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DISTRIBUTION FUNCTION DESCRIPTION OF PROBABILISTIC SETS AND ITS APPLICATION IN DECISION MAKING

Summary. The paper deals with some problems of decision making described and solved by the use of a concept of probabilistic set.

The distribution function description (representation) of probabilistic sets important from theoretical point of view is discussed as well as its application in decision making illustrated by means of numerical examples.

1. Introduction

Since the introducing by Zadeh [7] of fuzzy sets theory many papers have been published on this area also such with the aspects of the theory of probability [1,5,8].

Hirota [6] has introduced the idea of probabilistic set regarding the value of membership function of fuzzy set as a random variable depending on parameter. This concept seemed to be introduced because often the problems of ambiguity and subjectivity of observers might not be determined uniquely in [0,1]- interval.

The notion of probabilistic set has been proposed by using both probability and fuzzy sets theories and it includes of course the concept of classical fuzzy set introduced by Zadeh.

Hirota's paper [6] has considered the probabilistic sets from a measure-theoretical point of view. Because the probabilistic sets are a special case of random functions (random processes or random fields) a distribution function description (representation) of such sets in this paper has been introduced.

Section 3 describes the notion of probabilistic set and its distribution function description (representation). Two functions of probabilistic sets important from applicational point of view i.e. max and min functions and their distribution function description (representation) have been presented as well.

In Section 3 the decision making problem is formulated by using the concept of probabilistic set and its distribution function description (representation).

Numerical examples and concluding remarks are included in Sections 4 and 5.

2. Probabilistic sets and their distribution function description

Introducing the concept of probabilistic set the following notation is to be used [6].

Let $(\Omega, \mathcal{B}, P)$ be the probability space called here a parameter space, $(\Omega_c, \mathcal{B}_c) = ([0,1],$ Borel sets) be a characteristic space, and $\mathcal{M} = \{\mu \mid \mu : \Omega \to \Omega_c\}$ denotes a family of $(\mathcal{B}, \mathcal{B}_c)$ - measurable functions.

Now let us give the definition of probabilistic set [6]

Definition 1. A probabilistic set A on X is defined by a defining function μ_A

$$\mu_A : X \times \Omega \to \Omega_c \qquad (x,\omega) \mapsto \mu_A(x,\omega) \tag{1}$$

where $\mu_A(x,\cdot)$ is the $(\mathcal{B}, \mathcal{B}_c)$ - measurable function for each fixed $x \in X$.

A family of all probabilistic sets defined on X will be denoted by $\mathcal{P}(X)$

If $\mu_A(x,\cdot)$ is the $(\mathcal{B}, \mathcal{B}_c)$ - measurable function, it means, of course, that

$$\forall_{x \in X} \; \forall_{z \in \Omega_c} \left\{\omega : \mu_A(x,\omega) < z\right\} \in \mathcal{B} \tag{2}$$

On the other hand, $\mu_A(x,\omega)$ can be treated as a random process. This fact leads to the introduction of distribution function description of probabilistic set characterized by means of defining function $\mu_A(x,\omega)$. Let us consider now a multidimensional distribution function (n-dimensional) for any set of numbers $x_1, x_2, \dots, x_n \in X$ where the number n is chosen arbitrarily.

We can write it in the following form:

$$F_{\mu_A(x_1)\mu_A(x_2)\dots\mu_A(x_n)}(z_1, z_2, \dots, z_n) =$$

$$= P\left(\left\{\omega : \mu_A(x_1,\omega) < z_1,\ \mu_A(x_2,\omega) < z_2, \dots, \mu_A(x_n,\omega) < z_n\right\}\right) \tag{3}$$

or

$$F_{\mu_A(\underline{x})}(\underline{z}) = P\left(\left\{\omega : \mu_A(\underline{x},\omega) < \underline{z}\right\}\right) \quad \forall_{\underline{z} \in \underbrace{\Omega_c \times \Omega_c \times \dots \times \Omega_c}_{n\text{-times}}} \tag{3a}$$

where

$$\mu_A(\underline{x}) = \mu_A(x_1)\,\mu_A(x_2) \dots \mu_A(x_n)$$

$$\mu_A(\underline{x},\omega) = (\mu_A(x_1,\omega), \mu_A(x_2,\omega), \dots, \mu_A(x_n,\omega))$$

$$\underline{z} = (z_1, z_2, \dots, z_n).$$

The distribution function must obviously satisfy the following two condition:

