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SWITCHING NETWORKS WITH AUTOMATIC ERROR DETECTION

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# SWITCEING NETWORKS WITH AUTOMATIC ERROR DETECTION 

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#### Abstract

This work deals with the class of automata, which after detecting an error in its operation cannot return to the set of states in which it could be if it were not for the error. Those automata do not need specially designed checking devices for detecting and storing errors. Theorems on existence and examples of synthesis of such automata are given.


In digital oomputers networks an automatic check is often applied for detecting errors. Correctness of operation may be controlied by checking some properties of output datain dependence of determined properties of the input data, /e.g. the parity check in data transmitting, ohecking of arithmetic operations ci $[2],[5],[6],[9]$, and others/. Another method of ohecking, most frequently used, $1 s$ comparison of results obtained with two different networks or results obtained with one network at different times. Similar problems connected with operation of automata made of non-reliable elements are discussed by Neumann [8].

Checking in full is often troublesome in praotioe, requiring more complex ohecking circuits. Any simplifications in these circuits prevent thorough checking. On the other hand, very extended ohecking systems are rather disadvantageous as these may themselves become a source of errors.

The problem arises if it is possible to construct switching netrorks in which automatic error detection does not require special checking systems. Moreover, the check should be possibly most effective, 1 .e. appearance of an error in the circuit should be recorded in this circuit with the highest possible probability.

Yet it is not essential that we can reproduce the previous operation of the oircuit/e.g. to read partial results in memory/ or to find the source of error after this error appeared.

1. Finite automaton can be described by the function

$$
x^{\prime}, J^{\prime}=f(x, e)
$$

rhere

$$
\begin{aligned}
& x, x^{,} \quad \text { are the oirouit states, } \\
& e \\
& y^{\prime} \\
& \text { is the input state, } \\
& \text { is the output state. }
\end{aligned}
$$

Without losing generality, we may replace the pair of veriables $x^{\prime}, y^{\prime}$ by one vailable which is denoted by $x^{\prime}$. By analogy, $x$ replaces the pair $x, y$ /a state of the cirouit and of ourput preceding $x$, and $y^{\prime}$, respectively/. An equation for the automaton takes the form

$$
x^{\prime}=f(x, e)
$$

Where $x \in X, x^{\prime} \in X, \quad e \in E, X$ and $E$ are finite, and function $P$ is defined on $T \subset X \times E$. Synthesis of the automaton, for given equation $/ 2 /$, consista in finding mapping $I: X \rightarrow B^{n}, E \rightarrow B^{\text {III }}$, $\rho \rightarrow W \in W^{n}\left(B^{n} \times B^{m}\right)$, where $B^{k}$ denotes a set of all k-element sequences wich elements 0 or $1 ; m$, $n$ are natural numbers. Here $W\left(B^{n} \times B^{\text {m }}\right)$ denotes a set of expressions of $n+m$ zero-ons variables in a formalism determined by a set of basic elements which may be used for a construction of a given automaton. Similarly, $W^{n}\left(B^{n} \times B^{m}\right)$ denotes a set of $n-e l e m e n t ~ s e q u e n c e s ~ o f ~ t h e ~ e x p r e s s-~$ ions of $n+$ ril variables. We assume, for simplicity, that every system of $n$ expressions admissible in this formalism uniquely determines a diagram of the appropriate switohing network, i.e. a kind of elements to be used, method of connection, and cooperation.

Hence a diagram of the switohing network is determined by the equation

$$
\xi^{\prime}=\pi(\xi, \varepsilon),
$$

Where $\xi \in B^{n}, \xi^{\prime} \in B^{n} ; \varepsilon \in B^{m}$, and a value of the system of expressions $\pi$ is determined on the set $\Omega \subset B^{n} \times B^{m}$.

Here $\Omega$ is a set of all admissible combinations of the states of circuit and its inputs. Of oourse $I(T) \subset \Omega$, and $I(T) \neq \Omega$ is also possible.

