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# Optimising network flow for cost- and value-efficient operation of the supplier-to-foundry system

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## Abstract

Skillful control of a network flow, which creates a real bridge between the supplier and user, is one of the most important conditions for cost-efficient operation of an enterprise, foundry shop included. This paper describes modern principles of the network optimising for better distribution of the moulding sand, using modern methods of operational research and commonly available Excel calculation sheet equipped with an optimising tool called Solver.

**Keywords:** computer-aided production, network flow, optimising, logistics

## 1. Introduction

Contrary to the traditional system of materials handling, some examples of which have been discussed by the authors in former works [1,2], the problems related with network flows (which are a generalised form of various tasks that the handling system usually faces) admit the presence in a network, besides nodes $A^+$ (supplier) and $A^-$ (user), of nodes designated as $A^0$, which are considered transit or reloading nodes. In problems of the network flow one can quite successfully abandon the idea of some restrictions imposed by the presence (or absence) of the paths (arcs) joining individual nodes. One can also (or should rather in some cases) take additionally into account some constraints on flow capacity (flow rate) of the individual arcs (paths), this referring to the constraints on both minimum flow capacity (the, so called, "lower bounds") and maximum flow capacity (the, so called, "upper bounds ").

## 2. Methodology

The terminology used in network flows is most frequently that used in applied hydraulics. And so, the following terms generally apply:

- nodes representing the suppliers - these are the *sources*,
- nodes representing the users - these are the *sinks*,
- the size of the supply which the suppliers are capable of offering - these are the *source potentials* (which means that, according to the designations adopted previously, $a_i$ is the potential of an $i$-th source),
- the size of the demand as expressed by the users - these are the *sink potentials* (which means that, according to the designations adopted previously, $b_j$ is the potential of a $j$-th sink),
- the specific transport operations are the *flows*; the transit (reloading) node is the node in which the inflowing stream of liquid (in the case under discussion this will be the stream of moulding sand) will equal the outflowing stream of liquid.

When the individual constraints are determined for the individual nodes, all three types of the nodes, i.e. the *source, the sink,* and the *transit points*, can be treated in the same way, applying the inequality stated below:

*outflowing stream –inflowing stream* $\leq$ *node potential* (1)

on condition that the *source* potential is positive, the *sink* potential is negative, and that of the *transit point* is equal to zero.

As a first example, Figure 1 shows the *source* (of number k = 1) and the structure of respective constraints, assuming that *k* denotes the node number.

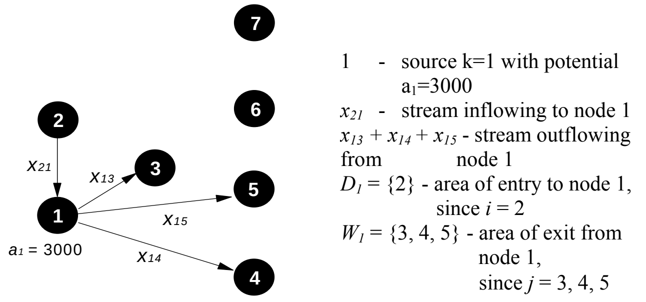



Fig. 1. Example of *source* with streams, potential and description

The general constraint resulting from inequality (1) assumes for the *source* the following form:

*outflowing stream - inflowing stream* $\leq$ *source potential* (2)

or otherwise:

$$\sum_{j \in W_k} x_{kj} - \sum_{i \in D_k} x_{ik} \leq a_k \tag{3}$$

which for the described *source* node k = 1 is cosistent with the inequality:

$$x_{13} + x_{14} + x_{15} - x_{21} \leq 3000 \tag{4}$$

As a second example, Figure 2 shows the *sink* (of number k = 5), discussed along with the structure of respective constraints.