1° the symmetry condition:

the equation

$$F_{\mu_A(x_{i_1})\,\mu_A(x_{i_2})\,\ldots\,\mu_A(x_{i_n})}(z_{i_1}, z_{i_2}, \ldots, z_{i_n}) =$$

$$= F_{\mu_A(x_1)\,\mu_A(x_2)\,\ldots\,\mu_A(x_n)}(z_1, z_2, \ldots, z_n) \tag{4}$$

holds for any permutation $i_1, i_2, \ldots, i_n$ of the numbers $1, 2, \ldots, n$

2° the compatibility condition

if $m < n$, then for any $z_{m+1}, z_{m+2}, \ldots, z_n$

$$\underset{m<n}{\forall} \quad F_{\mu_A(x_1)\,\mu_A(x_2)\,\ldots\,\mu_A(x_n)}(z_1, z_2, \ldots, z_m, +\infty, \ldots, +\infty) =$$

$$= F_{\mu_A(x_1)\,\mu_A(x_2)\,\ldots\mu_A(x_m)}(z_1, z_2, \ldots, z_m) \tag{5}$$

The similar conditions hold for n-dimensional density functions.For density function we have

$$F_{\mu_A(\underline{x})}(\underline{z}) = \int_{-\infty}^{z_1}\int_{-\infty}^{z_2}\ldots\int_{-\infty}^{z_n} f_{\mu_A(\underline{x})}(\underline{z})\, d\underline{z} \tag{6}$$

where $$d\underline{z} = dz_1\, dz_2\, \ldots\, dz_n$$

and we can put down:

$$\frac{\partial^n F_{\mu_A(x_1)\mu_A(x_2)\,\ldots\,\mu_A(x_n)}(z_1, z_2, \ldots, z_n)}{\partial z_1\, \partial z_2\, \ldots\, \partial z_n} =$$

$$= f_{\mu_A(x_1)\mu_A(x_2)\,\ldots\,\mu_A(x_n)}(z_1, z_2, \ldots, z_n) \tag{7}$$

assuming that distribution function is differentiable.

The symmetry and compatibility conditions are of the form

1°a $$f_{\mu_A(x_{i_1})\mu_A(x_{i_2}),\ldots\mu_A(x_{i_n})}(z_{i_1}, z_{i_2}, \ldots, z_{i_n}) =$$

$$= f_{\mu_A(x_1)\,\mu_A(x_2)\,\ldots\,\mu_A(x_n)}(z_1, z_2, \ldots, z_n) \tag{8}$$

$$2^\circ a \ \forall_{m<n} \int_{R^{n-m}} f_{\mu_A(x_1)\mu_A(x_2)\ldots\mu_A(x_n)}(z_1,z_2,\ldots,z_n)\, dz_{m+1} \ldots dz_n =$$

$$= f_{\mu_A(x_1)\mu_A(x_2)\ldots\mu_A(x_m)}(z_1,z_2,\ldots,z_m) \tag{9}$$

Because of the important meaning of max and min functions, we derive their distribution functions in the following considerations.