Let $R$ be a set of values of the pair $\xi, \mathcal{E}$ possible at normal operation of the netrork, i.e. if the network itself, as well as all the cooperating cirouits were operating all the time Without errors or if the elements of all circuits were reliable. Assume that $R=\mathrm{pr}_{1} R \times \mathrm{pr}_{2} R$ where $\mathrm{pr}_{1} R$ and $\mathrm{pr}_{2} R$ are projections of $R$ on $B^{n}$ and $B^{m}$, respeotively. It means that at normal operation of the netrork it is possible to introduce on its inputs any admissible signal disregarding the actual state of the network.

It follows, by definition of $R$, that if $\Omega-R \neq 0$, and since the state of the network and its inputs is $\xi, \varepsilon \in \Omega-R$, then either the network has operated erroneously or the input signal was erroneous.

The error may ocour in two ceses:
for $\xi^{\prime} \neq W(\xi, \varepsilon)$ /erroneous operation of the circuit itself/, and for $\varepsilon \in \mathrm{pr}_{2}(\Omega-R)$ /erroneous signal on the input/.

As we shall see further, such uniform treatment of the errors in network and in input signals allows to establish the requirements for oheoking the network and also for checking, at least to some extent, all the cooperating circuits that generate input signals. Then, if these networks have self-adjusting check system, they are checked dually. Advantages of such cheok are evident provided that not vary extended circuits are required. But, if for one or another reas on such check proves to be disadvantageous, our notion of error can be suitably narrowed. Essence of considerations presented in this paper remains unchanged, and they are easily applicable to such more limited case.

The probability of error depends upon a diagram of the network, this in turn is due to the mapping I for the automaton given by $/ 2 /$.

The error can be recorded by the network only after causing its transit to the state $\xi$ such that for arbitrary $\varepsilon$ we have $\xi, \varepsilon \in \Omega-$ R. Since the error should be kept registered for os long time as possible, for $\xi, \varepsilon \in \Omega-R$ should be $\pi(\xi, \varepsilon)=\xi^{\prime} \in \mathrm{pr}_{1}(\Omega-R)$. Actually, if $w(\xi, \varepsilon)=\xi^{\prime} \in \mathrm{pr}_{1} R$ then for arbitrary $\varepsilon^{\prime} \in \mathrm{pr}_{2} R$ we have $\xi^{\prime}, \varepsilon^{\prime} \in R$ which means that the network returns to $\varepsilon$ set of states of normal operation.

A diagram of the network should be chosen, if possible, so that In case of error the probability of transition to the state $\xi ' \in \mathrm{pr}_{1}(\Omega-\mathrm{R}) /$ in one or more steps/ is possibly great. For this network the probability of transition from arbitrary state of the set $\operatorname{pr}_{1}(\Omega-R)$, in which the network is found after its erroneous operation, to any state of the set $\mathrm{pr}_{1} R$ is possibly small.

In the present paper we shall deal with problems of synthesis of such automate not going, however, into detailed probabilistic analysis of the problem.
2. We shall deal with more specific oles of networks, viz. those which, after appearing an error after into stable state in which they remain during normal operation independently of the input state.

Assume that the network is described by expressions / $/ 3$ and $R=p r_{1} R \times p r_{2} R \subset \Omega$. We shall consider networks with the following properties:

> e1. if $\xi, \varepsilon \in R$ and $w(\xi, \varepsilon)=\xi^{\prime}$, then $\xi^{\prime} \in p r_{1} R$ e2. if $\xi, \varepsilon \in \Omega-R$ and $w(\xi, \varepsilon)=\xi^{\prime}$, then $\xi^{\prime} \in p r_{1}(\Omega-R)$
> e3. for every pair of states $\xi, \varepsilon \in \Omega-R$ and
for every infinite sequence of the input states $\varepsilon^{\prime}, \varepsilon^{\prime \prime}, \ldots, \varepsilon^{(n)}, \ldots$ exists $k$ such that if $\left.\xi^{\prime}=w(\xi, \varepsilon), \xi^{(n+1)}=\xi^{(n)}, \varepsilon^{(n)}\right)$, and $n>k$, then $\xi^{(n+1)}=\xi^{(n)}$.

