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## **Application of evolutionary algorithm for cast iron latent heat identification**

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#### Abstract

In the paper the cast iron latent heat in the form of two components corresponding to the solidification of austenite and eutectic phases is assumed. The aim of investigations is to estimate the values of austenite and eutectic latent heats on the basis of cooling curve at the central point of the casting domain. This cooling curve has been obtained both on the basis of direct problem solution as well as from the experiment. To solve such inverse problem the evolutionary algorithm (EA) has been applied. The numerical computations have been done using the finite element method by means of commercial software MSC MARC/MENTAT. In the final part of the paper the examples of identification are shown.

**Keywords:** Application of information technology to the foundry industry, Solidification process, Numerical techniques, Inverse problems, Evolutionary algorithms, Identification of latent heat

#### **1. Introduction**

At present, to solve the identification problem the evolutionary algorithms are among others used. Evolutionary algorithm (EA) is a search method that takes their inspiration from natural selection, survival and adaptation in biological world [1, 2, 3]. EA differ from more traditional optimization and identification techniques in that they involve a search from a population of solutions not, as in gradient method [4, 5, 6, 7, 8, 9, 10, 11, 12], from a single point. Each iteration of an EA involves a competitive selection that get out poor solutions. The solutions with high fitness function are recombined with other solutions by exchange of part of the solutions with another ones. Solutions are mutated by making a small change to a single element of solution. Recombination and mutation are also used.

The evolutionary algorithm operates on population of chromosomes (solutions) which contain the genes. Each chromosome is evaluating with use of the fitness function. The starting population is created randomly and next each chromosome is evaluated. Evolutionary operators change the genes values in some chromosomes. The offspring population is created as a result of a selection process. The stop criterion can be formulated as a maximum number of iterations or after achieving the predefined value of fitness function.

In the paper the evolutionary algorithm is used for an identification of parameters appearing in the mathematical description of solidification process.

#### 2. Direct problem

The energy equation describing the casting solidification has the following form [13, 14]

$$C(T)\frac{\partial T(x,t)}{\partial t} = \nabla \left[\lambda(T)\nabla T(x,t)\right]$$
(1)

where C(T) is the substitute thermal capacity [13],  $\lambda(T)$  is the thermal conductivity, *T*, *x*, *t* denote the temperature, geometrical co-ordinates and time.

The substitute thermal capacity can be written in the form [12]

$$C(T) = \begin{cases} c_{L}, & T > T_{L} \\ c_{p} - L \frac{df_{s}(T)}{dT}, & T_{s} \le T \le T_{L} \\ c_{s}, & T < T_{s} \end{cases}$$
(2)

where *L* is the volumetric latent heat,  $f_s$  is the volumetric solid state fraction at the considered point from the casting domain,  $T_L$ ,  $T_s$  are the liquidus and solidus temperature respectively,  $c_L$ ,  $c_s$ ,  $c_P = 0.5*(c_L + c_s)$  are the constant volumetric specific heats of molten metal, solid state and mushy zone sub-domain.

The considered equation is supplemented by the equation concerning a mould sub-domain

$$c_m \frac{\partial T_m(x,t)}{\partial t} = \lambda_m \nabla^2 T_m(x,t)$$
(3)

where  $c_m$  is the mould volumetric specific heat,  $\lambda_m$  is the mould thermal conductivity.

In the case of typical sand moulds on the contact surface between casting and mould the continuity condition in the form

$$\begin{cases} -\lambda(T)\mathbf{n} \cdot \nabla T(x,t) = -\lambda_m \mathbf{n} \cdot \nabla T_m(x,t) \\ T(x,t) = T_m(x,t) \end{cases}$$
(4)

can be accepted. On the external surface of the system the Robin condition

$$-\lambda_{m}\mathbf{n}\cdot\nabla T_{m}(x,t) = \alpha \Big[T_{m}(x,t) - T_{a}\Big]$$
<sup>(5)</sup>

is given ( $\alpha$  is the heat transfer coefficient,  $T_a$  is the ambient temperature). For time t = 0 the initial condition

$$t = 0: \quad T(x, 0) = T_0(x), \quad T_m(x, 0) = T_{m0}(x)$$
(6)

is also known.