Let $X_1,X_2,\ldots,X_n$ be probabilistic sets defined on the following finite spaces

$$\mathcal{X}^1 = \left\{x^1_{i_1}\right\} i_1 = \overline{1,K}\ ,\ \mathcal{X}^2 = \left\{x^2_{i_2}\right\} i_2 = \overline{1,L},\ldots,\ \mathcal{X}^n = \left\{x^n_{i_n}\right\} i_n = \overline{1,M} \tag{10}$$

where $K = \mathrm{card}(\mathcal{X}^1)$, $L = \mathrm{card}(\mathcal{X}^2)$, ..., $M = \mathrm{card}(\mathcal{X}^n)$.

Taking into account the probabilistic sets $X_1,X_2,\ldots,X_n$; $X_i \in \mathcal{P}(\mathcal{X}^i)$ expressed by their defining functions $\mu_{X_j}(x^j_{i_j},\omega)$ let us consider now the max and min functions of the form

$$\mu_{\max(X_1,X_2,\ldots,X_n)}(x^1_{i_1},x^2_{i_2},\ldots,x^n_{i_n},\omega) =$$

$$= \max(\mu_{X_1}(x^1_{i_1},\omega), \mu_{X_2}(x^2_{i_2},\omega), \ldots, \mu_{X_n}(x^n_{i_n},\omega)) \tag{11}$$

$$\mu_{\min(X_1,X_2,\ldots,X_n)}(x^1_{i_1},x^2_{i_2},\ldots,x^n_{i_n},\omega) =$$

$$= \min(\mu_{X_1}(x^1_{i_2},\omega), \mu_{X_2}(x^2_{i_2},\omega), \ldots, \mu_{X_n}(x^n_{i_n},\omega)) \tag{12}$$

The resolution of this problem is provided by the following theorem.

Theorem. (i) If $X_1,X_2,\ldots,X_n$ are probabilistic sets given by their distribution functions, then the distribution function of $\max(X_1,X_2,\ldots,X_n)$ is equal to,

$$F_{\mu_{\max(X_1,X_2,\ldots,X_n)}(x^1_{i_1},x^2_{i_2},\ldots,x^n_{i_n})}(w) =$$

$$= F_{\mu_{X_1}(x^1_{i_1})\mu_{X_2}(x^2_{i_2})\ldots\mu_{X_n}(x^n_{i_n})}(w,w,\ldots,w) \qquad w \in \Omega_0 \tag{13}$$

(ii) If $X_1,X_2,\ldots,X_n$ are probabilistic sets given by their distribution functions, then the distribution function of $\min(X_1,X_2,\ldots,X_n)$ is equal to,

$$F_{\mu_{\min(X_1,X_2,\dots,X_n)}(x_{i_1}^1,x_{i_2}^2,\dots,x_{i_n}^n)}(w) = \sum_{j=1}^{n} F_{\mu_{X_j}(x_{i_j}^j)} -$$

$$- \sum_{1\leqslant j<k\leqslant n} F_{\mu_{X_j}(x_{i_j}^j)\,\mu_{X_k}(x_{i_k}^k)}(w,w)+\dots+$$

$$+ (-1)^{n+1} F_{\mu_{X_1}(x_{i_1}^1)\mu_{X_2}(x_{i_2}^2)\dots\mu_{X_n}(x_{i_n}^n)}(w,w,\dots,w) \qquad w\in\Omega_o \qquad (14)$$

Proof

(i) Taking into account the following equality

$$P\left[\max(\mu_{X_1}(x_{i_1}^1,\omega),\mu_{X_2}(x_{i_2}^2,\omega),\dots,\mu_{X_n}(x_{i_n}^n,\omega))<w\right] =$$
$$= P\left[(\mu_{X_1}(x_{i_1}^1,\omega)<w)\cap(\mu_{X_2}(x_{i_2}^2,\omega)<w)\cap\dots\cap(\mu_{X_n}(x_{i_n}^n,\omega)<w)\right]$$

and the definition of the respective multidimensional distribution,we find that Eq. (13) holds.