Let $S$ be such a set of $\xi \in \operatorname{pr}_{1}(\Omega-R)$ that for every $\varepsilon$ we have $\xi=\xi^{\prime}=w(\xi, \varepsilon)$ for $\xi, \varepsilon \in \Omega$, ie. the network remains in state $\xi$ independently of the input state. From ed and es it follows that if $\xi, \varepsilon \in \Omega-R$ then, after a number of steps, the network will enter one of the states of the set $S$.

## Theorem

Let a given automaton be described by the equation

$$
x^{\prime}=f(x, \bar{e})
$$

where $x, e \in T=\mathbb{X} \times E$.
Furthermore,

$$
\begin{array}{ll}
K \subset B^{n}, & K=\operatorname{Min} K, *) \\
L \subset B^{m}, & L=\operatorname{Min} L .
\end{array}
$$

The sets $K$ and $X$, as well as $L$ and $E$, are of equal power $K$ and $X$ having more than one element each.
i is arbitrary one-one mapping $X$ on $K$, and $E$ on $L$.
$R=K \times L$
$S=\{\underbrace{00 \ldots 0}_{n}, \underbrace{11 \ldots 1}_{n}\}$
In such a case a system w exists which consists of $n$ normal expressions without negated variables. The system is such that for $i(x)$, $i(e) \in R$ we have

$$
i\left(x^{\prime}\right)=w(1(x), i(B))
$$

while $\Omega, R$, and $S$ satisfy conditions $e 1, e 2$, and $e 3$.

## Proof

Let us denote

$$
p_{\alpha}(\beta)=\prod_{k=1}^{n+m}\left(\beta_{k}+\bar{\alpha}_{k}\right)
$$

*Assume that $B^{n}$ is partly ordered in the following way: $\left(a_{1}, a_{2}, \ldots a_{n}\right) \leqslant\left(b_{1}, b_{2}, \ldots, b_{n}\right)$ if and only if $a_{1} \leqslant b_{1}$ for' $i^{\prime}=1,2, \ldots n$. By Kin $K$ a set of minimal elements of $K$ is denoted.

Where

$$
\alpha \in B^{n} \times B^{m}, \quad \beta \in B^{n} \times B^{m} .
$$

This is a conjunction of all variables with indices $k$ such that $\alpha_{k}=1$. He have $p_{\alpha}(\beta)=1$ if and only if $\beta \geqslant \alpha$. Any normal expression $v$ without negations is of the form.

$$
V(\beta)=\sum_{\alpha \in \mathbb{A}} p_{\alpha}(\beta) .
$$

We have $\gamma(\beta)=1$ if and only if exists $\alpha \in \mathbb{A}$ such that $\beta \geqslant \alpha$. Therefore by proper selection of $A$ it is possible to construct expression $/ 6 /$ taking on values 0 or 1 in arbitrary subsets of $B^{n} \times B^{\text {m }}$, provided that for arbitrary $\beta \in B^{n} \times B^{m}$ and $\gamma \in B^{n} \times B^{m}$ the following condition is satisfied:

$$
\text { if } \beta \leqslant \gamma, \text { then } \gamma(\beta) \leqslant \gamma(\gamma) \text {. }
$$

/7/
We are choosing e system of expressions wo that if $\xi, \varepsilon \in \Omega-R$ and exists $\xi_{1}, \varepsilon_{1} \in R$ such that $\xi_{1}, \varepsilon_{1} \leqslant \xi, \varepsilon$ then $*(\xi, \varepsilon)=11 \ldots 1$. For other $\xi, \varepsilon \in \Omega-R$ is $v(\xi, \varepsilon)=00 \ldots 0$. For a given functions $f$ and 1 , the values of $w$ are defined on the set $R$ by equation $/ 4 /$.