# 3. Experimental determination of substitute thermal capacity

To determine the course of substitute thermal capacity of cast iron the experimental researches have been realized [15]. The heat cast of hypo-eutectic grey cast iron of EN-GJL200÷EN-GJL250 class has been prepared. The charge material has been chosen according to the rules concerning the smelting of cast iron in the induction furnace. In the central part of the sampling casting the thermocouple PtRh-Pt has been installed. The thermocouple has been connected to the registering apparatus. The thermal and derivative analysis (TDA) has been done in order to determine the characteristic temperatures associated with the change transition. So, the heat processes proceeding in the solidifying metal connected with the latent heat emission of successive phases have been registered taking into account the cooling curve  $T_d(t) = T(x_d, t)$  and its time derivative  $\partial T_d(t)/\partial t$ . In Figure 1 the cooling curve at the central point of the sampling casting obtained by means of the experiment is shown.

Using the diagrams of the thermal and derivative analysis the values of temperature-dependent latent heat have been registered. Next, the substitute thermal capacity distribution for mushy zone containing the information about the austenite and eutectic phases has been described – Figure 2.



Of course, the physical condition in the form

$$\int_{T_{s}}^{T_{L}} C(T) dT = c_{P} (T_{L} - T_{s}) + L$$
(7)

must be fulfilled.



Fig. 2. Substitute thermal capacity of cast iron

So, in the case of cast iron solidification the following approximation of substitute thermal capacity can be taken into account (Figure 2, eq. (8))

$$C(T) = \begin{cases} c_{L}, & T > T_{L} \\ p_{1} = \frac{c_{L} + c_{s}}{2} + \frac{Q_{aus}}{T_{L} - T_{E}}, & T_{E} < T \le T_{L} \\ p_{2} = \frac{c_{L} + c_{s}}{2} + \frac{Q_{eu}}{T_{E} - T_{s}}, & T_{S} < T \le T_{E} \\ c_{s}, & T \le T_{s} \end{cases}$$
(8)

where  $T_E$  corresponds to the beginning of eutectic crystallization,  $Q_{aus}$ ,  $Q_{eu}$  are the latent heats connected with the austenite and eutectic phases evolution.

#### 4. An inverse problem solution

Let us the parameters appearing in the mathematical model of casting solidification are known except the segments  $p_1$ ,  $p_2$  creating the function C(T) – cf equation (8).

Additionally, it is assumed that the values  $T_{di}^{f}$  at the set of point  $x_i$  (sensor) selected from the casting-mould domain for times  $t^{f}$  are known

$$T_{di}^{f} = T_{d}(x_{i}, t^{f}), \qquad f = 1, 2, ..., F$$
(9)

To solve the inverse problem the least aquares criterion is applied [4, 15]

$$S(p_1, p_2) = \frac{1}{F} \sum_{f=1}^{F} (T_i^f - T_{di}^f)$$
(10)

where  $T_{di}^{f}$  (cf equation (9)) and  $T_{i}^{f} = T(x_{i}, t^{f})$  are the measured and estimated temperatures, respectively. The estimated temperatures are obtained from solution of the direct problem (cf chapter 1) by using the current available estimate for the unknown parameters.

In order to minimize the functional (10), the evolutionary algorithm has been used (e.g. [1, 2, 3]). Evolutionary algorithm minimizes the fitness function (functional (10)) with respect to parameters  $p_e$  [15]. A chromosome (vector) characterizes the solution

$$\mathbf{p} = \begin{bmatrix} p_1 & p_2 \end{bmatrix}^1 \tag{11}$$

where  $p_e$  are the genes containing information about the substitute thermal capacity function (cf equation (8)).

The genes are the real numbers on which constrains are imposed in the form

$$p_e^L \le p_e \le p_e^R, \quad e = 1, 2$$
 (12)

The evolutionary algorithm starts with an initial population. This population consists of *N* chromosomes  $\mathbf{p}^n$ , n = 1, 2, ..., N,

generated in random way – Figure 3. Every gene is taken from the feasible domain. For the assumed values of  $\mathbf{p}^n$ , n = 1, 2, ..., N, the direct problems described by equations (1) – (5) are solved. The next stage is an evaluation of the fitness function (10) for every chromosome  $\mathbf{p}^n$  and the selection is employed. The selection is performed in the form of ranking selection or the tournament selection [1, 2, 8, 9] and the evolutionary operators: mutation and crossover are applied. In this way the next population is created. The process is repeated until the chromosome, for which the value of the fitness function is zero, has been found or after the achieving the assumed number of populations.