(ii) Bearing in mind that

$$\min(\mu_{X_1}(x_{i_1}^1,\omega),\mu_{X_2}(x_{i_2}^2,\omega),\dots,\mu_{X_n}(x_{i_n}^n,\omega)) =$$

$$= -\max(-\mu_{X_1}(x_{i_1}^1,\omega), -\mu_{X_2}(x_{i_2}^2,\omega),\dots, -\mu_{X_n}(x_{i_n}^n,\omega))$$

the following holds

$$P\left[-\max(-\mu_{X_1}(x_{i_1}^1,\omega), -\mu_{X_2}(x_{i_2}^2,\omega),\dots, -\mu_{X_n}(x_{i_n}^n,\omega))<w\right] =$$

$$= P\left[\max(-\mu_{X_1}(x_{i_1}^1,\omega), -\mu_{X_2}(x_{i_2}^2,\omega),\dots, -\mu_{X_n}(x_{i_n}^n,\omega)) < -w\right] =$$

$$= 1 - P\left[(-\mu_{X_1}(x_{i_1}^1,\omega)\leqslant -w)\cap(-\mu_{X_2}(x_{i_2}^2,\omega)\leqslant -w)\cap\dots\cap(-\mu_{X_n}(x_{i_n}^n,\omega)\leqslant -w)\right] =$$

$$= 1 - P\left[(\mu_{X_1}(x_{i_1}^1,\omega)\geqslant w)\cap(\mu_{X_2}(x_{i_2}^2,\omega)<w)\cap\dots\cap(\mu_{X_n}(x_{i_n}^n,\omega)\geqslant w)\right] =$$

$$= P\left[(\mu_{X_1}(x_{i_1}^1,\omega)<w)\cup(\mu_{X_2}(x_{i_2}^2,\omega)<w)\cup\dots\cup(\mu_{X_n}(x_{i_n}^n,\omega)<w)\right] =$$

$$= \sum_{j=1}^{n} P(\mu_{X_j}(x^j_{i_j},\omega) < w) - \sum_{1\le j<k\le n} P\left[(\mu_{X_j}(x^j_{i_j},\omega) < w) \cap (\mu_{X_k}(x^k_{i_k},\omega) < w)\right] +$$

$$+ \dots + (-1)^{n+1} P\left[(\mu_{X_1}(x^1_{i_1},\omega) < w) \cap (\mu_{X_2}(x^2_{i_2},\omega) < w) \cap \dots \cap (\mu_{X_n}(x^n_{i_n},\omega) < w)\right]$$

Since the distribution function takes the form of Eq. (14)
The theorem is proved.

Assuming additionally the independency of $\mu_{X_1}(x^1_{i_1},\omega)$ for each $x^1_{i_1} \in \mathscr{X}_1$, the distribution functions of max and min functions can be rewritten as follows

$$F_{\mu_{\max(X_1,X_2,\dots,X_n)}(x^1_{i_1},x^2_{i_2},\dots,x^n_{i_n})}(w) = \prod_{j=1}^{n} F_{\mu_{X_j}(x^j_{i_j})}(w) \tag{15}$$

$$F_{\mu_{\min(X_1,X_2,\dots,X_n)}(x^1_{i_1},x^2_{i_2},\dots,x^n_{i_n})}(w) = \sum_{j=1}^{n} F_{\mu_{X_j}(x^j_{i_j})}(w) -$$

$$- \sum_{1\le j<k\le n} F_{\mu_{X_j}(x^j_{i_j})}(w)\, F_{\mu_{X_k}(x^k_{i_k})}(w) - \dots + (-1)^{n+1} \prod_{j=1}^{n} F_{\mu_{X_j}(x^j_{i_j})}(w) \tag{16}$$

3. Decision making in a fuzzy-probabilistic environment

Considering the decision making problem in the sense of Bellman and Zadeh [1] we are looking for decision (probabilistic) set D in the form

$$D = \bigcap_{i=1}^{n} X_i \tag{17}$$

where $X_i (i = 1,2,\dots,n)$ are probabilistic sets given on the same space

$$\mathscr{X}_1 = \mathscr{X}_2 = \dots = \mathscr{X}_n = \mathscr{X} \tag{18}$$

Some of the sets X_i may represent the constraints and the rest of them can represent the goals. From formal point of view it is not necessary to distinguish between goals and constraints.

