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FUZZY SETS IN INTERACTIVE COMPUTER MAN-MACHINE SYSTEMS

Summary. The paper deals with some problems of the use of the fuzzy set theory e.g.: linguistic approximation, approximate reasoning in designing of an interactive man-machine systems. The basic definition of linguistic variables constituting the main feature of this kind of systems has been presented in details and illustrated by means of a numerical example. The general structure of the system has been shown and one of the implementations has been presented as well.

1. Introduction

Recently some new ideas on interactive computer systems have been published [8, 13]. The list of human-oriented software programs is extensive (e.g. ELIZA or DEACON [5]). There is no doubt that they form a useful tool for solving a wide class of problems: the designing of complex devices or decision-making in many ill-defined processes.

An introduction of fuzzy set theory and the posibility theory [7, 8] leads to a general class of interactive systems, where a qualitative kind of information can be used and performed. These theories make it possible for us to discuss a new category of models, the so called verbal models [8, 9, 13], which may be systematically analyzed by the use of the fuzzy set theory. It is the aim of this paper to present the use of fuzzy sets in human oriented computer systems, introducing a fuzzy relational equation, a linguistic approximation and the concept of approximate reasoning as a basis of the construction of this kind of systems. At the very beinning we have summarized some theoretical results of a fuzzy relational equation, and a linguistic variable.

2. Linguistic variable and linguistic approximation

A linguistic variable, forming the basis of linguistic algorithms may be defined as follows [11].

DEFINITION 1

A linguistic variable is a system:

$$< L, T(L), X, G, M >$$ (1)

where L, T(L) (or shortly T),$\mathcal{X}$,G,M denote respectively:

  L - name of the variable,
  T - set of labels of a fuzzy subset of the universe of discourse,
  $\mathcal{X}$ - universe of discourse,
  G - syntactic rules, commonly defined as generative grammar, which de-
      fine the well-formed sentences in T,
  M - semantics, which consists of rules by which the meaning of   the
      terms in T can be determined.

  If X is a term in T, then its meaning (in the denotational sense) is
a subset of $\mathcal{X}$. The primary term in T is a term, whose meaning must be
defined a priori. It serves as a basis for the computation of the meaning
of the non-primary terms in T.

  Modifiers (hedges) such as: <u>very</u>, <u>more or less</u>, <u>slightly less</u>,<u>slightly</u>
<u>more</u> play a special role in semantics M. They may be defined as follows
[11]. For a given value of a linguistic variable t ∈ T defined by the
use of the membership function:

$$\mu_t \; : \; \mathcal{X} \to [0,1] \tag{2}$$

the modifier "m" works as follows:

$$\text{modifier } t = \mu_t^m \tag{3}$$

e.g.

$$\underline{\text{very}} \; t \; \triangleq \mu_t^2$$

$$\underline{\text{more or less}} \; t \; \triangleq \mu_t^{\frac{1}{2}} \tag{4}$$

<u>slightly less</u> $t = \mu_t^{0.75}$

<u>slightly more</u> $t = \mu_t^{1.25}$

  Another definition of modifiers may be used as well [4].

Example 1

  Let us illustrate a linguistic variable: pressure, expressing basic e-
lements of Def. 1.
L - pressure,
T = $\left\{$small,middle,big,more or less,small,very small,very very small...$\right\}$
$\mathcal{X}$ = $\left\{10,20,\ldots,100\right\}$
G is a grammar:

$$G = <V,\Sigma,P,\sigma>$$

where

   $V = \{$ small,middle,big,slightly,less,very,slightly more,more or less $\}$

   $\sum = \{$ value of linguistic variable,modifier,primaty term $\}$

with the following list of productions:

P :

   <value of linguistic variable>::=<primary term>|<modifier> <primary
                       term>|<modifier> <value of linguistic variable>

   <primary term>::= small| middle | big

   <modifier> ::= very| more or less| slightly less| slightly more

   δ = <value of linguistic variable

Example 2

   Taking into account a linguistic variable pressure as   discussed   in
Ex. 1 and the primary set <u>middle</u> defined by a membership function:

X	10	20	30	40	50	60	70	80	90	100
μ_{middle}	0	.1	.3	.5	1	.5	.3	.1	0	0

the values of linguistic variables created by means of   different   modi-
fiers are given by eq. (4) and illustrated in Fig. 1.

