PRACE Instytutu Maszyn Matematycznych PAN

Praca A 4 (21)

SELECTION OF OPTIMAL VALUES OF NONLINEAR CIRCUIT PARAMETERS BY MEANS OF DIGITAL COMPUTERS

by Ryszard ŁUKASZEWICZ

P R A C E

Instytutu Maszyn Matematycznych

P o l s k i e j   A k a d e m i i   N a u k

T. II Praca A 4/21/

SELECTION OF OPTIMAL VALUES OF NONLINEAR CIRCUIT PARAMETERS BY MEANS OF DIGITAL COMPUTERS

by Ryszard ŁUKASZEWICZ

Warszawa 1964

Instytut Maszyn Matematycznych
Praca A 4/21/

681.14-523.8:518.61:
621.3.064-526

SELECTION OF OPTIMAL VALUES OF NONLINEAR CIRCUIT PARAMETERS BY MEANS OF DIGITAL COMPUTERS

by Ryszard ŁUKASZEWICZ

Received December 1963

In the following paper the choice of optimal values of nonlinear circuit parameters is considered. The configuration of the circuit is known and the criterion given. The problem is treated as a certain stage of the designing procedure; evidence is given as to the effectiveness and economy of digital computers. A variant of the gradient method for finding optimum is discussed, providing a basis for writing an appropriate program for a digital computer. The proposed method of automatic selection of a step towards optimum increases the effectiveness of computations. Dependences, determining the direction of the step of the examined point near the constraint surface, are given. The discussed scheme of data is the basis for computing optimal parameters of servomechanisms with the aid of a universal optimization program.

The method of nonlinear circuit analysis, called a direct one, is presented in papers [1] and [2]. It consists of a description of circuit operation using differential equations of the first order in a normal form, and of their direct solution on a digital computer. A method is given for obtaining differential equations of the analysed circuit directly in the required form. On the basis of the SAKO autocode [3], [4] it is explained /in several examples/ how easily a program may be written for computing the examined linear or nonlinear circuit, using standard subroutines.

It should be emphasized that digital computers enable the analysis process to be automatic. The constructor's work should be lessened with the purpose of its reducing to writing only the subroutine that would comprise circuit differential equations as well as to choosing appropriate nonlinear subroutines from the subroutine library and the main program according to the criterion of the analysis.

1. THE CHOICE OF THE METHOD FOR SELECTING OPTIMAL CIRCUIT PARAMETERS

The paper discusses problems connected with circuit synthesis, i.e. the choice of optimal parameters of circuits with a known configuration according to the given criterion.

The purpose is to find the process of designing which satisfies the following postulates [5], available means being taken into consideration.

Basic postulates for estimating the effectiveness of the designing procedure.

1. Selection of optimal parameters, according to the given criterion.
2. Accuracy - circuit operation, determined as a result of designing, should correspond as closely as possible to real circuit operation.
3. Effectiveness in the sense of labor economy, costs and operating time.
4. Versatility - the applied method should include a possibly large class of circuit systems.

In the majority of cases a successive approximation is the way in which designing offices and laboratories are proceeding [5].

In the first stage of designing, the constructor chooses parameter values, taking advantage of his own experience and available methods of designing which generally are not very precise. Then, he makes the analysis of the circuit operation and compares the obtained characteristics with the assumed ones.

If assumptions are not satisfied, the constructor uses differences obtained among the characteristics and determines the changes of circuit parameter values that should be introduced in order to improve the circuit operation. After a certain number of iterations assumptions are satisfied and it usually is the end of the designing procedure which, however, is not synonymous with the obtaining of an optimal system. Such procedure is rather far from satisfying the above-mentioned postulates of the designing process.

ad. 1. The gain of an optimal system would demand a range of further iterations which is contradictory to point 3.

ad. 2. An increase of accuracy involves more time consuming computations, which is also contradictory to point 3.

ad. 4. If the general method is used, its adaptation to separate problems isn't required, which increases the effectiveness of the method. This being directly connected with point 3.

