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TOWARDS DECISION-MAKING IN A FUZZY PROBABILISTIC ENVIRONMENT AND A NEW IDEA OF FUZZY PROBABILISTIC CONTROLLER

Summary. Decision making problems tied together by means of the concept of probabilistic set are here presented.
First, the problem of decision making in the sense of Bellman and Zadeh in fuzzy probabilistic environment is mentioned. Then it is discussed a new idea of fuzzy probabilistic controller useful in control tasks of ill-known (weakly - defined) processes what enables to investigate the control strategies used by human operators.

1. Introduction

For the first time the Bellman and Zadeh [1] approach to decision - making in a fuzzy environment was employed and applied. Zadeh's [12]linguistic approach to fuzzy systems has motivated many works dealing with the synthesis and some aspects of the analysis of decision-making system called fuzzy logic controller demonstrated by several autors viz. Mamdani and Assilian [9], Mamdani [10], van Nauta Lemke and Kickert [8], Tong [11] and others.

The concept of probabilistic set [6] is defined regarding the value of membership function of fuzzy set as a random variable depending on parameter. This notion is introduced for the reason of the problems of ambiguity and subjectivity of the human observers operators that cannot be often determined uniquely in [0,1]- interval. Taking into account this notion and its distribution function representation [2,3] it is possible to consider algorithms of decision-making in a fuzzy probabilistic environment. In this paper there will be discussed some problems which may appear at initial stage to formalize the problems of construction of the decision-making algorithms useful for computational tractability.

The problems of decision-making in the sense of Bellman and Zadeh is outlined and the decision - making algorithms due to the decision-making systems (fuzzy probabilistic logic controller) are presented. Such decision-making systems which may be used to implement more general control policies in multidimensional case are discussed too.

2. Basic notions and definitions

The fundamental notion used in our considerations is the notion of probabilistic set. In the beginning we will define the following three terms [6]

$(\Omega,\mathcal{B},P)$ is a parameter space,

$(\Omega_0,\mathcal{B}_0) = ([0,1]$, Borel sets) is a characteristic spade,

$\mathcal{M} = \{\mu \mid \mu: \Omega \to \Omega_0 \ (\mathcal{B},\mathcal{B}_0)$ -measurable function$\}$ is a family of characteristic variables.

Now let us remind the definition of probabilistic set [6]

Definition 1 A probabilistic set A on the space X is defined by a defining function μ_A

$$\begin{array}{c}\mu_A : \mathfrak{X} \times \Omega \to \Omega_0 \\ \cup \qquad \cup \\ (x,\omega) \mapsto \mu_A(x,\omega)\end{array} \tag{1}$$

where $\mu_A(x,\cdot)$ is the $(\mathcal{B},\mathcal{B}_0)$-measurable function for each fixed $x \in \mathfrak{X}$. If μ_A is a defining function the of course, is always valid

$$\forall_{x\in\mathfrak{X}} \forall_{z\in[0,1]} \{\omega : \mu_A(x,\omega) < z\} \in \mathcal{B} \tag{2}$$

Choosing arbitrarily the numbers n and the points $x_1,x_2,\dots,x_n$ we can introduce the n-dimentional distribution functions F_n of probabilities in the form

$$\begin{aligned} &F_n(x_1,x_2,\dots,x_n;z_1,z_2,\dots,z_n) = \\ &P(\{\omega : \mu_A(x_1,\omega) < z_1,\ \mu_A(x_2,\omega < z_2,\dots,\ \mu_A(x_n,\omega) < z_n\}) = \\ &= P(\mu_A(x_1,\omega) < z_1,\ \mu_A(x_2,\omega) < z_2,\dots,\ \mu_A(x_n,\omega) < z_n) \end{aligned} \tag{3}$$

satisfying the symmetry and compatibility conditions.

The distribution function representation (description) will be used to obtain the distribution fumctions for the so called Max and Min functions of a collection of probabilistic sets defined as follows.

Let be $X_1,X_2,\dots,X_n$ the probabilistic sets defined on the respective spaces

$$\begin{aligned} \mathfrak{X}_1 &= \{x_{11},x_{12},\dots,x_{1i_1},\dots,x_{1K}\} \\ \mathfrak{X}_2 &= \{x_{21},x_{22},\dots,x_{2i_2},\dots,x_{2L}\} \\ \mathfrak{X}_n &= \{x_{n1},x_{n2},\dots,x_{ni_n},\dots,x_{nM}\} \end{aligned} \tag{4}$$

where $1 \le i_1 \le K$, $1 \le i_2 \le L, \dots,$ $1 \le i_n \le M$.