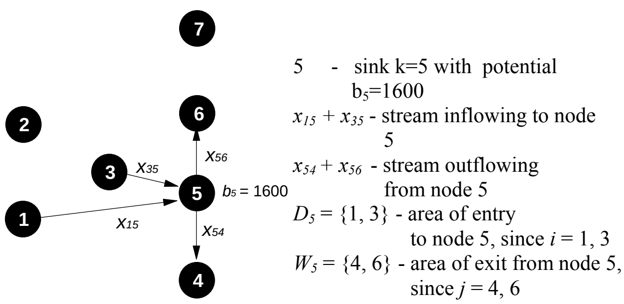



Fig. 2. Example of *sink* with streams, potential and description

The general constraint resulting from inequality (1) assumes for the *sink* the following form:

*outflowing stream – inflowing stream* $\leq$ *sink potential* (5)

or otherwise:

$$\sum_{j \in W_k} x_{kj} - \sum_{i \in D_k} x_{ik} \leq -b_k$$

which for the described *sink* node k=5 is consistent with the inequality:

$$x_{54} + x_{56} - (x_{15} + x_{35}) \leq -1600 \tag{6}$$

or after transformation:

$$x_{15} + x_{35} - x_{54} - x_{56} \geq 1600 \tag{7}$$

As mentioned previously, in *transit* (*reloading*) node, the stream of inflowing liquid equals the stream of outflowing liquid. An example of the transit node is shown in Figure 3.

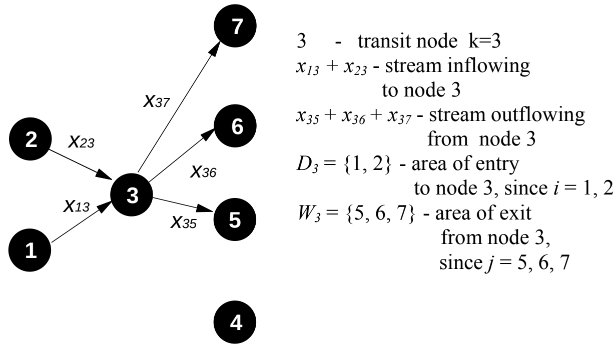



Fig. 3. Example of *transit* node with streams and description

Balancing the inflowing and outflowing streams in *transit* node according to the inequality (1):

*outflowing stream – inflowing stream = 0* (8)

can be easily described by a mathematical model

$$\sum_{j \in W_k} x_{kj} - \sum_{i \in D_k} x_{ik} = 0 \tag{9}$$

which for the *transit* node k = 3 is consistent with the following inequality:

$$x_{35} + x_{36} + x_{37} - (x_{13} + x_{23}) = 0 \tag{10}$$

Figure 4 shows the structure of the whole network used for supply of the moulding sand, including all constraints, where the number of constraints corresponds to the number of the network nodes. The *source* nodes – i.e. the suppliers, are the silica sand mines, denoted by nodes 1 and 2, while *sinks,* i.e. the users, are foundries denoted by nodes 4, 5, 6 and 7.

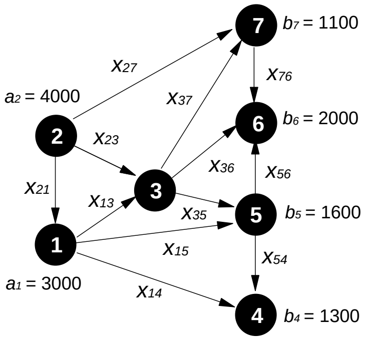



Fig. 4. The network of silica sand transportation system with constraints on respective nodes

It can be additionally stated that, besides the constraints on network nodes given in Figure 4, there are still the constraints operating on the network arcs, since on each of these arcs the flow should have a value positive or equal to zero, which means that:

$$x_{ij} \geq 0 \tag{11}$$

Detailed analysis of the network flow shows that some flow quantities can be restricted on a given arc ($i,j$) by "lower bounds" ($d_{ij}$) or by "upper bounds" ($g_{ij}$), which ultimately gives:

$$d_{ij} \leq x_{ij} \leq g_{ij}. \tag{12}$$

All values of $x_{ij}$, which are the solution for given constraints on the nodes, will make the, so called, acceptable flow, satisfying the demand of users (*sinks*) within the currently existing supply capacity of the suppliers (*sources*) and the available transport means. It should be added that, compared with the conventional transport means, where a solution (i.e. an acceptable flow) always exists, in the network flow it may happen so that the acceptable flow will be non-existent (which means that no solution can be found).