Taking into account the form of decision set we have to find the distribution function for the min function

$$\mu_D(x,\omega) = \mu_{\min(X_1,X_2,\dots,X_n)}(x,\omega) = \min(\mu_{X_1}(x,\omega),\mu_{X_2}(x,\omega),\dots,\mu_{X_n}(x,\omega)) \tag{19}$$

where $\mu_{X_i}(x,\omega)$ are defining functions of respective probabilistic sets X_i defined on the space $\mathbf{X}$.

The distribution function takes a form

$$F_{\mu_D(x)}(w) = F_{\mu_{\min(X_1,X_2,\dots,X_n)}(x)}(w) = \sum_{j=1}^{n} F_{\mu_{X_j}(x)}(w) -$$

$$\sum_{1\le j<k\le n} F_{\mu_{X_j}(x)}(w)\; F_{\mu_{X_k}(x)}(w) - \dots + (-1)^{n+1}\prod_{j=1}^{n} F_{\mu_{X_j}(x)}(w) \tag{20}$$

assuming the independency of all $\mu_{X_i}(x,\omega)$.

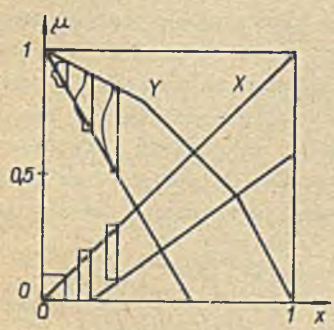

Fig. 1. Goal and constraint as probabilistic sets

Having goal or constraint defined as probabilistic set on the space $\mathbf{X}$, it means that for each $x \in \mathbf{X}$ the value of membership function is not determined uniquely in $[0,1]$ - interval but it is given by the respective distribution function or density function (Fig. 1). The border lines determine the boundaries of the respective density functions.

Considering for simplicity two probabilistic sets i.e.

$$X_1 = X, \quad X_2 = Y$$

the distribution function and the density function in the case of independent $\mu_X(x,\omega)$ and $\mu_Y(x,\omega)$ may be written as

$$F_{\mu_D(x)}(w) = F_{\mu_X(x)}(w) + F_{\mu_Y(x)}(w) - F_{\mu_X(x)}(w)\, F_{\mu_Y(x)}(w) \tag{21}$$

and

$$f_{\mu_D(x)}(w) = f_{\mu_X(x)}(w)\left[1 - F_{\mu_Y(x)}(w)\right] + f_{\mu_Y(x)}(w)\left[1 - F_{\mu_X(x)}(w)\right] \tag{22}$$

Having the distribution function $F_{\mu_D(x)}(w)$ or the density function $f_{\mu_D(x)}(w)$ we can carry out the moment analysis. Taking into account the first monitors one could decide which alternative can be chosen. For example we can easy obtain the mathematical expectation (mean value) $E[\mu_D(x)]$ and the variance $V[\mu_D(x)]$ of $\mu_D(x,\omega)$ for each $x \in \mathcal{X}$.

The problem of evaluation of the final decision $x^* \in \mathcal{X}$ can be solved in many ways e.g.

$$\begin{aligned}
&(i) && x^* = \left\{ x \in \mathcal{X} \;\middle|\; E[\mu_D(x)] \longrightarrow \max \right\} \\
&(ii) && x^* = \left\{ x \in \mathcal{X} \;\middle|\; \begin{matrix} E[\mu_D(x)] \longrightarrow \max \\ V[\mu_D(x)] \longrightarrow \min \end{matrix} \right\} \\
&(iii) && x^* = \left\{ x \in \mathcal{X} \;\middle|\; \frac{E[\mu_D(x)]}{\sqrt{V[\mu_D(x)]}} \longrightarrow \max \right\}
\end{aligned} \tag{23}$$

etc.

