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ON A FUZZY LOGIC CONTROLLER DESIGN METHOD

Summary. The paper presents some problems appearing in the initial stage of design of fuzzy logic controller and formalizes the tasks which might occur here, especially such as:

- completeness of collection of control rules,
- noncompetitivity of control rules,
- interaction between control rules.

The introduced performance indices make it possible to measure completeness and interaction. Numerical examples form an illustration of discussed considerations.

1. Introduction

In many papers dealing with the problems of fuzzy control, a fuzzy logic controller based on the compositional rule of inference has been discussed, e.g. the papers of Mamdani, Tong, Assilian [1] [2], [3] and applied to the broad class of complex, ill-defined processes. Also an approach to adaptive control algorithms has been made [3].

The aim of this paper is to present some problems appearing in the initial stage of design, and to formalize some problems which might occur here, introducing some definitons on this item.

2. Some problems of designing a fuzzy controller

Essentially, a fuzzy logic controller is formed on the basis of a collection of control rules:

- if X_1 then U_1

else(or)

- if X_2 then U_2

else(or)

⋮

- if X_N then U_N,

where X_i, U_i denotes fuzzy sets of the output of the process and control defined in the spaces of $\mathbb{X}$ and $\mathbb{U}$, respectively. N denotes the number of control rules. For the purpose of computer implementation, spaces $\mathbb{X}$ and $\mathbb{U}$ are discussed as having a finite number of elements:

$$\mathcal{X} = \{X_1, X_2, \ldots, X_n\} \tag{1}$$

$$\mathbb{U} = \{U_1, U_2, \ldots, U_m\} \tag{2}$$

Thus the relation R of a controller has a matrix representation, and for every fuzzy set of the output X', U' is calculated by the use of the composite rule of inference [5]:

$$U' = X' \circ R \tag{3}$$

where

$$R = \bigcup_{i=1}^{N} (X_i \to U_i) \tag{4}$$

i.e.

$$\mu_U(u_i) = \bigvee_{j=1}^{m} (\mu_{X'}(x_j) \wedge \mu_R(x_j, u_i)) \quad \forall_{u_i \in U} \tag{5}$$

The final result U_{opt} can be computed in a well known way [1], e.g.

$$U_{opt} = \frac{\sum_{i=1}^{n} \mu_{U'}(u_i) u_i}{\sum_{i=1}^{n} \mu_{U'}(u_i)} \tag{6}$$

The first obvious condition which should be satisfied is the following one:

- the fuzzy sets X_i, $i=1,2,\ldots,n$ should "cover" all the space $\mathcal{X}$, so that it would become possible to get a fuzzy control for each $X' \in \mathcal{F}(\mathcal{X})$, i.e.

$$\bigcup_{i=1}^{N} \text{supp}\, X_i = \mathcal{X} \qquad \text{supp}\, X_i = \{x \in \mathcal{X} \mid \mu_{X_i}(x) > 0\} \tag{7}$$

or a stronger one; satisfying a logical reality:

$$\bigcup_{i=1}^{N} \text{supp}\, X_{i\alpha} = \mathcal{X} \qquad \alpha \subset (0,1) \tag{8}$$

where $X_{i\alpha}$ stands for α-cut of the fuzzy set X_i.

The next problem is the interaction of the control rules, which could be defined as follows.

Def. 1

We say that in a fuzzy logic controller there exists an interaction if the following holds true:

$$X' = X_i \Rightarrow U' = X' \circ R \neq U_i \tag{9}$$

It might be interesting to notice that such an interaction exists in the classical controller (PID or PI), as a consequence of implementation, here it is a logical interaction.

Let us prove the following.

Theorem 1

If $\operatorname{supp} X_i \cap \operatorname{supp} X_j = \Phi (i \neq j)$ and U_i are normal, then there is no interaction.

Proof.