Definition 2. We call a probabilistic set M a Maximum function of probabilistic sets $X_1, X_2, \ldots, X_n$, if the following equation holds true

$$\mu_M(x_{1i_1}, x_{2i_2}, \ldots, x_{ni_n}, \omega) = \mathrm{Max}\left[\mu_{X_1}(x_{1i_1}, \omega), \mu_{X_2}(x_{2i_2}, \omega), \ldots, \mu_{X_n}(x_{ni_n}, \omega)\right] \tag{5}$$

Definition 3. A probabilistic set W is called a Minimum function of probabilistic set $X_1, X_2, \ldots, X_n$ if

$$\mu_W(x_{1i_1}, x_{2i_2}, \ldots, x_{ni_n}, \omega) = \mathrm{Min}\left[\mu_{X_1}(x_{1i_1}, \omega), \mu_{X_2}(x_{2i_2}, \omega), \ldots, \mu_{X_n}(x_{ni_m}, \omega)\right] \tag{6}$$

The distribution functions of the above defined function of the form

$$F_{\mu_M(x_{1i_1}, x_{2i_2}, \ldots, x_{ni_n})}(m) = P\left[\mu_M(x_{1i_1}, x_{2i_2}, \ldots, x_{ni_n}, \omega) < m\right] =$$

$$F_{\mu_{X_1}(x_{1i_1})\, \mu_{X_2}(x_{2i_2}) \ldots \mu_{X_n}(x_{ni_n})}(m, m, \ldots, m) \tag{7}$$

$$F_{\mu_W(x_{1i_1}, x_{2i_2}, \ldots, x_{mi_n})}(w) = P\left[\mu_W(x_{1i_1}, x_{2i_2}, \ldots, x_{mi_m}, \omega) < w\right] =$$

$$= \sum_{j=1}^{n} F_{\mu_{X_j}(x_{ji_j})}(w) - \sum_{1 \le j < k \le n} F_{\mu_{X_j}(x_{ji_j})\, \mu_{X_k}(x_{ki_k})}(w, w) +$$

$$+ \sum_{1 \le j < k < l \le n} F_{\mu_{X_j}(x_{ji_j})\, \mu_{X_k}(x_{ki_k})\, \mu_{X_l}(x_{li_l})}(w, w, w) - \ldots$$

$$\ldots + (-1)^{n+1} F_{\mu_{X_1}(x_{1i_1})\, \mu_{X_2}(x_{2i_2}) \ldots \mu_{X_m}(x_{ni_m})}(w, w, \ldots, w) \tag{8}$$

Assuming the independency of $\mu_{X_1}(x_{1i_1}, \omega)$ we get

$$F_{\mu_M(x_{1i_1}, x_{2i_2}, \ldots, x_{mi_n})}(m) = F_{\mu_{X_1}(x_{1i_1})}(m) \cdot F_{\mu_{X_2}(x_{2i_2})}(m) \ldots$$

$$\ldots {}^{F}\mu_{X_n}(x_{ni_n})^{(w)} \quad {}^{F}\mu_W(x_{1i_1}, x_{2i_2}, \ldots, x_{ni_n})^{(w)} = {}^{F}\sum_{j=1}^{n} {}^{F}\mu_{X_j}(x_{ji_j})^{(w)} -$$

$$-\sum_{1\le j<k\le n} {}^{F}\mu_{X_j}(x_{ji_j})^{(w)} \cdot {}^{F}\mu_{X_k}(x_{ki_k})^{(w)} + \sum_{1\le j<k<l\le n} {}^{F}\mu_{X_j}(x_{ji_j})^{(w)} \cdot$$

$$\cdot {}^{F}\mu_{X_k}(x_{ki_k})^{(w)} \; {}^{F}\mu_{X_l}(x_{li_l})^{(w)} - \ldots + (-1)^{n+1} \, {}^{F}\mu_{X_1}(x_{1i_1})^{(w)} \cdot \tag{9}$$

$$\cdot {}^{F}\mu_{X_2}(x_{2i_2})^{(w)} \ldots {}^{F}\mu_{X_n}(x_{ni_n})^{(w)} \tag{10}$$

The above presented distribution functions will play an essential role in the further considerations.

3. Decision-making in a fuzzy probabilistic environment

Let us consider a decision-making problem in the sense of Bellman and Zadeh [1] and let us look for decision set W fuzzy probabilistic in the form

$$W = \bigcap_{i=1}^{n} X_i \tag{11}$$

X_i $i = 1,2,\ldots,n$ are probabilistic sets defined on the same space $\mathcal{X}$ and some of them may represent constraints and the rest of them goals. The distribution function for

$$\mu_W(x,\omega) = \mathrm{Min}\left[\mu_{X_1}(x,\omega), \mu_{X_2}(x,\omega), \ldots, \mu_{X_n}(x,\omega)\right] \tag{12}$$

takes the form [2,3]

$${}^{F}\mu_W(x)^{(w)} = P\left[\mu_W(x) < w\right] = \sum_{j=1}^{n} {}^{F}\mu_{X_j}(x)^{(w)} - \sum_{1\le i<j\le n} {}^{F}\mu_{X_j}(x)\,\mu_{X_k}(x)^{(w,w)} +$$

$$+ \sum_{1\le j<k<l\le n} {}^{F}\mu_{X_j}(x)\,\mu_{X_k}(x)\,\mu_{X_l}(x)^{(w,w,w)} - \ldots +$$

$$+ (-1)^{n+1} \, {}^{F}\mu_{X_1}(x)\mu_{X_2}(x) \ldots \mu_{X_n}(x)^{(w,w,\ldots,w)} \tag{13}$$