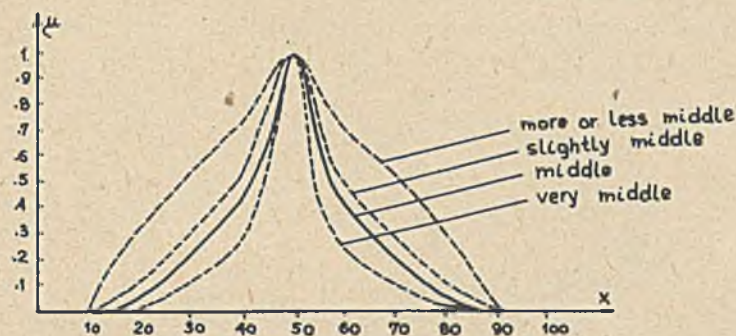

Fig. 1. Primary set <u>middle</u> and the values of linguistic variables created
                    by means of different modifiers

It is interesting to notice a characteristic feature of modifiers.   They
can be divided into two groups:

- fuzzification modifiers (m < 1, e.g. more or less)
- concentration modifiers (m > 1, e.g. very)

because the modifiers of the first group increase the degree of fuzziness
φ_t of primary sets, i.e.

$$\varphi_t < \varphi_{mt} \tag{5}$$

and using one of the second group the degree of fuzziness is decreased:

$$\varphi_t > \varphi_{mt} \tag{6}$$

where:

$$\varphi_t = \frac{1}{\underset{x \in \mathbb{X}}{Max \mu_t(x)}} \sum_{x \in \mathbb{X}} \mu_t(x) \tag{7}$$

One of the basic problems in approximate reasoning might be stated as follows:
- for a given fuzzy set with a membership function fit "best"
(in the sense of an appriopriate distance) primary sets and modifiers if the number of modifiers is given, i.e. approximating a fuzzy set by means of the best fitted value of a linguistic variable.
Let us denote:

$\left\{ t_i \right\}$ i=1.1,...,m - set of membership functions of primary fuzzy sets.

$\left\{ h_j \right\}$ j=1.2....,k - set of modifiers hedges

n - length of the chain of modifiers

μ - a given membership function

So we get the problem:

$$\underset{\substack{1 \leqslant i \leqslant m \\ 1 \leqslant j_1,\ldots,j_k \leqslant k}}{Min} \| \mu - \mu_{t_i}^{\overbrace{h_{j_1}\ldots h_{j_k}}^{n}} \| = \| \mu - \mu_{t_{i_0}}^{h_{j_{01}}\ldots h_{j_{0k}}} \| \tag{8}$$

where $\| \cdot \|$ denotes the distance.
The solution of the problem presented above needs a great amount of computation equal to:

$$m+m\cdot k+m\cdot k\cdot k+\ldots+m\cdot k^{n-1}\cdot k = m+m\cdot k+m\cdot k^2+\ldots+m\cdot k^n = m(1+k+k^2+\ldots+k^n) \tag{9}$$

see Fig. 2.
It is convenient to use a simpler, suboptimal strategy in order to find the values of the proper fuzzy set and of the modifiers. This has been illustrated in Fig. 3. This method needs the number of calculation to be equal to:

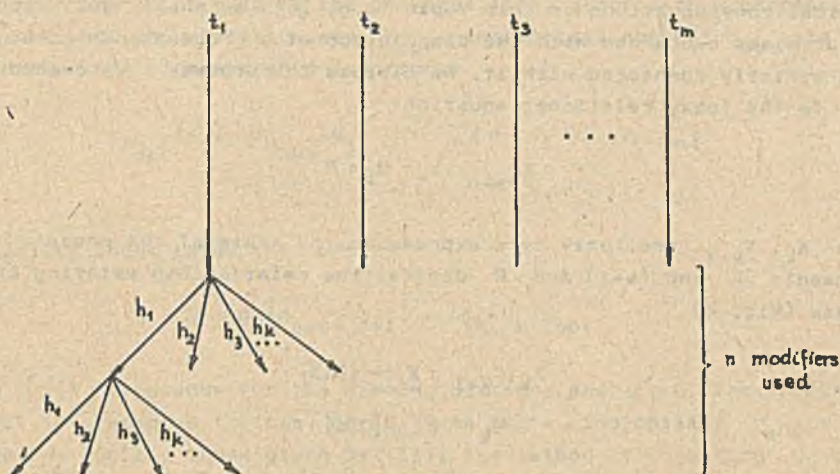$$m+m\cdot k + (n-1)\cdot k = m\cdot(1+k) + (n-1)\cdot k \tag{10}$$