Within the determined (existing) acceptable network flows, two main problems are examined:

- for which values of the acceptable flow one can obtain the lowest cost of flow $K_p$, with data available on the unit cost of flow $h_{ij}$ for individual arcs of the network ($i,j$) belonging to the set of network arcs Q. Since total cost of flow is a sum of the products of the unit costs $h_{ij}$ and the corresponding flow capacities $x_{ij}$ (decision variables) on individual network arcs ($i,j$), an optimum solution will be obtained through the task of linear optimising (under given conditions (2), (4), (8), (11) and (12)):

$$K_p = \sum_{i,j \in Q} h_{ij} x_{kj} \rightarrow min, \tag{13}$$

- for which values of the acceptable flow, the highest value of flow $W_p$ can be obtained, understood as a bulk mass of goods transported from the supplier (*source*) to the user (*sink*). An optimum solution will be obtained through the task of linear optimising (under given conditions (2), (4), (8), (11) and (12)):

$$Wp = \sum_{k \in A^+} \left( \sum_{j \in W_k} x_{kj} - \sum_{i \in D_k} x_{ik} \right) \rightarrow max. \tag{14}$$

It is worth noting that the value of flow is a sum of all the left members of the constraints on *source*. This means that for the network illustrated in Figure 4 one can obtain:

$$\begin{aligned} W_p &= x_{13} + x_{14} + x_{15} - x_{21} + x_{23} + x_{27} + x_{21} = \\ &= x_{13} + x_{14} + x_{15} + x_{23} + x_{27} \end{aligned} \tag{15}$$

# 3. The results

Below, an optimising task has been performed for a flow in a given network of the structure as shown in Figure 4 to obtain a flow of the lowest cost and maximum capacity, allowing for the size of supply expressed in tons (that is, the *source potential*), the size of demand expressed in tons (that is, the *sink potential*) and the unit cost of flow $h_{ij}$ amounting to:

- on arc 13 - 25 PLN/ton, ❍ on arc 14 - 55 PLN/ton
- on arc 15 - 34 PLN/ton, ❍ on arc 21 - 42 PLN/ton
- on arc 23 - 19 PLN/ton, ❍ on arc 27 - 29 PLN/ton
- on arc 35 - 38 PLN/ton, ❍ on arc 36 - 26 PLN/ton
- on arc 37 - 11 PLN/ton, ❍ on arc 54 - 51 PLN/ton
- on arc 56 - 48 PLN/ton, ❍ on arc 76 - 41 PLN/ton

After careful analysis of the examined network, the following constraints were taken into account:

- on arc 13 the flow should not go below 500 tons and above 700 tons,
- on arc 15 the flow should not go below 300 tons,
- on arc 23 the flow should not go above 600 tons.

With these data taken into account, the following constraints were obtained:

(W1) $x_{13} + x_{14} + x_{15} - x_{21}$ □ 3000 (the constraint on potential - that is, the supply of node 1)

(W2) $x_{23} + x_{27} + x_{21}$ □ 4000 (the constraint on potential - that is, the supply of node 2)

(W3) $x_{35} + x_{36} + x_{37} - x_{13} - x_{23} = 0$ (the constraint on potential of transit node 3)

(W4) $x_{14} + x_{54}$ □ 1300 (the constraint on potential - that is, the demand of node 4)

(W5) $x_{15} + x_{35} - x_{54} - x_{56}$ □ 1600 (the constraint on potential - that is, the demand of node 5)

(W6) $x_{36} + x_{56} + x_{76}$ □ 2000 (the constraint on potential - that is, the demand of node 6)