Such a system of expressions exists provided that do not exist both $\xi_{2}, \varepsilon_{2}<\xi_{3}, \varepsilon_{3}$ and expression $\eta_{k}$ contained in the system such that

$$
w_{k}\left(\xi_{2}, \varepsilon_{2}\right)>w_{k}\left(\xi_{3}, \varepsilon_{3}\right) .
$$

Suppose that such $\xi_{2}, \varepsilon_{2}, \xi_{3}, \varepsilon_{3}$ and $\nabla_{k}$ exist.
By $/ 8 /$, we have $\pi\left(\xi_{2}, \varepsilon_{2}\right) \nLeftarrow 00 \ldots 0$, hence there exists $\xi_{1}, \varepsilon_{1} \in R$ such that $\xi_{2}, \varepsilon_{2} \geqslant \xi_{1}, \varepsilon_{1}$. Hence $\xi_{1}, \varepsilon_{1} \leqslant \xi_{2}, \varepsilon_{2}<\xi_{3}, \varepsilon_{3}$. Since $R=K \times I=M i n K \times M \operatorname{Ln}=\mu \ln R$ we can not have $\xi_{3}, \varepsilon_{3} \in R$. Hence $\xi_{3}, \varepsilon_{3} \in \Omega-R$ and $\left(\xi_{3}, \varepsilon_{3}\right)=11 \ldots 1$ contrary to $/ 8 /$; therefore our assumption is false and the required system of expressions exists:

Since the equation $/ 4 /$ holds, the condition of is satisfied; In foot, if $\xi, \varepsilon \in R$ then $x=1^{-1}(\xi) \in X, \quad=1^{-1}(\varepsilon) \in E$, $f(x, \theta)=x^{\prime} \in X$. Thus $(\xi, \varepsilon)=\xi^{\prime}=1\left(x^{\prime}\right) \in K=p r_{1} R$.

If $\xi, \varepsilon \in \Omega-R$ and exists $\xi_{1}, \varepsilon_{1} \in R$ such that $\xi_{1}, \varepsilon_{1} \leqslant \xi, \varepsilon$, then $w(\mathcal{E}, \mathcal{E})=11 \ldots 1$. Since $K$ has more than one element, and $K=M i n K$, then $11 \ldots 1 \notin K$; hence for arbitrary $\varepsilon \in L$ we have 11...1, $\varepsilon \in \Omega-R$. For arbitrary $\xi, \varepsilon \in R$ we have $\xi, \varepsilon<11 \ldots 1, \varepsilon$. Hence $w(11 \ldots 1, \varepsilon)=11 \ldots 1 \in S$.

By analogy we can show that $00 \ldots 0 \notin K$ and $w(00 \ldots 0, \varepsilon)=00 \ldots 0 \in S$. Hence conditions ez and el are also satisfied.

If $\eta \in B^{\eta}$, and if $v(\eta)$ is normal expression containing negated variables, then the condition $/ 7 /$ may be unfulfilled. Every conjunction oocuring in $v$ may be written as

$$
p_{\pi, \nu}(\beta)=\prod_{k=1}^{n}\left(\beta_{k}+\bar{\pi}_{k}\right)\left(\bar{\beta}_{k}+\bar{\nu}_{k}\right)
$$

Where $\beta \in B^{n}, \quad \nu \in B^{n}, \quad \pi \in B^{n}, \quad \pi \nu=0$.
Here $p_{\pi, \gamma}(\beta)$ is a conjunction of non-negated variables $\beta_{k}$ with indices such that $\pi_{k}=1$, and negated variables $\bar{\beta}_{k}$ with indices such that $\nu_{k}=1$. If $\pi_{k}=v_{k}=0$, then neither variable $\beta_{k}$ nor its negation appear in the conjunction.