Fig. 2. Illustration of linguistic approximation in the case of n modi-
fiers used

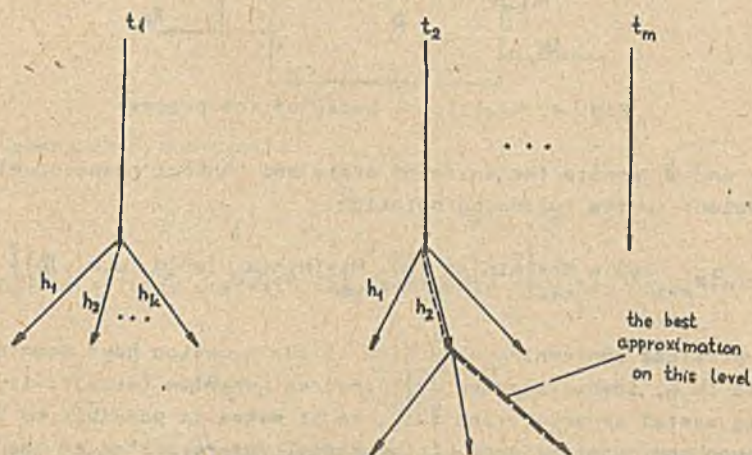Fig. 3. Illustration of modified linguistic approximation in the case of
n modifiers used

3. Fuzzy relational equations

Fuzzy relational equations form useful tool for describing complex,
ill-defined processes, where the representation of the process in catego-
ries of sets and relations is more adequate than in the categories of
points and functions. Because there is a great number of theoretical and

practical considerations on this topic (e.g. [6]) we shall   deal with the
main problems concerned with the description of a process and   some   pro-
blems strictly connected with it. We discuss the process   described   by
means fo the fuzzy relational equation:

$$X_{k+1} = X_k \circ U_k \circ R \tag{11}$$

where X_k, X_{k+1} are fuzzy sets expressing the state of the process in ti-
me moments k and (k+1) and R denotes the relationship existing in the
process (Fig. 4)

$$X_k, X_{k+1} : \mathbb{X} \longrightarrow [0,1]$$

$$U_k : \quad \mathbb{U} \longrightarrow [0,1] \tag{12}$$

$$R : \mathbb{U} \times \mathbb{X} \times \mathbb{X} \longrightarrow [0,1]$$
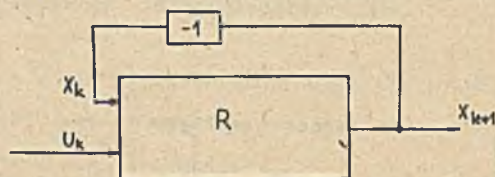

Fig. 4. Relational model of the process

where $\mathbb{X}$ and $\mathbb{U}$ denote the space of state and control respectively. Eq (11)
is equivalent to the following notation:

$$\mu_{X_{k+1}}(y) = \underset{x \in \mathbb{X}}{\text{Max}} \left\{ \text{Min} \left[\mu_{X_k}(x), \underset{u \in \mathbb{U}}{\text{Max}}(\text{Min}(\mu_{U_k}(u), \mu_R(u,x,y))) \right] \right\} \tag{13}$$

General problems concerning this kind of the equation have been discussed
in detail (e.g. identification [2], inverse   problem (sensitivity)   [1,7]
Eq. 11 is useful in prediction task, so it makes it possible to be   used
for a given and constant control to predict future states of the process.
If X_k and U are given, so using Max-Min composition we get:

$$X_{k+1} = X_k \circ U \circ R = X_k \circ G$$

$$X_{k+2} = X_{k+1} \circ U \circ R = X_{k+1} \circ G = X_k \circ G \circ G \tag{14}$$

$$\vdots$$

$$X_{k+p} = X_k \circ G \circ \dots \circ G = X_k \circ G^p$$

where p denotes the horizon of prediction.