Now consider for simplicity only two probabilistic set (Fig. 1) i.e.

$$X_1 = X, \quad X_2 = Y$$

The distribution function for defining function of decision set has the form

$$F_{\mu_W(x)}(w) = F_{\mu_X(x)}(w) + F_{\mu_Y(x)}(w) - F_{\mu_X(x)\,\mu_Y(x)}(w,w) \qquad (14)$$

The distribution function and the density function of μ_W in the case of independed μ_X and μ_Y may be written as

$$F_{\mu_W(x)}(w) = F_{\mu_X(x)}(w) + F_{\mu_Y(x)}(w) - F_{\mu_X(x)}(w) \cdot F_{\mu_Y(x)}(w) \qquad (15)$$

and

$$f_{\mu_W(x)}(w) = f_{\mu_X(x)}(w)\left[1 - F_{\mu_Y(x)}(w)\right] + f_{\mu_Y(x)}(w)\left[1 - F_{\mu_X(x)}(w)\right] \qquad (16)$$

As an illustration of the above presented considerations we have an interesting case when $\mu_X(x) = \frac{x}{\Delta}$ for $0 \leq x \leq \Delta$ and $\mu_Y(x,\omega)$ is uniformly distributed in the interval $\left[a(x),\, b(x)\right]$ as in Fig 2.

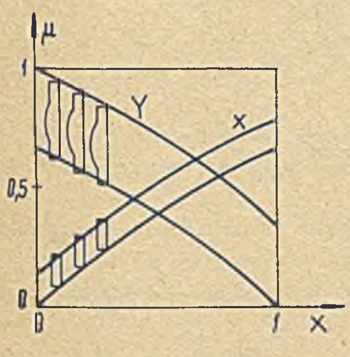

Fig. 1. Goal and constraint as as probabilistic sets

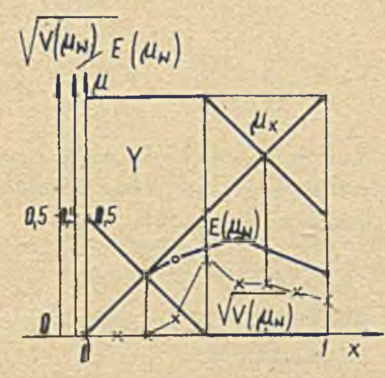

Fig. 2. Goal as common fuzzy set and constraint as probabilistic set

The density function is of the form

$$f_{\mu_W(x)}(w) = \delta\left(w - \frac{x}{\Delta}\right)\left[1 - \frac{w - a(x)}{b(x) - a(x)}\right] + \frac{1}{b(x) - a(x)}\left[1 - \mathbf{1}\left(w - \frac{x}{\Delta}\right)\right] \qquad (17)$$

The mathematical expectation (mean value) of μ_W is equal to

$$E\left[\mu_W(x)\right] = \frac{1}{b(x) - a(x)} \left[\frac{x}{\Delta} b(x) - \frac{1}{2}\frac{x^2}{\Delta^2} - \frac{1}{2} a^2(x)\right] \tag{18}$$

and for the variance of μ_W we have

$$V\left[\mu_W(x)\right] = \frac{1}{b(x) - a(x)} \left[\frac{x^2}{\Delta^2} b(x) - \frac{1}{3} a^3(x) - \frac{2}{3}\frac{x^3}{\Delta^3}\right] -$$

$$- \frac{1}{\left[b(x) - a(x)\right]^2} \left[\frac{x}{\Delta} b(x) - \frac{1}{2}\frac{x^2}{\Delta^2} - \frac{1}{2} a^2(x)\right]^2 \tag{19}$$

Numerical calculations for $\Delta = 1$ of $E\left[\mu_W(x)\right]$ and $V\left[\mu_W(x)\right]$ presented in Fig. 2. show the differences between $\mathrm{Min}\left\{E\left[\mu_X(x)\right];\ E\left[\mu_Y(x)\right]\right\}$ and $E\left[\mu_W(x)\right]$ for some $x \in \mathcal{X}$.