Fig. 3. Population and chromosome structure

In evolutionary computations the following evolutionary operators are applied [1, 2, 8, 9, 12]:

- uniform mutation operator which changes the genes values in chromosome by choosing the new ones in random way,
- nonuniform mutation operator which changes the genes values in chromosome using the Gauss distribution, the amplitude of such mutation in each generation is equal  $\sigma = 1/pop$ , where *pop* is number of generation,
- arithmetic crossover operator which creates new chromosome with genes which are the linear combination of two randomly chosen chromosomes.
- In Figure 4 the scheme of evolutionary algorithm is presented.



Fig. 4. Flow chart of evolutionary algorithm

#### 5. Results of computations

The casting-mould system shown in Figure 5 has been considered. At first, the direct problem has been solved. The following input data have been introduced:  $\lambda_L = 20 [W/(mK)]$ ,  $\lambda_S = 40 [W/(mK)]$ ,  $\lambda_m = 0.7 [W/(mK)]$ ,  $c_L = 5.859 \cdot 10^6 [J/(m^3K)]$ ,  $c_S = 5.422 \cdot 10^6 [J/(m^3K)]$ ,  $p_1 = 5.35 \cdot 10^7 [J/(m^3K)]$ ,  $p_2 = 1.586 \cdot 10^7 [J/(m^3K)]$  (c.f. equation (8)),  $c_m = 2.85 [MJ/(m^3K)]$ , pouring temperature  $T_0 = 1342^{\circ}$ C, liquidus temperature  $T_L = 1220^{\circ}$ C, border temperature  $T_E = 1126^{\circ}$ C, solidus temperature  $T_S = 1106^{\circ}$ C, initial mould temperature  $T_{m0} = 20^{\circ}$ C. The problem has been solved using finite element method by means of the commercial package MSC MARC/MENTAT.

In Figure 6 the discretization of the domain is shown, while Figure 7 illustrates the temperature distribution for times 10, 50, 150 and 200 s. In Figure 8 the course of cooling curves at the control point from casting sub-domain obtained from direct solution (variant 1) and experiment (variant 2) are shown.

Next, the inverse problem using evolutionary algorithm has been solved. In Table 1 the parameters applied in this algorithm are collected. The results obtained for variant 1 are presented in Table 2 and for variant 2 are presented in Table 3. In Figure 9 the comparison of cooling curves at the control point obtained from experiment and evolutionary computations are shown.

#### Table 1.

Evolutionary algorithm parameters

Number of generations	200
Number of chromosomes in each population	40
Probability of uniform mutation	40%
Probability of nonuniform mutation	30%
Probability of arithmetic crossover	40%
Probability of cloning	5%

Table 2. Results of computations using the EA – variant 1

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	Design variable	$p_1$	$p_2$	
	Exact value	$1.586 \cdot 10^7$	$5.35 \cdot 10^7$	
	Found value	$1.586 \cdot 10^7$	$5.34 \cdot 10^7$	
	Error %	0	0.19	

#### Table 3.

|--|

Design variable	$p_1$	$p_2$
Exact value	$1.586 \cdot 10^7$	$5.35 \cdot 10^7$
Found value	$1.420 \cdot 10^7$	$4.84 \cdot 10^7$
Error %	10.47	9.53







Fig. 6. Discretization of the domain

In the case of calculated cooling curve application (variant 1) the results of parameters  $p_1$ ,  $p_2$  estimation are very good, what more, the parameter  $p_1$  is exactly identified.

In the case of experimental cooling curve application (variant 2) the error of identification is decidedly bigger (Figure 9). It results, first of all, from the essential differences between the experimental and calculated cooling curves at the point considered from the casting domain (cf Figure 8) especially in the second stage of solidification process, this means after 150 seconds.



Fig. 7. Temperature distribution for time 10, 50, 150 and 200 s





#### 6. Conclusions

To identify the substitute thermal capacity of cast iron the evolutionary algorithm has been applied. In the case of exact cooling curve obtained from the direct problem soltion, the results of identification are very close to the assumed values describing the course of the substitute thermal capacity. In the case of experimental cooling curve the error of identification is bigger, but acceptable.

It should be pointed out that the evolutionary algorithm is time consuming and the error of identification is greater than in the case of gradient method application, but the solution of inverse problem is always obtained.

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