(W7) $x_{27} + x_{37} - x_{76}$ □ 1100 (the constraint on potential - that is, the demand of node 7)

(W8) $x_{13}$ □ 500 (the constraint on lower limit of the flow capacity on arc of index 13)

(W9) $x_{13}$ □ 700 (the constraint on upper limit of the flow capacity on arc of index 13)

(W10) $x_{15}$ □ 300 (the constraint on lower limit of the flow capacity on arc of index 15)

(W11) $x_{23} \leq 600$ (the constraint on upper limit of the flow capacity on arc of index 15).

After formułation of the above constraints, one can proceed to the creation of a new sheet, where in the block of cells C5:C16 it is necessary to enter the values of the unit cost of flows on given arcs of the network, the indeces of which have been entered to the block of cells A5:A16 (Fig. 5).

| | A | B | C | D | E |
|---|---|---|---|---|---|
| 1 | Przepływ w sieci | | | | |
| 3 | Łuk: | Wielkość przepływu | Koszt jedn. | Ograniczenia (przepustowość) | |
| 4 | indeks | na łuku | przepływu | Dolna granica | Górna granica |
| 5 | Łuk: 13 | 0 | 25 | 500 | 700 |
| 6 | Łuk: 14 | 0 | 55 | | |
| 7 | Łuk: 15 | 0 | 34 | 300 | |
| 8 | Łuk: 21 | 0 | 42 | | |
| 9 | Łuk: 23 | 0 | 19 | | 600 |
| 10 | Łuk: 27 | 0 | 29 | | |
| 11 | Łuk: 35 | 0 | 38 | | |
| 12 | Łuk: 36 | 0 | 26 | | |
| 13 | Łuk: 37 | 0 | 11 | | |
| 14 | Łuk: 54 | 0 | 51 | | |
| 15 | Łuk: 56 | 0 | 48 | | |
| 16 | Łuk: 76 | 0 | 41 | | |

Fig.5. Fragment of the sheet with data on the cost of flow and constraints on flow capacity

To cells D5 and E5 were entered the values of the lower and upper flow capacity limit for node of index 13, to cell D7 was entered the value of the lower flow capacity limit for arc of index 15, while to cell E9 was entered the value of the uper flow capacity limit for arc of index 23.

At the next stage were introduced the data on the size of potentials in individual nodes. To the block of cells G5:G11 were introduced the numbers of the nodes along with a description of their type (Fig. 6), the size of supply of source nodes (to block I5:I6), and the size of demand of sink nodes (to block J8:J11).

| | G | H | I | J |
|---|---|---|---|---|
| 3 | Numer węzła | Lewa strona | Wielkość | Wielkość |
| 4 | (rodzaj) | warunku | podaży | popytu |
| 5 | 1 (źródło) | 0 | 3000 | |
| 6 | 2 (źródło) | 0 | 4000 | |
| 7 | 3 (tranzyt) | 0 | | |
| 8 | 4 (ujście) | 0 | | 1300 |
| 9 | 5 (ujście) | 0 | | 1600 |
| 10 | 6 (ujście) | 0 | | 2000 |
| 11 | 7 (ujście) | 0 | | 1100 |
| 13 | Koszt przepływu: | 0 | | |
| 14 | | =SUMA.ILOCZYNÓW(B5:B16;C5:C16) | | |
| 15 | Wielkość przepływu: | 0 | | |
| 16 | | =H5+H6 | | |

Fig.6. Constraints and objective functions defined for the problems of a network flow

To the block of cells H5:H11 enter the *formulae* defining the left members of the first seven constraints (W1:W7). And thus:

- to *cell* H5 enter *formula* defining the left member of constraint (W1), that is: =B5+B6+B7-B8,
- to *cell* H6 enter *formula* defining the left member of constraint (W2), that is: =B9+B10+B8,
- to *cell* H7 enter *formula* defining the left member of constraint (W3), that is: =B11+B12+B13-B5-B9,
- to *cell* H8 enter *formula* defining the left member of constraint (W4), that is: =B6+B14,
- to *cell* H9 enter *formula* defining the left member of constraint (W5), that is: =B7+B11-B14-B15,
- to *cell* H10 enter *formula* defining the left member of constraint (W6), that is: =B12+B15+B16,
- to *cell* H11 enter *formula* defining the left member of constraint (W7), that is: =B10+B13-B16.