As a final result we must make up decision which nonfuzzy x_{opt} should be chosen. It could be performed in many ways, especially,

$$\text{i} \qquad x_{opt} = \left\{x \in \mathcal{X} \;\middle|\; E\left[\mu_W(x)\right] \longrightarrow \mathrm{Max}\right\} \tag{20}$$

$$\text{ii} \qquad x_{opt} = \left\{x \in \mathcal{X} \;\middle|\; \begin{array}{l} E\left[\mu_W(x)\right] \longrightarrow \mathrm{Max} \\ V\left[\mu_W(x)\right] \longrightarrow \mathrm{Min} \end{array}\right\} \tag{21}$$

$$\text{iii} \qquad x_{opt} = \left\{x \in \mathcal{X} \;\middle|\; \frac{E\left[\mu_W(x)\right]}{V\left[\mu_W(x)\right]} \longrightarrow \mathrm{Max}\right\} \tag{22}$$

etc.

Having the mathematical expectation and the variance of μ_W it is possible to use also other probability criteria like Chebyshev's inequality, Bienaymé's inequality etc.

4. Construction of fuzzy probabilistic decision making algorithms in control problems

Now we will discuss the decision making system which may be employed in many control tasks of the complex processes [4,5].
We will assume that in the simplest case (single input - single output) a fuzzy probabilistic decision making system (fuzzy probabilistic logic controller) may be formed on the basis of a collection (set) of control rules

$$\left\{\text{if } X_i \text{ then } U_i\right\}_{i=1,2,\dots,N} = \left\{X_i \Rightarrow U_i\right\}_{i=1,2,\dots,N} = \left\{X_i \times U_i\right\}_{i=1,2,\dots,N} \tag{23}$$

where X_i and U_i are probabilistic sets of the state and control variables defined by the use of respective discrete spaces $\mathcal{X}_1 = \mathcal{X}$ und $\mathcal{X}_2 = \mathcal{U}$ i.e.

$$\mu_{X_i} : \mathcal{X} \times \Omega \longrightarrow \Omega_{c,x}$$
$$(x,\omega) \longmapsto \mu_{X_i}(x,\omega)$$
$$\tag{24}$$
$$\mu_{U_i} : \mathcal{U} \times \Omega \longrightarrow \Omega_{c,u}$$
$$(u,\omega) \longmapsto \mu_{U_i}(u,\omega)$$

It means, of course, that the ambiguity and subjectivity of human operators of the process may not be determined uniquely in $[0,1]$ - interval.

For singular implication we have

$$\mu_{X_i \Rightarrow U_i}(x_j,u_k,\omega) = \mathrm{Min}\left[\mu_{X_i}(x_j,\omega),\ \mu_{U_i}(u_k,\omega)\right] \tag{25}$$

The "memory" of fuzzy probabilistic controller created by the set of rules may be written in the form of fuzzy probabilistic relation

$$R = \bigcup_{i=1}^{N} (X_i \Rightarrow U_i) = \bigcup_{i=1}^{N} R_i \tag{26}$$

for which we have

$$\mu_R(x_j,u_k,\omega) = \underset{1\leqslant i\leqslant N}{\mathrm{Max}}\ \mu_{X_i \Rightarrow U_i}(x_j,u_k,\omega)$$

$$= \underset{1\leqslant i\leqslant n}{\mathrm{Max}}\ \mathrm{Min}\left[\mu_{X_i}(x_j,\omega),\ \mu_{U_i}(u_k,\omega)\right] \tag{27}$$

Now having distribution functions of $\mu_{X_i}(x_j,\omega)$ and $\mu_{U_i}(u_k,\omega)$ for each $x_j \in \mathcal{X}$ and $u_k \in \mathcal{U}$ we can find distribution function of $\mu_R(x_j,u_k,\omega)$ assuming independency of $\mu_{X_i}(x_j,\omega)$

$$F_{\mu_R(x_j,u_k)}(w) = \prod_{i=1}^{N}\left[F_{\mu_{X_i}(x_j)}(w) + F_{\mu_{U_i}(u_k)}(w) - F_{\mu_{X_i}(x_j)}(w)\cdot F_{\mu_{U_i}(u_k)}(w)\right] \tag{28}$$

Then for each set of the state of the process X' (nonfuzzy, fuzzy or probabilistic set) the value of control variable may be computed using the compositional rule of inference

$$U' = X' \circ R \tag{29}$$

what in the case of probabilistic sets leads to the equations

$$\mu_{U'}(u_k,\omega) = \operatorname*{Max}_{x_j \in X} \operatorname{Min}\left[\mu_{X'}(x_j,\omega),\ \mu_R(x_j,u_k,\omega)\right] \tag{30}$$

and assuming the independency of $\mu_{X'}$ and μ_R

$$^F\mu_{U'(u_k)}(w) = \prod_{x_j \in X}\Big[{}^F\mu_{X'(x_j)}(w) + {}^F\mu_{R(x_j,u_k)}(w) - {}^F\mu_{X'(x_j)}(w) \cdot {}^F\mu_{R(x_j,u_k)}(w)\Big] \tag{31}$$

Having the distribution function of the probabilistic set U' it is possible to carry out the moment analysis what forms one of the most important features distinguishing this method from respective considerations in previous papers mentioned above.

