ denotes filtered $y_F(n)$ or unfiltered corrupted $x(n)$ signal. Value of the quality index QI_F is inversely proportional to the quality of the measurement signal. Both estimation results groups of QI_F for filtered and unfiltered signals are listed in Table 2. Power of the wideband desired signal is described using DSENR and varies from 0 to 5. For all stationary experiment conditions the algorithm improves quality of corrupted measurement signal. Significant degradation of filtering quality can be stated for certain DSENR values; the higher the engine speed, the lower border of significant degradation which is approximately equal to 1.0 for engine speed of 67 [Hz].

Tab.2.

Quality index QI_F for stationary and non-stationary engine-induced vibrations

Quality Index QI_F [m^2s^{-4}]					
Engine speed [Hz]	Filtered Signals				Corrupted Signals
	Desired wideband signal to engine noise ratio				
	0	0.1	1	5	
42	**0.19**	0.19	0.20	0.23	**0.91**
50	**0.26**	0.26	0.27	0.46	**1.49**
58	**0.32**	0.32	0.34	0.63	**4.28**
67	**0.49**	0.49	0.94	25.50	**23.48**
Volatile	**0.89**	0.94	0.94	1.99	**4.68**

5.4. Time/Frequency domain analysis of non-stationary engine-induced vibrations

Validation of the filtering algorithm was additionally performed using a non-stationary measurement signal which has been prepared as a compound of all parts of stationary engine-induced signals, each 10 [sec] long for certain engine speed values (Figure 5a). Such experiment allows validating accuracy of the algorithm while engine speed is changing and wideband desired signal is added to the non-stationary disturbance signal. Time-frequency characteristic of such non-stationary measurement signal and results of adaptive filtering which shows significant improvement of non-stationary measurements are presented in Figure 5b. High performance of the algorithm for non-stationary disturbance has been also estimated using quality index QI_F (18) (Table 2). Similarly to stationary vibrations dedicated cases, also filtering of non-stationary engine-induced vibrations is recommended for DSENR not higher than 1.0.

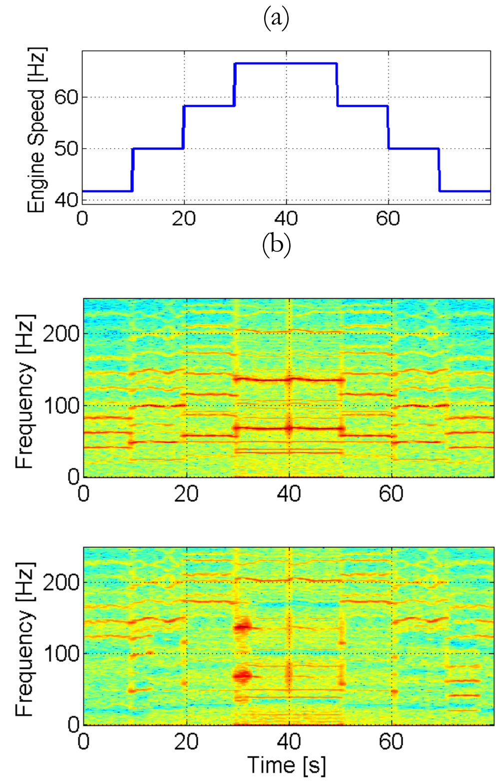

Figure 5 Time-frequency analysis of adaptive filtering for non-stationary engine-induced vibrations and desired signal to engine noise ratio equal to 1: a) desired engine speed, b) time-frequency filtering results.

6. Conclusions

Engine-induced vibrations have significant impact on the quality of measurements taken in vehicles. Most of road vehicles are equipped with engines which can significantly deteriorate accuracy of measurements while running. The adaptive filtering LMS based algorithm including multi-notch filter was proposed. Validation of the filtering algorithm was performed using offline and online methods. An offline measurement analysis showed that an optimal filter, which efficiently suppresses nonlinear engine-induced vibrations, exists. Developments of the adaptive algorithm improve filtering efficiency in case of stationary and non-stationary engine-induced acceleration signals as well as additional wideband desired signal.

Further improvements need to be included to increase accuracy of the algorithm. The algorithm may be used also in applications of fundamental frequency tracking and engine speed estimation.

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