These topics will be the subject of further investigations.

4. Numerical examples

1. Let us consider two independent probabilistic sets X and Y in the case when $\mu_X(x,\omega)$ is uniformly distributed in the interval $[a(x), b(x)]$, (the boundaries of density function are dependend on x) and $\mu_Y(x,\omega)$ is a common membership function given as (Fig. 2):

$$\mu_Y(x) = \begin{cases} 1 & \text{for} \quad x \leq x_0 \\ 0 & \text{for} \quad x > x_0 \end{cases} \tag{24}$$

or

$$\mu_Y(x) = \mathbb{1}(x - x_0)$$

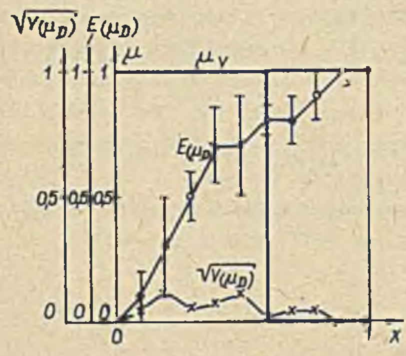

Fig. 2. Goal as probabilistic set and conshaint as common nonfuzzy set

The distribution function of $\mu_D(x,\omega)$ has a simple form

$$F_{\mu_D(x)}(w) = \begin{cases} F_{\mu_X(x)}(w) & \text{for} \quad x \leq x_0 \\ 1 & \text{for} \quad x > x_0 \end{cases} \tag{25}$$

The mathematical expectation mean value for this case can be found for each $x \in X$ as

$$E[\mu_D(x)] = \begin{cases} \dfrac{a(x) + b(x)}{2} & \text{for } x \leqslant x_0 \\ 0 & \text{for } x > x_0 \end{cases} \tag{26}$$

The variance of $\mu_D(x,\omega)$ in this example depends only on the length of the interval $[a(x), b(x)]$ and is an increasing function od the length i.e.

$$V[\mu_D(x)] = \begin{cases} \dfrac{[b(x) - a(x)]^2}{12} & \text{for } x \leqslant x_0 \\ 0 & \text{for } x > x_0 \end{cases} \tag{27}$$

The values of dispersion $\sqrt{V[\mu_D(x)]}$ are shown in Fig. 2. Now the problem of evaluation of the final decision $\bar{x} \in X$ should be solved according to (i) (ii), or (iii).

2. Now we will consider more general case when $\mu_X(x,\omega) = \frac{x}{\Delta}$ for $\omega \in \Omega$ and $0 \leqslant x \leqslant \Delta$, $\mu_Y(x,\omega)$ is exponentially distributed in the interval $[0,1]$ as in Fig. 3.

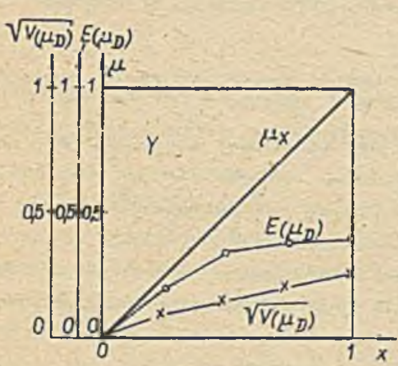

Fig. 3. Goal as common fuzzy set and constraint as probabilistic set

The density function of $\mu_D(x,\omega)$ has the form

$$f_{\mu_D(x)}(w) = \delta\left(w - \frac{x}{\Delta}\right)\left[1 - \frac{1-e^{-\lambda(x)w}}{1-e^{-\lambda(x)}}\right] +$$

$$+ \frac{\lambda(x)e^{-\lambda(x)w}}{1-e^{-\lambda(x)}}\left[1 - \mathbf{1}\left(w - \frac{x}{\Delta}\right)\right]$$