The knowledge of distribution function $\mu_{U'}$ allows to calculate monitors of the probabilistic set U' (viz. membership function, first monitor, second monitor etc.) facilitating the solution of a problem of a final decision-making [2,3].

Now consider a numerical example illustrating the idea of fuzzy probabilistic logic controller.

Let fuzzy sets of the state and control be given by means of the following membership functions:

	x_1	x_2	x_3	x_4
μ_{X_1}	1	0,5	0	0
μ_{X_2}	0	1	0,5	0
μ_{X_3}	0	0,5	1	1

	u_1	u_2	u_3	u_4
μ_{U_1}	1	0,5	0,5	0
μ_{U_2}	0	1	1	0
μ_{U_3}	0	0	0,5	1

The fuzzy logic controller consisting of 3 control rules has the fuzzy relation equal to

$$\mu_R = \begin{bmatrix} 1 & 0{,}5 & 0{,}5 & 0 \\ 0{,}5 & 1 & 1 & 0{,}5 \\ 0 & 0{,}5 & 0{,}5 & 1 \\ 0 & 0 & 0{,}5 & 1 \end{bmatrix} \tag{32}$$

Taking for the same sets of the state and control the respective distribution functions we get easy the respective matrix $F_{\mu_R(x_j,u_k)}(w)$.

Assuming that the input set is equal to $\mu_{X'}(x_j) = \begin{bmatrix} 0 & 0 & 1 & 0 \end{bmatrix}$ what is equivalent to

$$F_{\mu_{X'}(x_j)}(w) = \begin{bmatrix} F_{\mu_{X'}(x_1)}(w) & F_{\mu_{X'}(x_2)}(w) & F_{\mu_{X'}(x_3)}(w) & F_{\mu_{X'}(x_4)}(w) \end{bmatrix}$$

$$= \begin{bmatrix} \begin{cases} 0 \text{ for } w<0 \\ 1 \text{ for } w\geqslant 0 \end{cases} & \begin{cases} 0 \text{ for } w<0 \\ 1 \text{ for } w\geqslant 0 \end{cases} & \begin{cases} 0 \text{ for } w<1 \\ 1 \text{ for } w\geqslant 1 \end{cases} & \begin{cases} 0 \text{ for } w<0 \\ 0 \text{ for } w\geqslant 0 \end{cases} \end{bmatrix} \tag{33}$$

The output set U' obtained by the use of compositional rule of inference is equal to

$$\mu_{U'}(u_k) = \begin{bmatrix} 0 & 0,5 & 0,5 & 1 \end{bmatrix} \tag{34}$$

and the equivalent output obtained by means of distribution functions is equal to

$$F_{\mu_{U'}(u_k)}(w) = \begin{bmatrix} F_{\mu_{U'}(u_1)}(w) & F_{\mu_{U'}(u_2)}(w) & F_{\mu_{U'}(u_3)}(w) & F_{\mu_{U'}(u_4)}(w) \end{bmatrix}$$

$$= \begin{bmatrix} \begin{cases} 0 \text{ for } w<0 \\ 1 \text{ for } w\geqslant 0 \end{cases} & \begin{cases} 0 \text{ for } w<0,5 \\ 1 \text{ for } w\geqslant 0,5 \end{cases} & \begin{cases} 0 \text{ for } w<0,5 \\ 1 \text{ for } w\geqslant 0,5 \end{cases} & \begin{cases} 0 \text{ for } w<1 \\ 1 \text{ for } w\geqslant 1 \end{cases} \end{bmatrix} \tag{35}$$

This expected result shows also that the presented fuzzy probabilistic algorithm is constructed correctly.

5. Fuzzy probabilistic logic controller in multidimentional case

We will assume that in multid mensional case control algorithm may be also formed on the basis of a collection set of controll rules as in Fig.3

$$\left\{ \text{if } X_{1(i)} \text{ then if } X_{2(i)} \text{ then if } \dots X_{p(i)} \text{ then } \bar{Y}_{(i)} \right\} = \bar{R}_i \tag{36}$$

where $X_{1(i)}, X_{2(i)}, \dots X_{p(i)}$ are inputs of the i-th rule and the outputs of i-th rule may be written in the form of

$$\bar{X}_{p+1(i)} = \bar{Y}_{(i)} = X_{p+1(i)} \times X_{p+2(i)} \times \dots \times X_{p+r(i)}$$

$$X_{1(i)} \in \mathcal{F}(\mathcal{X}_1),\quad X_{2(i)} \in \mathcal{F}(\mathcal{X}_2), \dots,\ X_{p+r(i)} \in \mathcal{F}(\mathcal{X}_{p+r}) \tag{37}$$

$$\bar{R}_i \in \mathcal{F}(\mathcal{X}_1 \times \mathcal{X}_2 \times \mathcal{X}_3 \times \dots \times \mathcal{X}_{p+r})$$