Another important task is the one-step local control of a process. Let us assume that the optimal state X_{opt} and optimal control U_{opt} are given in the form of membership functions:

$$\mu_{X_{opt}}(x_i), \mu_{U_{opt}}(u_j) \qquad X = \left\{x_1, x_2, \dots, x_n\right\}$$
$$1 \leqslant i \leqslant n \; 1 \leqslant j \leqslant p \quad \mathbb{U} = \left\{u_1, u_2, \dots, u_p\right\}$$

The quality index takes the form:

$$Q = \rho_H(X_{opt}, X_{k+1}) + \alpha \rho_N(U_k, U_{opt}) \tag{15}$$

where $\rho_H(\cdot, \cdot)$ stands for the Hamming distance and $\alpha \geqslant 0$. The problem of control is stated as follows: For a given state find optimal U_k i.e. minimize the quality index given by (15). The method of computing U_k is as follows:

1. Compute

$$Q(\tilde{U}_j) = \rho_H(X_{opt}, X_k \circ \tilde{U}_j \circ R) + \alpha \rho_H(\tilde{U}_j, U_{opt}) \tag{16}$$
$$j=1,2,\dots,p \quad \alpha \geqslant 0$$

where

$$\tilde{U}_j : \mathbb{U} \longrightarrow [0,1]$$

with the membership function:

$$\mu_{U_j}(u_i) = \delta_{u_i, u_j} \quad i=1,2,\dots,p \tag{17}$$

2. Put U_k as a fuzzy set with the membership function equal to:

$$\mu_{U_k}(u_i) = 1 - \frac{Q(\tilde{U}_i)}{\underset{1 \leqslant i \leqslant p}{\text{Max}} Q(\tilde{U}_i)} \tag{18}$$

The considerations given above are used in the construction of an interactive system.

4. General structure of an interactive computer man-machine system

In this section we present the structure of an interactive man-machine system, which can be used in the case of processing a qualitative form of information. The main blocks of the system are depicted in the Fig. 5 Let us briefly discuss some of them.
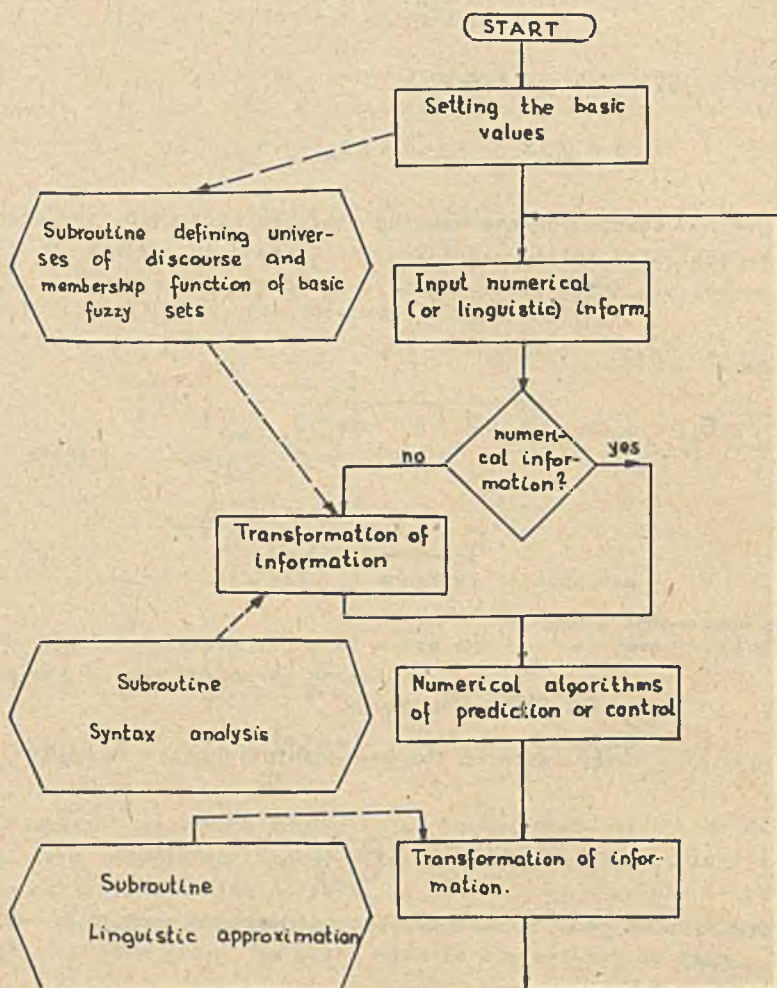
Fig. 5. General structure of an interactive man-machine system