The process of designing by the method of successive approximations may be hastened by means of various computing machines. However, this concerns only computations. The decision about the change of parameters is still to be taken in order to reach a more effective device operation. In the case of more complex nonlinear circuits, it is much more difficult to find the dependence that would help to take this decision. Those dependences should be formulated independently for various circuit structures, which makes it difficult to satisfy condition 3.

Digital computers, thanks to their properties of performing logical operations, high operation speed and great computation accuracy, permit to automatize the process of designing. They simultaneously satisfy all above-mentioned postulates. The fulfillment of postulates points 2 and 3 results from the above-mentioned properties. Postulates 1 and 4 are realized on digital computers selecting optimal parameters as follows.

The optimum /i.e. minimum or maximum/ of the function

$$P = P(x_i) \qquad /1/$$

is to be found, where: function $P(x_i)$ is determined by the optimization criterion; x_i - values of selected parameters; $i = 1,2,3,...,$.

It is assumed that certain constraints are not overstepped. They are given in the form of inequality

$$R_s(x_i) \geqslant 0 \qquad /2/$$

where: $s = 1, 2, 3, ...,$

$R_s(x_i)$ - constraint functions brought about by the given properties of the system, by subset characteristics and so on.

Functions $P(x_i)$ and $R_s(x_i)$ are in the majority of cases nonlinear.

Gradient and random test methods are most often applied for solving the above problem.

The random test method is particularly suitable to problems in which several minimums in the function $P(x_i)$ are expected in the space concerned. This method enables to observe the behaviour of function P within the whole space.

When using the gradient method, the step is directed towards the steepest descent of the function leading to minimum. Thanks to this, the method is much faster than that of the random test. On the other hand, when using it, after having reached the minimum, there is no indication whether the obtained optimum is local or absolute. Therefore, when choosing the gradient method to test a function which may have several local minimums, the computations should be repeated starting successively from various points.

Further, the gradient method will be considered from the viewpoint of satisfying the demands for the designing process by means of digital computers. Let's assume that functions P and R_s are continual, their derivatives being partial in respect to all independent variables in the whole space of seeking for optimum.

A variant of the gradient method is given below for writing an appropriate program of computations determined by dependences /1/ and /2/. Two basic stages are distinguished in it. The first concerns the seeking of optimum in an unconstrained space. In the case of function P being a convex one a new economical method is proposed of an automatic choice of step directed by a gradient. In the second, stage the step being also chosen automatically, leads towards optimum along a tangent to the constrained surface.

2. AUTOMATIC SELECTION OF STEP IN THE DIRECTION DETERMINED BY A GRADIENT IN AN UNCONSTRAINED SPACE

Assume we start from point r inside the space determined by the dependence $R(x_i) \geqslant 0$.

The direction of the vector gradient in this point is computed according to the dependence

$$\text{Grad } P = \sum_{i=0}^{n} \frac{\partial P}{\partial x_i} \bar{x}_i \approx \sum \frac{\Delta P}{\Delta x_i} \bar{x}_i \qquad /3/$$

where: $\bar{x}_i$ – unitary vectors directed along the coordinate axis

$\frac{\partial P}{\partial x_i}$ – values of partial derivatives

$\frac{\Delta P}{\Delta x_i}$ – approximate values of partial derivatives for an appropriately small increment of parameters Δx_i

P – values $\Delta P = P_{x_{i,r}} + \Delta x_i - P_{x_{i,r}}$ – corresponding to increments Δx_i are found by means of solving the system of equations describing the operation of the investigated system for values of parameters $x_{i,r}$ and $x_{i,r} + \Delta x_i$, and by means of the algorithm determined by the accepted criterion of optimization by computing $P_{x_{i,r}}$ and $P_{x_{i,r} + \Delta x_i}$

A certain step is made in the direction determined by the gradient of function P, providing the greatest change of this function value in the neighbourhood of point r, according to the dependence

$$x_{i,r+1} = x_{i,r} + h \frac{\Delta P}{\Delta x_i} \qquad /4/$$

The value of step h in a most general case is selected automatically by means of a control of the value of angle between gradient vectors being determined successively [6]. If this angle is smaller than a certain accepted value Θ, the step h is doubled; if the angle is too large h is twice reduced.