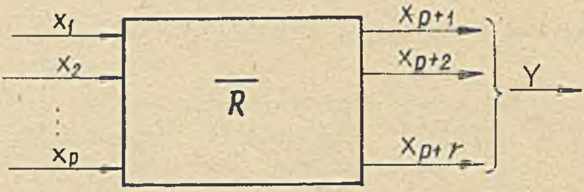

Fig. 3. Diagramatic representation of fuzzy probabilistic controller in multidimensional case

For the membership functions we have

$$\mu_{X_{1(i)}} : \mathcal{X}_1 \to [0,1]$$

$$\mu_{\bar{R}_i} : \mathcal{X}_1 \times \mathcal{X}_2 \times \dots \times \mathcal{X}_{p+r} \to [0,1] \qquad (38)$$

To translate the complex implication most authors use the Min operator and we will also use this operator for cartesian product of the outputs.

The relation creating the "memory" of the controller may be written in the form

$$\bar{R} = \bigcup_{i=1}^{U} (X_{1(i)} \Rightarrow X_{2(i)} \Rightarrow \dots \Rightarrow X_{p(i)} \Rightarrow \bar{Y}_{(i)}) = \bigcup_{i=1}^{N} \bar{R}_i \qquad (39)$$

N - denotes the number of the rules

The membership function of such relation is given by the formula

$$\mu_{\bar{R}}(x_{1i_1}, x'_{1i_2}, \dots, x_{pi_p}, x_{p+1i_{p+1}}, \dots, x_{p+ri_{p+r}}) =$$

$$= \max_{1 \le i \le N} \min \Big[\mu_{X_{1(i)}}(x_{1i_1}), \mu_{X_{2(i)}}(x_{2i_2}), \dots, \mu_{X_{p(i)}}(x_{pi_p}), \mu_{X_{p+1}}(x_{p+1i_{p+1}}),$$

$$\dots, \mu_{X_{p+r(i)}}(x_{p+ri_{p+r}})\Big] \qquad (40)$$

Having inputs $X'_1, X'_2, \dots, X'_p$ it is possible to use the compositional rule of inference. Hence

$$\bar{Y}' = X'_p \circ \dots \circ X'_2 \circ X'_1 \circ \bar{R} \qquad (41)$$

and for the membership function of multidimensional output we have

$$\mu_{\overline{Y}'}(x_{p+1i_{p+1}},\dots, x_{p+ri_{p+r}}) = \underset{x_{pi_p}\in X_p}{\mathrm{Max}}\ (\mathrm{Min}\ \mu_{X_p'}(x_{pi_p}),\dots \tag{42}$$

$$\dots, \underset{x_{1i_1}\in X_1}{\mathrm{Max}}\ (\mathrm{Min}\ \mu_{X_1'}(x_{1i_1}), \mu_{\overline{R}}(x_{1i_1},x_{2i_2},\dots,x_{pi_p},x_{p+1i_{p+1}},\dots x_{p+ri_{p+r}}))$$

what leads to the following equation

$$\mu_{\overline{Y}'}(x_{p+1i_{p+1}},\dots,x_{p+ri_{p+r}}) = \underset{\substack{x_{1i_1}\in X_1\\ x_{2i_2}\in X_2\\ \vdots\\ x_{pi_p}\in X_p}}{\mathrm{Max}}\ (\mathrm{Min}\ \mu_{X_p'}(x_{pi_p}),\dots$$

$$\dots,\mu_{X_1'}(x_{1i_1}), \mu_{\overline{R}}(x_{1i_1},x_{2i_2},\dots,x_{pi_p},x_{p+1i_{p+1}},\dots,x_{p+ri_{p+r}})) \tag{43}$$

The output $\overline{Y}'$ is given by the respective relation which has the membership function

$$\mu_{\overline{Y}'}(x_{p+1i_{p+1}},\dots,x_{p+ri_{p+r}})$$

Having this relation we can look for the final fuzzy sets of respective outputs. This problem can be perfomed in many ways, especially by the respective projection i.e.

$$\mu_{X_{p+k}'}(x_{p+ki_{p+k}}) = \underset{\substack{x_{p+1i_{p+1}}\in X_{p+1}\\ \vdots\\ x_{p+k-1i_{p+k-1}}\in X_{p+k-1}\\ x_{p+k+1i_{p+k+1}}\in X_{p+k+1}\\ \vdots\\ x_{p+ri_{p+r}}\in X_{p+r}}}{\mathrm{Max}}\ \mu_{\overline{Y}'}(x_{p+1i_{p+1}},\dots x_{p+ri_{p+r}}) \tag{44}$$