Fig. 6. Fuzzy relation R describing the process

$$\begin{bmatrix} 1 & .5 & .3 & .2 & .1 & 0 \\ .4 & .6 & 1 & .4 & .3 & .1 \\ .3 & .4 & .7 & 1 & .4 & .3 \\ .1 & .3 & .5 & 1 & .3 & .1 \\ 0 & .3 & .4 & .8 & 1 & .9 \\ 0 & .2 & .3 & .7 & .9 & 1 \end{bmatrix} U=U_1$$

$$\begin{bmatrix} 1 & .9 & .8 & .7 & .5 & .3 \\ 1 & .8 & .5 & .2 & .1 & 0 \\ .9 & 1 & .9 & .5 & .4 & .3 \\ .9 & .9 & 1 & .7 & .3 & .1 \\ .9 & .9 & 1 & .9 & .4 & .2 \\ .1 & .2 & .5 & .6 & 1 & .5 \end{bmatrix} U=U_2$$

$$\begin{bmatrix} .3 & .5 & .9 & 1 & .8 & .7 \\ .4 & .6 & .8 & 1 & .9 & .7 \\ .3 & .5 & 1 & .6 & .3 & .2 \\ .7 & .8 & 1 & .4 & .2 & 0 \\ .9 & .9 & 1 & .8 & .5 & .2 \\ .9 & 1 & .5 & .2 & 0 & 0 \end{bmatrix} U=U_3$$

$$\begin{bmatrix} .3 & .7 & .8 & .9 & 1 & 1 \\ .5 & .8 & .9 & 1 & .2 & .2 \\ .6 & .9 & 1 & .9 & .4 & .4 \\ .5 & .8 & 1 & .6 & .3 & .1 \\ .4 & .5 & .9 & 1 & .3 & .1 \\ .4 & .7 & 1 & .3 & .1 & .1 \end{bmatrix} U=U_4$$

$$\begin{bmatrix} 1 & .4 & .3 & .2 & 0 & 0 \\ .8 & 1 & .5 & .3 & .1 & .1 \\ .4 & .5 & 1 & .2 & .2 & .1 \\ .3 & .8 & 1 & .3 & 0 & 0 \\ .7 & 1 & .8 & .5 & 0 & 0 \\ .5 & .9 & 1 & .1 & 0 & 0 \end{bmatrix} U=U_5$$

Fig. 7. Fuzzy relation R describing the process

<u>Setting of the basic values</u> - In this block the primary fuzzy sets and modifiers are defined and the spaces X and U are created.

<u>Transformation of information</u> - Input information is transformed into numerical values by means of defined fuzzy sets and modifiers. Output information can be numerical in character or form the value of a linguistic variable. In the second case the subroutine of linguistic approximation is used. The algorithms of prediction and control have been presented in the previus section. A computer programme based on the general scheme given in Fig. 5.has been realised using FORTRAN 1900 (Fig. 6). Spaces and U consist of 6 and 5 elements:

$$X = \left\{ x_1, x_2, x_3, x_4, x_5, x_6 \right\}$$
$$U = \left\{ u_1, u_2, u_3, u_4, u_5 \right\}$$

and relation R describing the process is presented in Fig. 7. Definitions of basic sets and modifiers (hedges) are given in Fig. 8. Some results obtained by means of this programme are presented in Fig. 9.

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    DEFINED BASIC SETS AND HEDGES
    --------------