In the solution of a network task, two main objective functions were applied. The first function minimalised the overall cost of flow $K_p$ and according to formula (13) to cell H13 the formula *Sum of products =SUMA.ILOCZYNÓW*(B5:B16; C5:C16) was entered, while the task of the second function was to maximise the flow capacity $W_p$ *,* and according to relationships (14) and (15) to cell H15 the formula =H5+H6 was entered.

As a first step, the transportation capacity was optimised to minimise the overall cost of flow $K_p$. To achieve this goal, a *Solver* module was operated, a cell with the objective function was selected (cell H13), the type of optimising procedure was established (Min), and addresses of the decision variables were defined (block of cells B5:B16) along with the respective constraints (Fig. 7).

After filling in the dialogue window *Solver-Parameters* - *Solver-Parametry* make active the press key *Options* - *Opcje* and in dialogue window *Solver-Options* - *Solver-Opcje* declare the non-negative character of the decision variables (*Accept Non-negative Variables* - *Przyjmij nieujemne*) and select the linear model (*Accept Linear Model* - *Przyjmij model liniowy*), return to dialogue window *Solver-Parameters* - *Solver-Parametry* and make active option *Solve* - *Rozwiąż*, which will start up the task solving process.

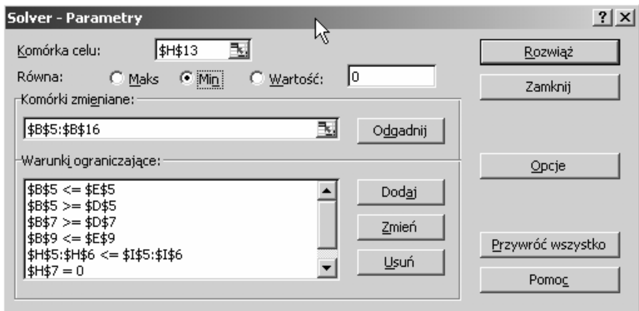
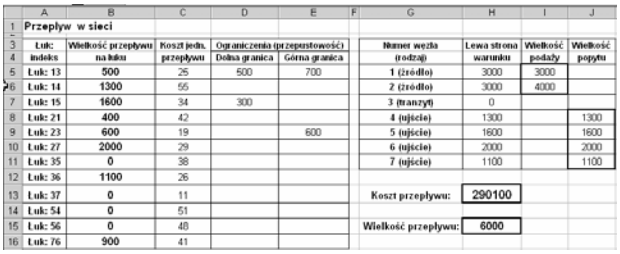



Fig. 7. Filled in dialogue window *Solver-Parameters* - *Solver-Parametry* for optimum solution rendering minimum flow cost

The outcome is an optimum solution (Fig. 8), which entered into the network is shown in Fig. 9.