Where $1 \leq k \leq r$

Assuming that all inputs and outputs are probabilistic sets we can obtain the respective formulas for fuzzy probabilistic logic controller. Taking into account the compositional rule of inference we have to find the distribution function of output $\bar{Y}'$ having the distribution functions of all inputs $X_1', X_2', \ldots, X_p'$. In other words we have to pass for distribution functions of the inputs the following equation

$$\mu_{\bar{Y}'}(x_{p+1i_{p+1}}, \ldots, x_{p+ri_{p+r}}, \omega) = \max_{x_{pi_p} \in X_p} \operatorname{Min} \mu_{X_p'}(x_{pi_p}, \omega),$$

$$\ldots, \max_{x_{1i_1} \in X_1} \operatorname{Min} \mu_{X_1'}(x_{1i_1}, \omega), \mu_{\bar{R}}(x_{1i_1}, x_{2i_2}, \ldots, x_{pi_p}, x_{p+1i_{p+1}}, \ldots, x_{p+ri_{p+r}}, \omega) \tag{45}$$

From formula given above we get

$$\mu_{\bar{Y}'}(x_{p+1i_{p+1}}, \ldots, x_{p+ri_{p+r}}, \omega) = \max_{\substack{x_{1i_1} \in X_1 \\ x_{2i_2} \in X_2 \\ \vdots \\ x_{pi_p} \in X_p}} (\operatorname{Min} \mu_{X_p'}(x_{pi_p}, \omega), \ldots$$

$$\ldots, \mu_{X_2'}(x_{2i_2}, \omega), \mu_{X_1'}(x_{1i_1}, \omega), \mu_{\bar{R}}(x_{1i_1}, x_{1i_2}, \ldots, x_{pi_p}, x_{p+1i_{p+1}}, \ldots, x_{p+ri_{p+r}}, \omega)) \tag{46}$$

First, we find the distribution function of the fuzzy probabilistic relation $\bar{R}$ which has a form

$$^F\mu_R(x_{1i_1}, x_{2i_2}, \ldots, x_{pi_p}, x_{p+1i_{p+1}}, \ldots, x_{p+ri_{p+r}})^{(w)} =$$

$$= \prod_{i=1}^{N} \left[\sum_{j=1}^{p+r} {}^F\mu_{X_{ji}}(x_{ji_j})^{(w)} - \sum_{1 \le j < k \le n} {}^F\mu_{X_{ji}}(x_{ji_j})^{(w)} \cdot {}^F\mu_{X_{ki}}(x_{ki_k})^{(w)} + \right.$$

$$+ \sum_{1 \le j < k < l \le n} {}^F\mu_{X_{ji}}(x_{ji_j})^{(w)} \cdot {}^F\mu_{X_{ki}}(x_{ki_k})^{(w)} \cdot {}^F\mu_{X_{li}}(x_{li_l})^{(w)} - \ldots$$

$$\ldots + (-1)^{p+r}\, {}^F\mu_{X_{1i}}(x_{1i_1})^{(w)}\, {}^F\mu_{X_{2i}}(x_{2i_2})^{(w)} \ldots {}^F\mu_{X_{p+ri}}(x_{p+ri_{p+r}})^{(w)} \tag{47}$$

Using the compositional rule of inference for distribution functions of inputs i.e. $F_{\mu_{X_1'}}, F_{\mu_{X_2'}}, \ldots, F_{\mu_{X_p'}}$ and for given $F_{\mu_{\bar{R}'}}$ we can find the distribution function $F_{\mu_{\bar{Y}'}}$.

Assuming the independency of $\mu_{X_j'}$ and $\mu_{\bar{R}}$ we have

$$F_{\mu_{\bar{Y}'}}(x_{p+1i_{p+1}},\ldots,x_{p+ri_{p+r}})^{(w)} = \prod_{x_{pi_p}\in \mathcal{X}_p} \cdots \prod_{x_{1i_1}\in \mathcal{X}_1} F_{\mu_{X_1'}}(x_{1i_1})^{(w)} + $$

$$+ F_{\mu_{\bar{R}}}(x_{1i_1},x_{2i_2},\ldots,x_{pi_p},x_{p+1i_{p+1}},\ldots,x_{p+ri_{p+r}})^{(w)} -$$

$$- F_{\mu_{X_1'}}(x_{1i_1})^{(w)} \cdot F_{\mu_{\bar{R}}}(x_{1i_1},x_{2i_2},\ldots,x_{pi_p},x_{p+1i_{p+1}},\ldots,x_{p+ri_{p+r}})^{(w)} \qquad (48)$$

or

$$F_{\mu_{\bar{Y}'}}(x_{p+1i_{p+1}},\ldots,x_{p+ri_{p+r}})^{(w)} = \prod_{\substack{x_{pi_p}\in \mathcal{X}_p \\ \vdots \\ x_{qi_1}\in \mathcal{X}_1}} \Big(\sum_{j=1}^{p} F_{\mu_{X_j}}(x_{ji_j})^{(w)} +$$