    SMALL
    ------------
X1     1,000
X2     0,600
X3     0,400
X4     0,200
X5     0,000
X6   · 0,000

       MIDDLE
    ------------
X1     0,000
X2     0,300
X3     1,000
X4     0,300
X5     0,000
X6     0,000

        BIG
    ------------
X1     0,000
X2     0,200
X3     0,400
X4     0,600
X5     0,700
X6     1,000


VERY              ---2,000
    MORE OR LESS  ---0,500
```

Fig. 8. Definition of basic sets and modifiers (hedges)

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  --- ----------- ------ --
PREDICTION


  .........
  INITIAL VALUES
  ---------
  STATE OF THE PROCESS
  ---------
      X1      1.000
      X2      0.000
      X3      0.000
      X5      0.000
      X6      0.000
      LINGUISTIC APPROXIMATION     VERY              SMALL
  CONTROL OF THE PROCESS
  ---------
      U1      0.000
      U3      0.000
      U3      1.000
      U4      0.000
      U5      0.000
      PREDICTION
      .........
STEP OF PREDICTION  1
      X1      0.300
      X2      0.500
      X3      0.900




      X4      1.000
      X5      0.800
      X6      0.700
      LINGUISTIC APPROXIMATION     MORE OR LESS      BIG
STEP OF PREDICTION  2
      X1      0.800
      X2      0.800
      X3      1.000
      X4      0.800
      X5      0.500
      X6      0.500
      LINGUISTIC APPROXIMATION     MORE OR LESS      SMALL
STEP OF PREDICTION  3
      X1      0.700
      X2      0.800
      X3      1.000
      X4      0.800
      X5      0.800
      X6      0.700
      LINGUISTIC APPROXIMATION     MORE OR LESS      BIG
STEP OF PREDICTION  4
      X1      0.800
      X2      0.800
      X3      1.000
      X4      0.800
      X5      0.800
      X6      0.700
      LINGUISTIC APPROXIMATION     MORE OR LESS      BIG
STEP OF PREDICTION  5
      X1      0.800
      X2      0.800
      X3      1.000
      X4      0.800
      X5      0.800
      X6      0.700
      LINGUISTIC APPROXIMATION     MORE OR LESS      BIG
STEP OF PREDICTION  6
      X1      0.800
      X2      0.800
      X3      1.000
      X4      0.800
      X5      0.800
      X6      0.700
      LINGUISTIC APPROXIMATION     MORE OR LESS      BIG
STEP OF PREDICTION  7
      X1      0.800
      X2      0.800
      X3      1.000
      X4      0.800
      X5      0.800
      X6      0.700
      LINGUISTIC APPROXIMATION     MORE OR LESS      BIG
      CONTROL
  .........
  OPTIMAL STATE
  ---------
      X1      0.100
      X2      0.200
      X3      0.500
      X4      0.900
      X5      1.000
      X6      1.000
      OPTIMAL CONTROL
  ---------
      U1      1.000
      U2      0.400
      U3      0.200
      U4      0.000
      U5      0.000
      STATE OF THE PROCESS
  ---------
      X1      0.500
      X2      0.800
      X3      1.000
      X4      0.400
      X5      0.200
      X6      0.000
      CONTROL
  ---------
      U1      0.909




      U2      0.306
      U3      1.000
      U4      0.579
      U5      0.000
  --------------------------------
```

Fig. 9. Examples of results obtained by means of programme

5. Concluding remarks

The concept of the fuzzy set theory forms a formal frame of expressing vague, ambiguous and ill-defined concepts. A human-oriented computer system,.using the methods of these theories e.g. fuzzy relational equations, linguistic approximation, possibility theory can operate on a qualitative kind of information giving results of linguistic or numerical character, which are used in many areas, where the human factor forms an important element of the system. Otherwise, such a system can form a proper tool for linguistic simulation and for the modelling of complex, not well-defined systems.

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ZBIORY ROZMYTE W INTERAKTYWNYCH SYSTEMACH KOMPUTEROWYCH

S t r e s z c z e n i e

W pracy rozpatrzono niektóre problemy dotyczące   zastosowania   teorii
zbiorów rozmytych (np. aproksymacji lingwistycznej, rozumowania   przybli-
żonego) w projektowaniu interaktywnych systemów komputerowych. Szczegóło-
wo przedstawiono definicję i interpretację zmiennej lingwistycznej grają-
cej istotną rolę w tego rodzaju systemach, jak również   podano   przykład
numeryczny. Praca zawiera ogólną strukturę systemu wraz z jedną z moż-
liwych implementacji.

РАСПЛЫВЧАТЫЕ МНОЖЕСТВА В ИНТЕРАКТИВНЫХ КОМПЬЮТЕРНЫХ СИСТЕМАХ

Р е з ю м е

В работе рассмотрено некоторые проблемы применения теории   расплывчатых
множеств например лингвистической апроксимации, приближенного рассуждения
в проектированию интерактивных компьютерных систем. Дано точное определение
и интерпретацию лингвистической переменной играющую основную роль в этого
рода системах. Дан числовой пример. В работе представлено общую   структуру
системы с одной из возможных интерпретаций.