Przepływ w sieci

| Łuk: indeks | Wielkość przepływu na łuku | Koszt jedn. przepływu | Ograniczenia (przepustowość) Dolna granica | Górna granica | Numer węzła (rodzaj) | Lewa strona warunku | Wielkość podaży | Wielkość popytu |
|---|---|---|---|---|---|---|---|---|
| Łuk: 13 | 500 | 25 | 500 | 700 | 1 (źródło) | 3000 | 3000 | |
| Łuk: 14 | 1300 | 55 | | | 2 (źródło) | 3000 | 4000 | |
| Łuk: 15 | 1600 | 34 | 300 | | 3 (tranzyt) | 0 | | |
| Łuk: 21 | 400 | 42 | | | 4 (ujście) | 1300 | | 1300 |
| Łuk: 23 | 600 | 19 | | 600 | 5 (ujście) | 1600 | | 1600 |
| Łuk: 27 | 2000 | 29 | | | 6 (ujście) | 2000 | | 2000 |
| Łuk: 35 | 0 | 38 | | | 7 (ujście) | 1100 | | 1100 |
| Łuk: 36 | 1100 | 26 | | | | | | |
| Łuk: 37 | 0 | 11 | | | Koszt przepływu: | 290100 | | |
| Łuk: 54 | 0 | 51 | | | | | | |
| Łuk: 56 | 0 | 48 | | | Wielkość przepływu: | 6000 | | |
| Łuk: 76 | 900 | 41 | | | | | | |

Fig. 8. Optimum solution rendering minimum flow cost

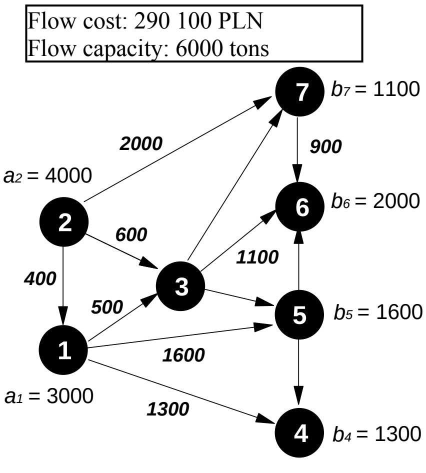



Fig.9. The network with flows rendering minimum overall cost

At the next stage, the size of the moulding sand transportation system can be optimised to obtain maximum flow capacity $W_p$. To achieve this goal, operating on module *Solver,* assume that the cell with objective function is cell H15, and its value is a maximum value (Maks) (Fig.10). The definitions of the addresses of the decision variables and of the constraints are the same as in the case of optimising done previously on the flow cost.

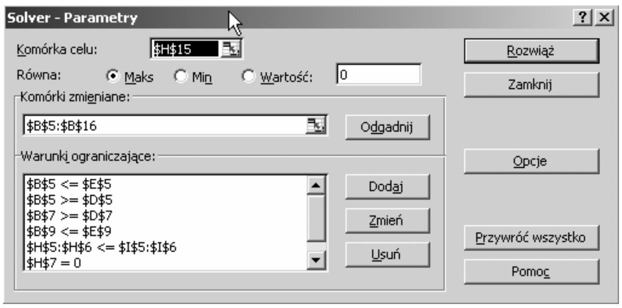
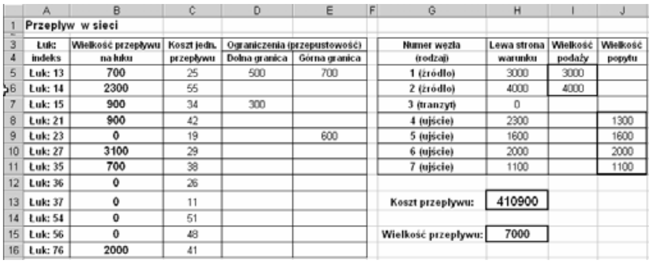



Fig. 10. Filled in dialogue window *Solver-Parameters - Solver-Parametry* for optimum solution rendering maximum flow capacity

The obtained optimum solution rendering maximum flow capacity in the network is shown in Fig. 11.