Let us have

$$
\hat{\beta}=\beta_{1}, \beta_{2}, \ldots, \beta_{n}, \bar{\beta}_{1}, \bar{\beta}_{2}, \ldots, \bar{\beta}_{n}=\beta, \bar{\beta} \quad / 10 /
$$

The alternation $/ 9 /$ can be then expressed in the form $/ 5 /$. Denote $\pi, v=\hat{\alpha} \in B^{\text {en }}$. Then

$$
\begin{aligned}
& p_{\pi, \nu}(\beta)=p_{\hat{\alpha}}(\beta)=\prod_{k=1}^{n}\left(\beta_{k}+\bar{\Pi}_{k}\right) \prod_{k=1}^{n}\left(\bar{\beta}_{k}+\bar{v}_{k}\right)= \\
= & \prod_{k=1}^{n}\left(\hat{\beta}_{k}+\bar{\alpha}_{k}\right) \prod_{k=1}^{n}\left(\hat{\beta}_{k+n}+\overline{\hat{\alpha}}_{k+n}\right)=\prod_{k=1}^{n}\left(\hat{\beta}_{k}+\overline{\hat{\alpha}}_{k}\right)=p_{\hat{\alpha}}\left(\hat{\beta}_{\beta}\right) .
\end{aligned}
$$

The expression $p_{\hat{\alpha}}(\hat{\beta})$ can be obtained from / $/ 9$ by formal substitution of $\hat{\beta}_{k}$ in place of $\beta_{k}$, and of $\hat{\beta}_{k+n}$ in place of $\bar{\beta}_{k}$. The image $\mathscr{\nu}\left(B^{n}\right)$ of set $B^{n}$ after transformation /10/
v: $\beta \rightarrow \beta, \bar{\beta}=\hat{\beta}$ is a subset of $B^{2 n}$ containing all sequences $\hat{\beta}$ with $n$ ones, and such that $\left(\hat{\beta}_{1}, \hat{\beta}_{2}, \ldots, \hat{\beta}_{n}\right) \cdot\left(\hat{\beta}_{n+1}, \hat{\beta}_{n+2}, \ldots \hat{\beta}_{2 n}\right)=0$. Therefore $v\left(B^{n}\right)=\operatorname{Min} v\left(B^{n}\right)$.

Let us consider the switching networks for which diagrams are expressible by systems of normal expressions. If for such network a negating element is one of basic components, then it may occur that values of its input and output signals will be equal as an effect of erroneous operation of this element. In such oases it is convenient to assume independence of signals on inputs and outputs of negating elements. Then by mapping $\nu$ the output normal expression may be reduced to an expression without negated variables. Then using the notations of our Theorem the assumptions $K=M i n K$, and $L=M i n I$ are satisfied.

If in circuits described by normal expressions either there is no negating element or for another res on signals corresponding to a variable and negated variable can not have the same value, the problem of synthesis of the circuit satisfying conditions et - el greatly simplifies. Since for each zero-one function determined on $B^{n+m}$ exists its normal equivalent of $n+m$ variables, then the problem reduces to determining properly a value of expressions that describe the circuit for $\xi, \varepsilon \in \Omega-R$. In particular, let us consider normal expressions that are alternations of full conjunctions /ie. conjunctions of $n+m$ negated or non-negated variables/ expressed by

$$
\nabla(\xi, \varepsilon)=\sum_{\alpha, \beta \in R}\left[\prod_{k=1}^{n}\left(\xi_{k}+\bar{\alpha}_{k}\right)\left(\bar{\xi}_{k}+\alpha_{k}\right) \prod_{k=1}^{m}\left(\varepsilon_{k}+\bar{\beta}_{k}\right)\left(\bar{\varepsilon}_{k}+\beta_{k}\right)\right] .
$$

Every system of $n$ such expressions describes an automaton for which conditions et - e 3 are filfillisi and for which $S=\{\underbrace{00 \ldots 0}_{n}\}$ for $00 \ldots 0, \operatorname{pr}_{1}(\Omega-R)$. If the circuit has to cooperate with n
other circuits for which $S=\{00 \ldots 0\}$, and if it has to response properly to an erroneous combination on the input, then there must be $\underbrace{00 \ldots 0}_{m+n} \in \Omega-R$.

## Examplo

The problem is to find a system of normal expressions/3/ Without negated variables for an automaton satisfying conditions ef - e3 and described by the following Moore's [7] diagram :


The ralues for the output states are given so as they should be in a designed circuit.