$$+ F_{\mu_{\bar{R}}}(x_{1i_1},\ldots,x_{pi_p},x_{p+1i_{p+1}},\ldots,x_{p+ri_{p+r}})^{(w)} -$$

$$- \sum_{1\le j<k\le p} F_{\mu_{X_j}}(x_{ji_j})^{(w)} \cdot F_{\mu_{X_k}}(x_{ki_k})^{(w)} -$$

$$- \sum_{j=1}^{p} F_{\mu_{X_j}}(x_{ji_j})^{(w)} \cdot F_{\mu_{\bar{R}}}(x_{1i_1},\ldots,x_{pi_p},x_{p+1i_{p+1}},\ldots,x_{p+ri_{p+r}})^{(w)} +$$

$$+ \ldots + (-1)^{p+1} F_{\mu_{X_1}}(x_{1i_1})^{(w)} \cdots F_{\mu_{X_p}}(x_{pi_p})^{(w)} \cdot$$

$$\cdot F_{\mu_{\bar{R}}}(x_{1i_1},\ldots,x_{pi_p},x_{p+1i_{p+1}},\ldots,x_{p+ri_{p+r}})^{(w)} \qquad (49)$$

Having $F_{\mu_{\bar{Y}'}}$ we would like to obtain

$$F_{\mu_{X_{p+1}}}(x_{p+1i_{p+1}})^{(w)},\ldots, F_{\mu_{X_{p+r}}}(x_{p+ri_{p+r}})^{(w)}$$

This problem cannot be solved uniquely and should be the subject of further investigations. Some interesting results can be obtained assuming that

$$\mu_{X'_{p+r}}(x_{p+ki_{p+k}},\omega) = \max_{\substack{x_{p+1i_{p+1}} \in \mathscr{X}_{p+1} \\ \vdots \\ x_{p+k-1i_{p+k-1}} \in \mathscr{X}_{p+k-1} \\ x_{p+k+1i_{p+k+1}} \in \mathscr{X}_{p+k+1} \\ \vdots \\ x_{p+ri_{p+r}} \in \mathscr{X}_{p+r}}} \mu_{Y'}(x_{p+1i_{p+1}},\ldots,x_{p+ri_{p+r}},\omega)$$

where $1 \leq k \leq r$

5. Concluding remarks

Introducing the basic notions and definitions two aspects of decision-making have been indicated.

First, the decision-making in sense of Bellman and Zadeh in a fuzzy probabilistic environment has been discussed. The main obtained result is the distribution function of decision set and when having this fuction it is easy to calculate the first monitors. Then evaluation of final decision is possible.

In the second part some problems concerning the construction of fuzzy probabilistic algorithms (fuzzy probabilistic logic controllors) treated as a kind of decision making systems are considered.
Comparing the fuzzy logic controller and fuzzy probabilistic logic controller it is stated that 'the concept of fuzzy logic controller is embedded in the concept of fuzzy probabilistic logic controller [4]. The formulas obtained for the multidimensional case of fuzzy probabilistic logic controller make also possible the computational tractability of the above mentioned decision-making algorithms.

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ПРИНЯТИЕ РЕШЕНИЙ В РАСПЛЫВЧАТО-ВЕРОЯТНОСТНОЙ СРЕДЕ В СОПОСТАВЛЕНИИ С НОВОЙ КОНЦЕПЦИЕЙ РАСПЛЫВЧАТО-ВЕРОЯТНОСТНОГО РЕГУЛЯТОРА

Р е з ю м е

Представлена проблема принятия решений на основе идеи вероятностного множества.

В первой части рассмотрена проблема принятия решений в смысле Бельмана и Задеха в расплывчато-вероятностной среде. Затем представлено новую концепцию расплывчато-вероятностного регулятора, которого применение целесообразно в области управления слабо распознанными (слабо определёнными) процессами, дающего возможность исследовать стратегии управления применяемые оператором.

PODEJMOWANIE DECYZJI W ROZMYTO-PROBABILISTYCZNYM OTOCZENIU A NOWA KONCEPCJA ROZMYTO-PROBABILISTYCZNEGO REGULATORA

S t r e s z c z e n i e

Przedstawiono problemy podejmowania decyzji w powiązaniu z ideą zbioru probabilistycznego.

W pierwszej części poruszono problem podejmowania decyzji w sensie Bellmana i Zadeha w rozmyto-probabilistycznym otoczeniu. Następnie przedyskutowano nową koncepcję rozmyto-probabilistycznego regulatora użytecznego w problemach sterowania słabo poznanymi (słabo zdefiniowanymi) procesami, umożliwiającego badanie strategii sterowania stosowanej przez operatora.