Przepływ w sieci

| Łuk: indeks | Wielkość przepływu na łuku | Koszt jedn. przepływu | Ograniczenia (przepustowość) Dolna granica | Górna granica | Numer węzła (rodzaj) | Lewa strona warunku | Wielkość podaży | Wielkość popytu |
|---|---|---|---|---|---|---|---|---|
| Łuk: 13 | 700 | 25 | 500 | 700 | 1 (źródło) | 3000 | 3000 | |
| Łuk: 14 | 2300 | 55 | | | 2 (źródło) | 4000 | 4000 | |
| Łuk: 15 | 900 | 34 | 300 | | 3 (tranzyt) | 0 | | |
| Łuk: 21 | 900 | 42 | | | 4 (ujście) | 2300 | | 1300 |
| Łuk: 23 | 0 | 19 | | 600 | 5 (ujście) | 1600 | | 1600 |
| Łuk: 27 | 3100 | 29 | | | 6 (ujście) | 2000 | | 2000 |
| Łuk: 35 | 700 | 38 | | | 7 (ujście) | 1100 | | 1100 |
| Łuk: 36 | 0 | 26 | | | | | | |
| Łuk: 37 | 0 | 11 | | | Koszt przepływu: | 410900 | | |
| Łuk: 54 | 0 | 51 | | | | | | |
| Łuk: 56 | 0 | 48 | | | Wielkość przepływu: | 7000 | | |
| Łuk: 76 | 2000 | 41 | | | | | | |

Fig. 11. Optimum solution rendering maximum flow capacity

It is easy to note that the proposed optimum solution (Fig.12) differs quite considerably from the solution optimising flow cost (Fig. 9) as regards the obtained flow capacities admissible on given arcs of the network.

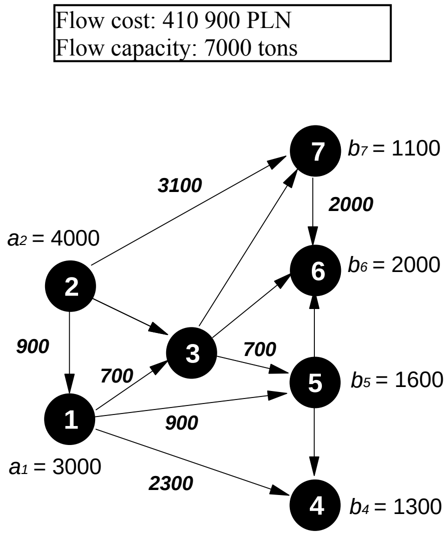



Fig.12. The network with flows rendering maximum capacity

Often it happens so that the constraints on flow capacity (usually of an "upper" character) affect also other nodes in the network. For example, if in the problem solved now, the transit node no. 3 had the upper flow capacity limit reduced to a value of 600 tons, the solutions obtained previously would not apply. So, to solve this problem, it would be necessary to add to the existing constraints still another constraint, namely $x_{13}+x_{23} \leq 600$ and re-solve the task again. The left member of this constraint is placed in cell H12, and the whole constraint is added to constraints present in the dialogue *window Solver - Parameters - Solver-Parametry*.

Figure 13 shows an optimum solution obtained after adding the constraint on flow capacity in node no. 3 to the upper limit of 600 tons rendering minimum flow cost, while Figure 14 shows an optimum solution rendering maximum flow capacity. Comparing now the obtained optimum solutions with the solutions which do not allow for the constraint on flow capacity in node no. 3, it is easy to note that the structure of flow has been preserved but the capacity of flows on individual arcs has changed.