Since the oircuit has no inputs, equations $/ 2 /$ and $/ 3 /$ can De written in simpler form

$$
\begin{aligned}
& x^{\prime}=f(x), \\
& \xi^{\prime}=W(\xi) .
\end{aligned}
$$

If $W$ is a system of expressions without negated variables, then condition /7/ must be satisfied. Hence, for

$$
K=\{1(a, 00), \quad 1(b, 01), \ldots, 1(f, 10)\}
$$

we have $K=M i n K$. Indeed, since $w$ is isotone, taking e.g. $1(a, 00) \leqslant 1(b, 01)$ we have

$$
1(a, 00) \leqslant 1(b, 01) \leqslant 1(c, 01) \leqslant \ldots \leqslant 1(1,10) \leqslant 1(a, 00)
$$

or

$$
1(a, 00)=1(b, 01)=\ldots=1(f, 10)
$$

By analogy, it can be shown that for arbitrary $\xi_{1} \in K, \xi_{2} \in K$ the inequality $\xi_{1} \leqslant \xi_{2}$ does not hold.

The least va?ue of $n$, for which $K \subset B^{n}$, equals $4: K$ is a set of four-element sequences each containing two ones.

Omitting the mappings obtainable by permutation of variables $\xi_{1}, \xi_{2}, \xi_{3}, \xi_{4}$, we have the following possible mappings:


From the Moore's diagram we obtain the table of values of $w$ for $i_{1}$ :


Extension of the values $w$ onto $\Omega /$ third column of the table/ is unique and follows only from the condition that $W$ is isotone /7/. Minimal normal expressions for this table, obtained e.g. by the method given in [10], are as follows:

$$
\begin{aligned}
& \xi_{1}^{\prime}=\xi_{2} \xi_{4}+\xi_{2} \xi_{3}+\xi_{1} \xi_{3}, \\
& \xi_{2}^{\prime}=\xi_{3} \xi_{4}+\xi_{1} \xi_{3}+\xi_{1} \xi_{2}, \\
& \xi_{3}^{\prime}=\xi_{3} \xi_{4}+\xi_{2} \xi_{3}+\xi_{1} \xi_{4}, \\
& \xi_{4}^{\prime}=\xi_{2} \xi_{4}+\xi_{1} \xi_{4}+\xi_{1} \xi_{2} .
\end{aligned}
$$

Hence the conditions el - es are satisfied.
For function $1_{2}$ we have also a unique solution

$$
\begin{aligned}
& \xi_{1}^{\prime}=\xi_{2} \xi_{4}+\xi_{3} \xi_{4}+\xi_{2} \xi_{3}, \\
& \xi_{2}^{\prime}=\xi_{1} \xi_{3}+\xi_{2} \xi_{3}+\xi_{1} \xi_{2}, \\
& \xi_{3}^{\prime}=\xi_{3} \xi_{4}+\xi_{1} \xi_{3}+\xi_{1} \xi_{4}, \\
& \xi_{4}^{\prime}=\xi_{2} \xi_{4}+\xi_{1} \xi_{4}+\xi_{1} \xi_{2} .
\end{aligned}
$$

In this pase the conditions of - el are satisfied for all the simplest systems of expressions; for error detection no oomplioatron of the oirouit is required.

## Example 2

The problem is to find a system of normal expressions /3/ Without negated variables for an automaton satisfying conditions et - es, the Moore's diagram of which is the following:


As before, minimal $n$ equals 4.
Let us take the following mapping 1 :

| $x$ |  |  | $1(x)$ |  |
| :--- | :--- | :--- | :--- | :--- | :--- |
| $a$ | 0 | 0 | 1 | 1 |
| $b$ | 0 | 1 | 0 | 1 |
| $c$ | 1 | 0 | 1 | 0 |
| $a$ | 1 | 1 | 0 | 0 |

Table of values of w:


Values of $w$ on $\Omega$ are determined by the method given in proof of the Theorem.