Determining the expectation (mean value) of $\mu_D(x,\omega)$ we have

$$E[\mu_D(x)] = \frac{1}{1-e^{-\lambda(x)}}\left[\frac{1}{\lambda(x)}\left(1-e^{-\lambda(x)\frac{x}{\Delta}}\right) - \frac{x}{\Delta}e^{-\lambda(x)}\right]$$

For the variance of $\mu_D(x,\omega)$ we obtain

$$V[\mu_D(x)] = \frac{1}{1-e^{-\lambda(x)}}\left\{\frac{2}{\lambda^2(x)} - \frac{x^2}{\Delta^2}e^{-\lambda(x)} - \frac{2}{\lambda(x)}e^{-\lambda(x)\frac{x}{\Delta}}\left[\frac{x}{\Delta} + \frac{1}{\lambda(x)}\right]\right\} -$$

$$- \frac{1}{\left(1-e^{-\lambda(x)}\right)^2}\left\{\frac{1}{\lambda(x)}\left(1 - e^{-\lambda(x)\frac{x}{\Delta}}\right) - \frac{x}{\Delta}e^{-\lambda(x)}\right\}^2$$

Numerical calculation of $E[\mu_D(x)]$ and $V[\mu_D(x)]$ for $\lambda(x) = 1$ and $A=1$ are given also in Fig. 3. These results show that there are differences between min $\{E[\mu_X(x)], E[\mu_Y(x)]\}$ and $E[\mu_D(x)]$ for some $x \in \mathcal{X}$.

5. Concluding remarks

The distribution function description (representation) of probabilistic sets proposed here, seems to be suitable in obtaining a solution of some problems of decision making.
The obtained numerical results show the differences between the values $E[\mu_D(x)]$ and min $\{E[\mu_X(x)], E[\mu_Y(x)]\}$ for determined regions od X. It can have an influence on choosing an optimal alternative.

There is also a possibility to obtain some interesting results for other operations on probabilistic sets then "$\cap$" and "$\cup$" for example

$$U = X \cdot Y$$

where

$$\mu_U(x,\omega) = \mu_X(x,\omega) \cdot \mu_Y(x,\omega)$$

etc.

and for given distribution functions or density functions it is possible to compare the respective results.

The distribution functions of max and min functions are also very useful in the decision making system called fuzzy probabilistic controller where they are both used simultaneously. The above mentioned decision making system will be considered in separate papers [4].

The first moniters obtained by means of distribution function or density function may be used to the probability criteria useful in decision making, like Chebyshev's inequality and others. This analysis will be the subject of further investigation.

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ИНТЕГРАЛЬНОЕ ОПИСАНИЕ ВЕРОЯТНОСТНЫХ МНОЖЕСТВ И ЕГО ПРИМЕНЕНИЕ В ПРИНЯТИ РЕШЕНИЙ

Р е з ю м е

В работе представлены проблемы принятия решений описываемые и решаемые на основе вероятностных множеств.

Рассмотрено интегральное описание вероятностных множеств важное для теории и практического применения в процессе принятия решений. Рассуждения проиллюстрировано численными примерами.

DYSTRYBUANTOWY OPIS ZBIORÓW PROBABILISTYCZNYCH I JEGO ZASTOSOWANIE W PODEJMOWANIU DECYZJI

S t r e s z c z e n i e

W pracy przedstawiono problemy podejmowania decyzji opisane i rozwiązane w oparciu o koncepcję zbioru probabilistycznego.

Przedyskutowano dystruantowy opis (reprezentację) zbiorów probabilistycznych ważny z teoretycznego punktu widzenia, jak również jego zastosowania w podejmowaniu decyzji zilustrowano przykładami numerycznymi.