| | A | B | C | D | E | F | G | H | I | J |
|---|---|---|---|---|---|---|---|---|---|---|
| 1 | Przepływ w sieci | | | | | | | | | |
| 2 | Dodany warunek ograniczający przepustowość dla węzła 3 | | | | | | | | | |
| 3 | Łuk: | Wielkość przepływu | Koszt jedn. | Ograniczenia (przepustowość) | | | Numer węzła | Lewa strona | Wielkość | Wielkość |
| 4 | indeks | na łuku | przepływu | Dolna granica | Górna granica | | (rodzaj) | warunku | podaży | popytu |
| 5 | Łuk: 13 | 500 | 25 | 500 | 700 | | 1 (źródło) | 3000 | 3000 | |
| 6 | Łuk: 14 | 1300 | 55 | | | | 2 (źródło) | 3000 | 4000 | |
| 7 | Łuk: 15 | 1600 | 34 | 300 | | | 3 (tranzyt) | 0 | | |
| 8 | Łuk: 21 | 400 | 42 | | | | 4 (ujście) | 1300 | | 1300 |
| 9 | Łuk: 23 | 100 | 19 | | 600 | | 5 (ujście) | 1600 | | 1600 |
| 10 | Łuk: 27 | 2500 | 29 | | | | 6 (ujście) | 2000 | | 2000 |
| 11 | Łuk: 35 | 0 | 38 | | | | 7 (ujście) | 1100 | | 1100 |
| 12 | Łuk: 36 | 600 | 26 | | | | 3 (tranzyt) | 600 | 600 | |
| 13 | Łuk: 37 | 0 | 11 | | | | Koszt przepływu: | 302600 | | |
| 14 | Łuk: 54 | 0 | 51 | | | | | | | |
| 15 | Łuk: 56 | 0 | 48 | | | | Wielkość przepływu: | 6000 | | |
| 16 | Łuk: 76 | 1400 | 41 | | | | | | | |

Fig. 13. Optimum solution rendering minimum flow cost allowing for an additional constraint on flow capacity limit

| | A | B | C | D | E | F | G | H | I | J |
|---|---|---|---|---|---|---|---|---|---|---|
| 1 | Przepływ w sieci | | | | | | | | | |
| 2 | Dodany warunek ograniczający przepustowość dla węzła 3 | | | | | | | | | |
| 3 | Łuk: | Wielkość przepływu | Koszt jedn. | Ograniczenia (przepustowość) | | | Numer węzła | Lewa strona | Wielkość | Wielkość |
| 4 | indeks | na łuku | przepływu | Dolna granica | Górna granica | | (rodzaj) | warunku | podaży | popytu |
| 5 | Łuk: 13 | 600 | 25 | 500 | 700 | | 1 (źródło) | 3000 | 3000 | |
| 6 | Łuk: 14 | 2300 | 55 | | | | 2 (źródło) | 4000 | 4000 | |
| 7 | Łuk: 15 | 1000 | 34 | 300 | | | 3 (tranzyt) | 0 | | |
| 8 | Łuk: 21 | 900 | 42 | | | | 4 (ujście) | 2300 | | 1300 |
| 9 | Łuk: 23 | 0 | 19 | | 600 | | 5 (ujście) | 1600 | | 1600 |
| 10 | Łuk: 27 | 3100 | 29 | | | | 6 (ujście) | 2000 | | 2000 |
| 11 | Łuk: 35 | 600 | 38 | | | | 7 (ujście) | 1100 | | 1100 |
| 12 | Łuk: 36 | 0 | 26 | | | | 3 (tranzyt) | 600 | 600 | |
| 13 | Łuk: 37 | 0 | 11 | | | | Koszt przepływu: | 408000 | | |
| 14 | Łuk: 54 | 0 | 51 | | | | | | | |
| 15 | Łuk: 56 | 0 | 48 | | | | Wielkość przepływu: | 7000 | | |
| 16 | Łuk: 76 | 2000 | 41 | | | | | | | |

Fig. 14. Optimum solution rendering minimum flow cost allowing for an additional constraint on flow capacity limit

As we can see, after introducing the additional constraint on flow capacity limit, another optimum solution is obtained for an optimum (minimum) flow cost, which at present amounts to 302 600 PLN (Fig. 13).

# 4. Summary

Using a linear programming system with simplex optimising is a rational method aiding optimum decisions to solve the problems of a network flow, which include various processes of the moulding materials supply to foundries. The optimising tasks of a network flow can be successfully solved on an Excel calculation sheet with the built-in Solver tool, provided the task constraints and objective functions have been properly defined.

Another approach to the problem of constraints on flow capacity limit for individual nodes is introducing additional arcs representing these nodes and adopting the flow capacity constraints on these arcs.

By solving the problems of a network flow it is also possible to allow for the flow increase or decrease on the arcs of a network.

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