This is evidently an efficient method, as it ensures steps near the curve of the steepest descent /or growth/ leading to optimum. However, for a certain class of function P this method is not economical. It requires most time consuming computation of gradient direction every time when determining the angle between successive gradient vectors.

In the paper a more efficient method is presented of an automatic choice of step h in the direction determined by the gradient in the case when the function P is convex, /i.e. its second derivative does not change the sign/. This method is based upon an examination of the relation $\left|\frac{\Delta' P - \Delta'' P}{\Delta' P}\right|$, at the increment of function P arguments $h \cdot \frac{\Delta P}{\Delta x_i}$, where $\Delta' P$ is expressed by the dependence $\Delta' P = \sum_{i=0}^{n} \left(\frac{\Delta P}{\Delta x_i}\right)^2 \cdot h$

and $\Delta''P$ is the inorement of the examined function P . Then, depending on the result, the value of step h is inoreased or reduced.

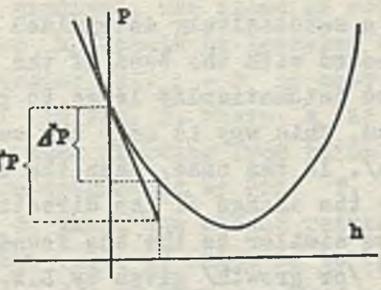

Fig. 1. Automatic selection of step while seeking for optimum by the gradient method.

This method may be easily explained by the diagram /fig. 1/ showing the function P(h), where h defines the increment of arguments in the direction determined by the gradient, using the formula

$$\Delta' x_i = h \cdot \frac{\Delta P}{\Delta x_i} \qquad /5/$$

Let's accept the initial value h . If it is too small, i.e. the relation $\left|\frac{\Delta'P - \Delta''P}{\Delta'P}\right|$ is smaller than a certain constant γ which is smaller than unity /for instance 0,1 - 0,3/, the value h is to be doubled and the relation $\left|\frac{\Delta'P - \Delta''P}{\Delta'P}\right|$ examined once more. This is repeated till the moment when the inequality reaches $\left|\frac{\Delta'P - \Delta''P}{\Delta'P}\right| \geq \gamma$. In the case when the initial value h is too great, one acts conversely, i.e. the value h is being divided by half till the moment of satisfying the condition $\left|\frac{\Delta'P - \Delta''P}{\Delta'P}\right| < \gamma$.

This conditions the value of step h to the bend of the run of function P appearing when realizing this step. Such a control of the shift of the examined point in the direction determined by the gradient, significantly lessens the appearance of undesirable effects, as for instance, the entering the space where function P is not determined, or those connected with its specific of computation.

Moreover, when a successively determined change of the gradient direction is connected with the bend of the run of function P, the proposed method automatically leads to point out the way of seeking for optimum. This way is near the curve of the steepest descent /or growth/. In the case, when the examined function bend does not accompany the change of the direction of the gradient, the optimum will be similar to the one found by the method of the steepest descent*) /or growth/ given by L.W. Kantorowicz [7].

The method proposed by the author provides a relatively quick automatic choice of the value of the step in the direction determined by the gradient. This is due to the fact that each change of the step value is connected only with a single computing of the value of function P independent of the number of selected parameters.

The step towards optimum, as above described, is realized starting from successively determined points r + 1, r + 2, ..., according to the dependence /4/ till the moment when:

a/ it is ascertained that the neighbourhood of minimum with the assumed accuracy δ is reached. An adequate criterion may be

$$P_{r+1} - P_r < \delta \qquad /6/$$

The fulfillment of this inequality is synonymous with the end of computation,

b/ it is ascertained that the constraint determined by the dependence $R(x_i) \geqslant 0$ has been overstepped. In this case one acts as below /sec. 3/.

