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THE CONCEPT OF FUZZY RELIABILITY OF SYSTEMS

Summary. The theory of reliability playing an important role is based mainly on probability theory. There are some problems which could be discussed basing on subjective and objective kind of information.

The paper deals with the concept of fuzzy reliability, reformulating some basic notions in the sense of fuzzy set theory (e.g. fuzzy failure, reliability, mean time between failure) and presenting the idea of calculating the reliability of complicated structures. The proposed concepts are compared with others established by means of probability theory.

1. Introduction

Nowadays the theory of reliability plays an important role and has a valuable place in engineering branches of knowledge ([1], [2], [3]). Although it has worked out a wide range of mathematical tools, especially probabilistic, for handling reliability aspects of the system in a formal way, there are some problems which could be discussed basing on subjective and objective kind of information.

It is a well-known fact that in the case of many objects, usually complicated ones, the characteristics of reliability e.g. intensivity of reliability λ are not known exactly or could not be expressed precisely but commonly their values are established under the influence of human way of thinking, fuzzy in its character.

The paper deals with the concept of fuzzy reliability, defining some basic notions. We reformulate them in the sense of fuzzy set theory, introducing fuzzy failure, reliability, mean time between failure. The second part of paper presents the idea of calculating the reliability of complicated system, if the structure and elements of it are given (e.g. parallel and serial structure).

Numerical examples of electrical circuits illustrate the proposed idea pointing out their main features comparing simultaneously the obtained results with others established by means of probability theory.

2. Basic notations and notions

Commonly while discussing the reliability and failure, we take into account a collection of features of the considered object which generally could be measurable and nonmeasurable.

If we distinguish:

n_1 - the number of measurable features,
n_2 - the number of nonmeasurable ones,

then the notion of failure used very often is defined as follows:

Def. 1

$$\underset{1\le i\le n_1}{\exists}\; c_{mi} \notin \left[\underline{c}_{mi},\, \bar{c}_{mi}\right] \quad \text{or} \quad \underset{1\le j\le n_2}{\exists}\; c_{nj} = 0 \tag{1}$$

$\left[\underline{c}_{mi},\, \bar{c}_{mi}\right]$ - the interval of tolerance (in many cases chosen subjectively) of the i-th measurable feature c_{mi}, c_{nj} denotes the j-th nonmeasurable feature and we have:

$$c_{nj} = \begin{cases} 0, & \text{if the feature is out of order} \\ 1, & \text{if the feature is correct} \end{cases} \tag{2}$$

Next the reliability of the object is expressed as the probability of its correct running, i.e.

Def. 2

$$R(t) = P\left\{ \underset{1\le i\le n_1}{\forall}\; c_{mi} \in \left[\underline{c}_{mi}(t),\, \bar{c}_{mi}(t)\right],\; \underset{1\le j\le n_2}{\forall}\; c_{nj} = 1 \,\middle|\, \phi(t),\, \underset{t\in[0,T]}{\forall} \right\} \tag{3}$$

where $\phi(t)$ stands for the conditions of environment expressed as stochastic process:

$$\phi(t) = \left[\phi_1(t),\, \phi_2(t), \ldots\right] \tag{4}$$

Next let us reformulate def. 1,2, introducing the concept of fuzzy set theory [4], which enables not to make a distinction between measurable and nonmeasurable features:

Def. 3. The failure of the object we mean the existence of probabilistic event that fuzzy set of feature doesn't form an element the family of fuzzy tolerance, i.e.

$$\underset{1\le i\le n}{\exists}\; \underset{\sim}{c}_{mi} \notin \mathcal{F}_{1i}(\mathcal{X}) \tag{5}$$

where

$$\mathscr{X} = \begin{cases} \mathbb{R} \text{ (or subline of } \mathbb{R}\text{), if the feature "i" is measurable one.} \\ \{x_1, x_2, x_3, \ldots, x_N\} \text{ if the feature "i" is nonmeasurable and the introduced scale is weakest one (nominal scale)} \end{cases}$$

$$\mathcal{F}_{1i}(\mathscr{X}) = \left\{ \underset{\sim}{X} \in \mathcal{F}(\mathscr{X}) \,\middle|\, \underset{\sim}{X} \subseteq \underset{\sim}{X}_{G_i}, \underset{\sim}{X}_{G_i} \in \mathcal{F}(\mathscr{X}) \right\} \tag{6}$$

$$Fz_{1i}(\mathscr{X}) = \left\{ \mu_{\underset{\sim}{X}} : \mathscr{X} \to [0,1] \;\middle|\; \mu_{\underset{\sim}{X}}(x) \le \mu_{\underset{\sim}{G}_i}(x),\ \forall x\, \mathscr{X} \right\} \tag{7}$$

$\mathcal{F}(\mathscr{X})$ - the family of all fuzzy sets defined on $\mathscr{X}$,