Let

$$X' = X_{i_o} \qquad 1 \leq i_o \leq N$$

We get

$$\mu_{U'}(u_j) = \bigvee_{i=1}^{m} \left[\mu_{X_{i_o}}(x_i) \wedge \mu_R(x_i, u_j)\right] =$$

$$= \bigvee_{i=1}^{m} \left\{\mu_{X_{i_o}}(x_i) \wedge \left[\bigvee_{k=1}^{N} (\mu_{X_k}(x_i) \wedge \mu_{U_k}(u_j))\right]\right\} =$$

$$= \bigvee_{i=1}^{m} \left\{\mu_{X_{i_o}}(x_i) \wedge \left[\bigvee_{\substack{k=1 \\ k \neq i_o}}^{N} (\mu_{X_k}(x_i) \wedge \mu_{U_k}(u_j)) \vee (\mu_{X_{i_o}}(x_i) \wedge \mu_{U_{i_o}}(u_j))\right]\right\} = \tag{10}$$

Hence:

$$\mu_{U'}(u_j) = \bigvee_{i=1}^{m} (\mu_{X_{i_o}}(x_i) \wedge \mu_{U_{i_o}}(u_j)) = \mu_{U_{i_o}}(u_j) \tag{11}$$

i.e.

$$X_{i_o} \circ R = U_{i_o}$$

We introduce the following measure of interactivity of the control rules of a fuzzy controller.

Def. 2

We define the degree of interactivity of the control rules as follows:

$$d_{int} = \max_{1 \leq i \leq N} \frac{\varphi_{X_i \circ R}}{\varphi_{U_i}} = \max_{1 \leq i \leq N} \frac{\varphi_{U'_i}}{\varphi_{U_i}} \tag{12}$$

where,

$$\varphi_A = \frac{1}{\max\limits_{1 \leq i \leq m} \mu_A(x_i)} \sum_{i=1}^{m} \mu_A(x_i) \tag{13}$$

In a fuzzy logic controller the following form of implication could be used:

$$R_i = X_i \rightarrow U_i = X_i \times U_i \tag{14}$$

Another definitions of implication have been used as well [5]:

$$R'_i = X_i \rightarrow U_i = [X_i \cap U_i] \cup \bar{X}_i \tag{15}$$

$$R''_i = X_i \rightarrow U_i = U_i \times \bar{X}_i \tag{16}$$

It might be useful to be proved:

Theorem 2

For fuzzy implications (14), (15) the following holds true:

$$R \subseteq R' \tag{17}$$

Thus the degree of interaction attains a lower value, if the fuzzy implication given by eq. (14) is used.

The following example presents the values of the degree of interaction of the control rules:

$$X_1 = [1 \;\; .8 \;\; .3 \;\; .2 \;\; .1] \qquad U_1 = [1 \;\; .7 \;\; .5 \;\; .1 \;\; 0]$$

$$X_2 = [.6 \;\; .7 \;\; 1 \;\; .7 \;\; .6] \qquad U_2 = [.8 \;\; 1 \;\; .3 \;\; .2 \;\; 0]$$

Hence the values of the degree of interaction are the following ones:

for (14) $d_{int} = 1.6$

for (15) $d_{int} = 1.7$

for (16) $d_{int} = 2.$

Because of the fact that control rules are created by the human operator or group of experts, they may be formed with tespect to the different criteria of optimality (e.g. high degree of accuraccy, low consumption of control energy) sometimes competitive in their sense. Thus the method of investigation of the degree of competitivity of each rule may be useful. We introduce:

Def. 3

We call a set of rules a noncompetitive one, if the following holds true:

$$\varphi_{X_i \cap X_j} < \delta \Rightarrow \varphi_{U_i \cap U_j} < \varepsilon \qquad \underset{i,j=1,2,\dots,N}{\forall} \quad \varepsilon, \delta \geq 0 \quad (18)$$

or

$$\varphi_{X_i \cap X_j} < \varphi_{X_k \cap X_m} \Rightarrow \varphi_{U_i \cap U_j} < \varphi_{U_k \cap U_m} \qquad \underset{i,j=1,2,\dots,N}{\forall} \quad (19)$$

i.e. the order in the space $\mathbb{X}$ is preserved in the space $\mathbb{U}$.