---

*) The method of the steepest descent consists in determining the gradient of function P, and in a step in the opposite direction till the minimal value of function P is reached. In this point a new gradient direction is defined and analogous to the former the minimal value of function P is found, and so on.

3. CONSTRAINTS - STEPS ALONG THE TANGENT

If, while realizing a certain successive step h in the direction determined by the gradient, the space of constraint $R(x_i) = 0$ is overstepped, the step h should be lessened so as to be found on the surface of constraint $R(x_i) = 0$ or near it. Let us consider the diagram $R(h)$ in Fig. 2, where h is determined by the dependence /5/. Assume, that starting from point r_1 the step h_1 is made in the direction determined by Grad P, and point r_2 is reached where $R(h_1) < 0$. Then, one should return to point r_1 and make a new step h_2, the value of which is to be found by means of a linear interpolation /or of higher order/ according to the dependence

$$h_2 = h_1 \frac{R(o)}{R(o) - R(h_1)} \qquad /7/$$

Being on the surface of constraint /or near it/ the step is to be directed towards optimum along the tangent to this surface. Let us be in point $r(x_i)$ on the surface $R(x_i) = 0$. Further seeking for optimum is performed by a step in the direction determined by the projection of vector Grad $P(x_{i,r})$ on the plane tangent to the surface $R(x_i) = 0$ in point r /or to the surface $R(x_i) = K$, near the surface $R(x_i) = 0$/.

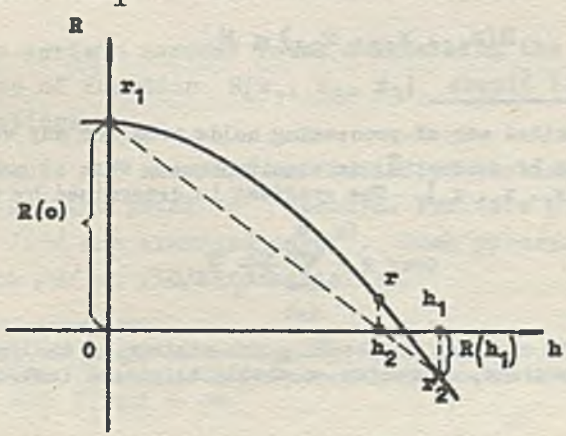

Fig. 2. The diagram of the constraint curve R(h).

An appropriate way of proceeding will be most easily explained using Fig. 3 presenting space $\{x_n\}$ in the case*) $n = 3$, i.e. space $\{x_1, x_2, x_3\}$. Let us be in point $\vec{r}$, on the surface of constraint $R(x_1, x_2, x_3) = 0$. In this point we define vector $\vec{G}_1$ of the gradient function $P(x_1, x_2, x_3)$ and vector $\vec{N}$**) normal to the surface of constraint $R(x_1, x_2, x_3) = 0$. Vector $\vec{S}$, being tangent to this surface, is found as follows:

First, components of the projection of vector $\vec{G}_1$ against vector $\vec{N}$ are computed according to the dependence

$$\vec{G}_{1N} = \frac{(\vec{G}_1 \cdot \vec{N})}{N^2} \cdot \vec{N} \qquad /8/$$

Then, point $\vec{d}$ should be found, vector $\vec{G}_{1N} = -\vec{G}_{1N}$ being led out from point $\vec{c}$. Vector $\vec{S}$ tangent to the surface of constraint $R(x_1, x_2, x_3) = 0$ is determined by the expression $\vec{S} = \vec{G}_1 - \vec{G}_{1N}$.

Modulus value of vector $\vec{S}$ is chosen by means of its twofold increase or decrease depending on results of the examination of conditions ***)

$$P(x_{1d}, x_{2d}, x_{3d}) < P(x_{1r}, x_{2r}, x_{3r}) \qquad /9/$$

and

$$R(x_{1d}, x_{2d}, x_{3d}) \leqslant M \qquad /10/$$

---

* Further described way of proceeding holds true for any value n.