Expressions: for $\xi$ are

$$
\begin{aligned}
& \xi_{1}^{\prime}=\xi_{2} \xi_{4}+\xi_{1} \xi_{3} \\
& \xi_{2}^{\prime}=\xi_{3} \xi_{4}+\xi_{1} \xi_{3}+\xi_{1} \xi_{2} \xi_{4} \\
& \xi_{3}^{\prime}=\xi_{2} \xi_{4}+\xi_{1} \xi_{2}+\xi_{1} \xi_{3} \xi_{4} \\
& \xi_{4}^{\prime}=\xi_{3} \xi_{4}+\xi_{1} \xi_{2}
\end{aligned}
$$

Other possible 1 is

| $x$ |  |  | $i(x)$ |  |
| :--- | :--- | :--- | :--- | :--- |
| $a$ | 0 | 0 | 0 | 1 |
| $b$ | 0 | 0 | 1 | 0 |
| 0 | 0 | 1 | 0 | 0 |
| $d$ | 1 | 0 | 0 | 0 |

Table of values of $W$


Finally, we obtain from the table a system of simplest normal expressions

$$
\begin{aligned}
& \xi_{1}^{\prime}=\xi_{2}+\xi_{3} \xi_{4} \\
& \xi_{2}^{\prime}=\xi_{3} \\
& \xi_{3}^{\prime}=\xi_{4}+\xi_{1} \xi_{3} \\
& \xi_{4}^{\prime}=\xi_{1}
\end{aligned}
$$

Values of $w$ are defined on $\Omega-R$ so as to obtain possibly the simplest circuit, of course, taking into account that the function is isotone. The obtained circuit is simpler than the one obtained by applying the method given in proof of the Theorem; it requires, however, greater average number of steps for transition from any state $\xi \in \operatorname{pr}_{1}(\Omega-R)$ to the stable one.

Moore's diagram for the obtained circuit is


Sxamplo3

The problem is to construct an automaton as in Example 2 if expressions $w_{k}$ oan be conjunctions of alternations of non-negated variables or alternations of conjunctions of negated variables. This formalism corresponds to a ferrite core technique used in the Institute of Mathematical Machines of the Polish Academy of Sclences; here 1 means the pulse, and 0 means the lack of pulse /cf. $[11] /$.

Table of function 1

| $x$ |  |  | $1(x)$ |  |
| :--- | :--- | :--- | :--- | :--- | :--- |
| $a$ | 0 | 1 | 1 | 0 |
| $b$ | 1 | 0 | 1 | 0 |
| $c$ | 0 | 1 | 0 | 1 |
| $d$ | 1 | 0 | 0 | 1 |

Table of values of $w$


The system of simplest expressions $w$ in the formalism mentioned

$$
\begin{aligned}
& \xi_{1}^{\prime}=\left(\xi_{2}+\xi_{4}\right)\left(\xi_{2}+\xi_{3}\right) \\
& \xi_{2}^{\prime}=\left(\xi_{1}+\xi_{4}\right)\left(\xi_{1}+\xi_{3}\right) \\
& \xi_{3}^{\prime}=\bar{\xi}_{2} \bar{\xi}_{3}+\bar{\xi}_{1} \bar{\xi}_{4} \\
& \xi_{4}^{\prime}=\bar{\xi}_{2} \bar{\xi}_{4}+\bar{\xi}_{1} \bar{\xi}_{3}
\end{aligned}
$$

The above Table shows that $S=\{1100\}$ and conditions el - e 3 are satisfied.

Moore's diagram of the obtained cirouit:


Author suggests the problem: Give a method of systhesis of the simplest finite automata satisfying conditions et - e3 for an arbitrary function $/ 2 /$. The automata are to be desoribed by systems of normal expressions /3/ with negated variables or Without them. The method should comprise the choice of a function 1 as well as a manner of extending the values of $w$ onto the set $\Omega$.

## Conclusion

Considerations discussed in the paper have been applied in practice for constructing small networks used in the Institute of Mathematical Machines for examining technical properties of basic logical elements. Easy detection of errors in such networks proved to be useful for investigations. In the author's opinion those automata can be used in digital computers, but this requires the development of methods for designing such networks.

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Watyeda

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