Let us present a numerical example, The control rules are as follows.

	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8
X_1	1	.8	.5	.2	0	0	0	0
X_2	.2	.3	.6	1	.4	.1	0	0
X_3	0	0	0	.1	.3	.5	.8	1

	u_1	u_2	u_3	u_4	u_5	u_6	u_7	u_8	u_9
U_1	0	0	0	0	0	.2	.3	.7	1
U_2	0	.1	.2	.5	1	.5	.3	.2	0
U_3	1	.6	.3	.1	0	0	0	0	0
U_3	0	0	.1	.3	.6	1	.4	.3	.2

Hence we get:

$$\varphi_{X_1 \cap X_2} = 1.2 \quad \varphi_{U_1 \cap U_2} = .7$$

$$\varphi_{X_1 \cap X_3} = .1 \quad \varphi_{U_1 \cap U_3} = 0$$

$$\varphi_{X_2 \cap X_3} = .5 \quad \varphi_{U_2 \cap U_3} = .4$$

so the rules $X_1 \to U_1$, $X_2 \to U_2$, $X_3 \to U_3$ are noncompetitive. If the rule $X_3 \to U_3$ is replaced by $X_3 \to U_3^*$, then the set of rules $X_1 \to U_1$ $X_1 \to U_2$ $X_3 \to U_3$ is a competitive one ($\varphi_{U_1 \cap U_2} = .7$, $\varphi_{U_1 \cap U_3^*} = 1$, $\varphi_{U_2 \cap U_3^*} = 2$).

The satisfaction of def. 3 for each collection of control rules makes it possible to investigate their competivity and to eliminate a competitive ones, or to share them with respect to competitive criteria. If there exist different competitive sets of rules, the fuzzy matrix of the controller is created as follows:

$$R = \bigcap_{p=1}^{N_o} \bigcup_{i=1}^{n_p} (X_{ip} \to U_{ip}) \tag{20}$$

where X_{ip}, U_{ip} denotes the i-th rule in p-th group and N_o is equal to the number of groups:

$$n = \sum_{p=1}^{N_o} n_p \tag{21}$$

3. Conclusions

The problem dealt here may be important to assure a formal validity of a collection of control rules. Some indices of performance can form a tool for investigations of the:

- completeness of the control rules,
- their noncompetivity,
- the degree of their interactions.

The paper gives only a formal approach to the design of a controller from the logical point of view (without discussing implementation problems [4]). Further investigations in this range could extend the knowledge on fuzzy controllers and work out an optimal way of constructing algorithms based on vague, fuzzy, linguistic statements.

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METODA PROJEKTOWANIA REGULATORA ROZMYTEGO

S t r e s z c z e n i e

W pracy przedstawiono niektóre problemy, które mogą wystąpić na początkowym etapie projektowania regulatora rozmytego oraz dokonano formalnego opisu zadań, które mogą się tu pojawić, takich jak:

- zupełność,
- niesprzeczność,
- interakcja,

reguł stosowania. Jednocześnie wprowadzono odpowiednie wskaźniki jakości. Przykłady numeryczne tworzą ilustrację przeprowadzonych rozważań.

МЕТОД РРОЕКТИРОВАНИЯ РАСПЛЫВЧАТОГО РЕГУЛЯТОРА

Р е з ю м е

В работе даны некоторые проблемы, которые могут выступать на этапе проектирования расплывчатого регулятора. Представлено формулировку описания задач, которые могут здесь возникнуть. Одновременно введено соответственные показатели качества. Числовые примеры илюстритуют проведённые рассуждения.