** The direction of vector $\vec{N}$ is simultaneously that of gradient of the function $R(x_1, x_2, x_3)$. The gradient is determined by the dependence

$$\text{Grad } R = \sum_{i=1}^{n} \frac{\partial R}{\partial x_i} \vec{x}_i$$

*** Dependence /9/ concerns the seeking for minimum of the function P. While seeking for maximum, character < should be placed instead of > .

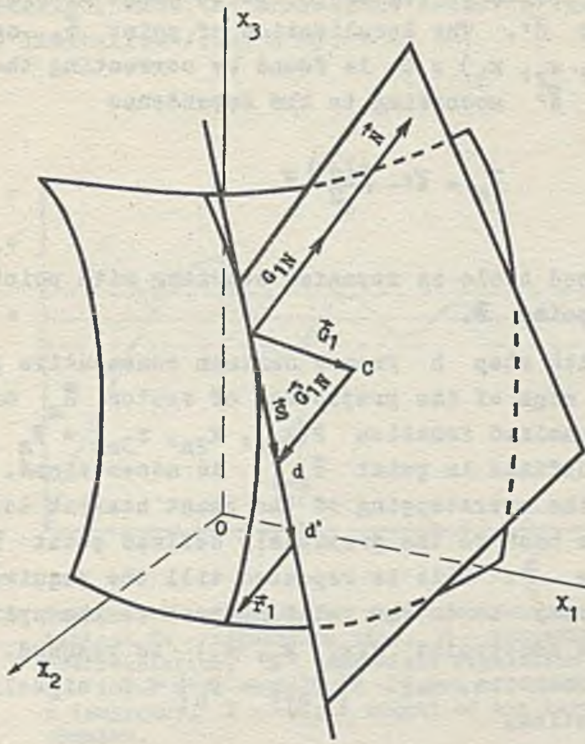

Fig. 3. Defining tangent direction to the limitation curve.

where M is a certain assumed value determining the tolerance in which the value of function $R(x_1, x_2, x_3)$ should be included during computations.

When conditions /9/ and /10/ are satisfied the lenght of vector $\bar{S}$ is doubled and the point $\bar{d}'$ reached. For this point conditions /9/ and /10/ are examined anew*). Such proceeding lasts till conditions /9/ or /10/ are satisfied.

---

*When doubling the lenght of vector $\bar{S}$, which results in reaching, for instance, point $\bar{d}'$, condition /9/ concerns the value of function P in points $\bar{d}'$ and $\bar{d}$ and so on.

Assume the length of vector $\vec{S}$ to be found determining coordinates of point $\vec{d}'$. The localization of point $\vec{r}_1$ near the constraint $R(x_1, x_2, x_3) = 0$ is found by correcting the localization of point $\vec{d}'$ according to the dependence

$$\vec{r}_1 = \vec{d}' - \frac{R(\vec{d}')}{N^2}\vec{N} \qquad /11/$$

The described cycle is repeated starting with point $\vec{r}_1$ and so on, like for point $\vec{r}$.

Starting with step h from a certain consecutive point $\vec{r}_n$ the change of the sign of the projection of vector $\vec{G}_n$ of the gradient of the examined function $P(x_{1n}, x_{2n}, x_{3n}) = P_n$ on the tangent $\vec{S}_{n-1}$, defined in point $\vec{r}_{n-1}$, is ascertained. This is equivalent to the overstepping of the point nearest to the optimum. Then, one goes back to the previously defined point $\vec{r}_{n-1}$ and tries the step $\frac{h}{2}$. This is repeated till the required approach with the accuracy E to the point nearest to the optimum on the surface of the constraint $R(x_1, x_2, x_3)$ is reached. The fulfilling of this condition, e.g. $|P_{n+1} - P_n| < E$ signalizes the end of the computation.

4. THE PROBLEM OF UNIVERSAL PROGRAM OPTIMIZATION

A program prepared for a digital computer and based on the gradient method concerns the problem of optimization which is generally formulated by dependences /1/ and /2/. If such a program is appropriately written, the constructor's task can be reduced to adjoining the following subroutines: computation of the projected system operation, optimized function P, limitation of R, and the preparation of adequate numerical data.

An exemplary universal program of servomechanism parameter optimization has been developed. Minimal time regulation being accepted as criterion, with the admissible error E and a given overregulation limitation /fig. 4 *)/.

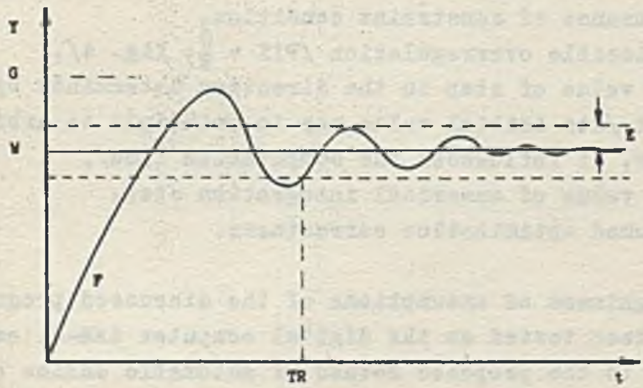

Fig. 4. Outline of criterions of the servomechanism optimizer. TR - regulation time, G - admissible overregulation, E - admissible regulation error, W - input step signal, F - curve of the run of a temporary, Y - output signal of the servomechanism.

The following scheme of input data was accepted.

NUMERICAL DATA:

Scale - scale of computation,

x_i - selected variable parameters of servomechanism /i = 0, 1, 2, ... being the index of a consecutive parameter/,

Δx_i - increments of servomechanism parameter variables used for gradient computation /their values are estimated usually as 1% - 3% of the expected value of the appropriate parameter after the end of optimization/,

---

*) In principle any signal may be the input signal W, if its time function may be computed by means of an appropriate subroutine.

y_k – initial conditions of differential equation system that describe servomechanism operation,

NP – the number of selected parameters,

N – the numer of differential equations,

EPS – admissible regulation error,

M – tolerance of constraint condition,

PIK – admissible overregulation /PIK = $\frac{G}{W}$; fig. 4/,

H – the value of step in the direction determined by the gradient /its initial value may in principle be arbitrary, however, it influences the computation time/,

HRK – the value of numerical integration step,

E – assumed optimization correctness.

The rightness of assumptions of the discussed program realization has been tested on the digital computer ZAM-2, especially in reference to the proposed method of automatic choice of step and shifting of the examined point near the space of constraint. Moreover, it has been found that for solving the problem of universal program optimization, the computer should be a floating point one /the problem of choosing the computation scale would then be cancelled/, with a larger operational storage and greater speed of operation than ZAM-2. Polish digital computers which satisfy the conditions required are ZAM-3 and ZAM-41, the latter being now under realization on the Institute of Mathematical Machines of the Polish Academy of Sciences.

References

1. ŁUKASZEWICZ R.: Direct Method of Circuit Analysis on Digital Computers, Prace IMM, Warsaw 1962:2, A17.

2. ŁUKASZEWICZ R.: Analysis of Nonlinear Circuits by the Aid of Digital Computers, Prace IMM, Warsaw 1963:2, A 2/19/.

3. ŁUKASZEWICZ L.: SAKO - An Automatic Coding System, Annual Review in Automatic Programming, Pergamon Press, 1961:2, 161-176.

4. MAZURKIEWICZ A.W.: Arithmetic Formulae and the Use of Subroutines in SAKO, Pergamon Press, 1961:2, 177-195.

5. BROWN R.R.: A Generalized Computer Procedure for the Design of Optimum Systems, Communication and Electronics, July 1959.

6. DENNIS J.B., NEASE R.F., SAUNDERS R.M.: System Synthesis with the Aid of Digital Computers, AIEE transaction, November 1959.

7. Collective work edited by SOŁODOVNIKOV V.: Primienienije vyčislitelnoj techniki dla avtomatizacji proizvodstva, Mazgiz, Moskva 1961.

W.D.N. Zam. 159/o/64. Nakład 400 egz.

A 4 (21)