$\underset{\sim}{X}_{G_i}$ - border fuzzy set of tolerance given by its membership function

$$\mu_{\underset{\sim}{G}_i} : \mathscr{X} \to [0,1] \tag{8}$$

(n-number of considered features)

Putting down:

(i) for measurable feature:

$$Fz_{1i}(\mathscr{X}) = \chi : \mathscr{X} \to \{0,1\} \,\Big|\, \inf_{x \in \mathscr{X}} \operatorname{supp} G_{i_\alpha} = \underline{c}_{mi}, \sup_{x \in \mathscr{X}} \operatorname{supp} G_{i_\beta} = \bar{c}_{mi} \,\Big|\, \alpha, \beta \in [0,1] \tag{9}$$

we get Eq.1 (see Fig. 1)

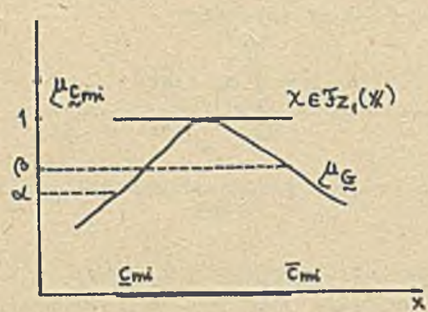

Fig. 1. Tolerance interval as a special case of border fuzzy set of tolerance $\underset{\sim}{G}$

(ii) similarly for nonmeasurable feature we get:

$$Fz_{1i}(\mathscr{X}) = \left\{ \chi : \mathscr{X} \to \{0,1\} \right\} \tag{10}$$

Next we introduce the definition of reliability as follows:

$$R_f(t) = P\left\{ \forall_{1\leq i\leq n} \; \underset{\sim}{C}_{mi} \in \mathcal{F}_{t_i}(\mathcal{X}) \,\middle|\, \phi(t), t\in [0,T]\right\} \tag{11}$$

where

$$\mathcal{F}_{t_i}(\mathcal{X}) = \left\{ X \in \mathcal{F}(\mathcal{X}) \,\middle|\, \underset{\sim}{X} \subseteq X_{G_i}\right\} \tag{12}$$

i.e.

$$Fz_{t_i}(\mathcal{X}) = \left\{ \mathcal{X}\times [0,T] \rightarrow [0,1] \,\middle|\, \mu_{\underset{\sim}{X}}(x,t) \leq \mu_{\underset{\sim}{G}_i}(x,t)\right\} \tag{13}$$

Then for each instant time moment fixing index i ($i = i_0$) we have:

$$R_f(t) = \int_{\mathcal{X}} \mu_{\underset{\sim}{G}}(x,t)dx \; R(t) \tag{14}$$

where

$$R(t) = P\left\{\tau \geq t\right\} \tag{15}$$

stands for reliability function of element.

Putting down

$$\varphi_{\underset{\sim}{G}}(t) = \int_{\mathcal{X}} \mu_{\underset{\sim}{G}}(x,t)dx \tag{16}$$

which is called a degree of fuzziness (or cardinal number) of fuzzy set we obtain:

$$R_f(t) = R(t)\,\varphi_{\underset{\sim}{G}}(t)^{*} \tag{17}$$

Another important index of reliability such as mean time between failures m_{TBF} is defined in the form of integral:

$$m_{TBF} = \int_0^{\infty} R_f(t)dt \tag{18}$$

* if $\varphi_{\underset{\sim}{G}}(o) \neq 1$ then

$$R_f(t) = \frac{\varphi_{\underset{\sim}{G}}(t)}{\varphi_{\underset{\sim}{G}}(o)} R(t)$$

Example. At the design stage of the system fuzzy set of tolerance of capacitor C was arbitrary given as follows:

t \ C [nF] 10^8 [h]	.8	.9	.1	1.1	1.2
0	0	0	1	0	0
2	.2	.7	1	.9	.5
2.5	.5	ỷ9	1	1	.6
3	1	1	1	1	1

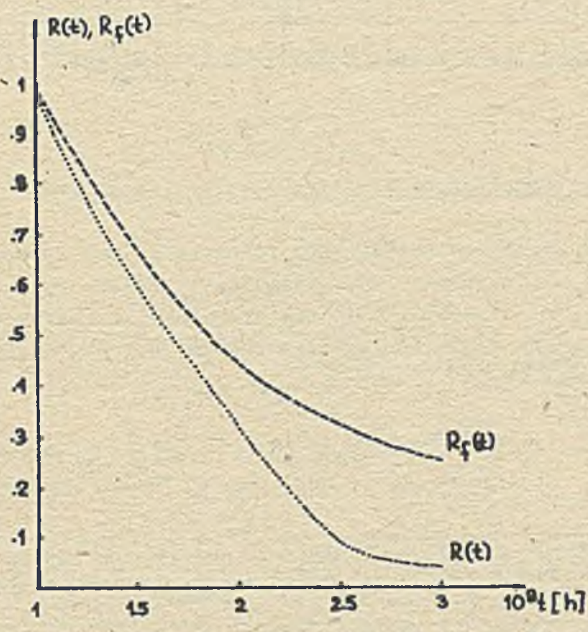

Fig. 2. Reliability functions R(t), $R_f(t)$ of the element

It is easy to see that degree of fuzziness of the tolerance interval grows up in time, what is in accordance with the intuition; we cannot predict the future state of capacitor clearly, in detail.
We assume the reliability function of capacitor is given in the form:

$$R(t) = \exp(-\lambda t) \tag{19}$$

where $\lambda = .01 \cdot 10^{-6}\left[\frac{1}{h}\right] = 10^{-8}\left[\frac{1}{h}\right]$. Obtained results of computations of $R_{\underset{\sim}{r}}(t)$ and $R(t)$ are depiced in Fig. 2. Taking into account the relation given by Eq. 17 and noticing that usually $\varphi_{\underset{\sim}{G}}$ is an increasing function of time, it is interesting to see that $R_{\underset{\sim}{r}}(t)$ should be a decreasing function of time, so the following condition needs to be satisfied:

$$\frac{dR_{\underset{\sim}{r}}(t)}{dt} \leqslant 0 \tag{20}$$

Assuming additionally the exponential shape of reliability function and differantiability of $\varphi_{\underset{\sim}{G}}(t)$ we have:

$$R'(t)\,\varphi_{\underset{\sim}{G}}(t) + R(t)\,\varphi'_{\underset{\sim}{G}}(t) \leqslant 0 \tag{21}$$

$$\varphi'_{\underset{\sim}{G}}(t)/\varphi_{\underset{\sim}{G}}(t) \leqslant \lambda \tag{22}$$

Of course, if $\underset{\sim}{G}$ is a normal fuzzy set $(\sup_x \mu_{\underset{\sim}{G}}(x,t) = 1)$ and $\varphi_{\underset{\sim}{G}}=1$ then $R_{\underset{\sim}{r}}(t) = R(t)$.

3. Reliability function of the structure of the system

Now let us state the reliability function of the structure of elements. At first let us remember the extension principle [5]. For fuzzy set of tolerance $\underset{\sim}{C}_{mi}$ $i = 1,2,\ldots,K$, if the structure of the system is time invariant and given by means of function F, fuzzy set of tolerance of the system C is equal to:

$$\underset{\sim}{C} = F(\underset{\sim}{C}_{m1}, \underset{\sim}{C}_{m2},\ldots,\underset{\sim}{C}_{mk}) \tag{23}$$

so

$$\mu_{\underset{\sim}{C}}(y,t) = \sup_{(x_1,x_2,\ldots,x_k)\in F^{-1}(y)} \left[\min\left(\mu_{\underset{\sim}{C}_{m1}}(x_1,t), \mu_{\underset{\sim}{C}_{m2}}(x_2,t),\ldots, \mu_{\underset{\sim}{C}_{mk}}(x_k,t)\right)\right] \tag{24}$$

and

$$R_{\underset{\sim}{r}}(t) = R(t)\,\varphi_{\underset{\sim}{C}}(t) \tag{25}$$

holds true, where $R(t)$ depends on the structure of the system and reliability function of each of the element

$$R(t) = F\left[R_1(t), R_2(t),\ldots,R_k(t)\right] \tag{26}$$

Example. Let us discuss the reliability function of two structures of systems widely used, i.e. serial and parallel. We assume that intensivity of faults of each of the element of the system is invariant in time and we have:

$$R_1(t) = e^{-\lambda_1 t} \tag{27}$$

and

$$R_2(t) = e^{-\lambda_2 t} \tag{28}$$

Similarly $\underset{\sim}{C}_1(t)$ and $\underset{\sim}{C}_2(t)$ are given as well and do not depend upon time.

x	10	20	30	40	50	60	70	80	90	100	110	120	130
$\mu_{\underset{\sim}{C}_1}$	0	.2	.4	.8	1	.3	.2	.1	.1	0	0	0	0
$\mu_{\underset{\sim}{C}_2}$	.2	.4	1	.9	.7	.5	.4	.2	.2	.1	0	0	0

So we get:

$$R(t) = R_1(t).R_2(t) \tag{29}$$

and

$$\underset{\sim}{C} = \underset{\sim}{C}_1 + \underset{\sim}{C}_2 \tag{30}$$

i.e.

$$R(t) = e^{-\lambda t}, \quad \lambda = \lambda_1 + \lambda_2 \tag{31}$$

$$\mu_{\underset{\sim}{C}}(z) = \sup_{(x,y)\in F^{-1}(z)} \left[\min\left(\mu_{\underset{\sim}{C}_1}(x) \wedge \mu_{\underset{\sim}{C}_2}(y)\right)\right] \tag{32}$$

where F is a sum of x and y: $z = F(x,y) = x+y$.
The membership function of $\underset{\sim}{C}$ is equal as follows.

x	10	20	30	40	50	60	70	80	90	100	110	120	130
μ_C	0	.2	.2	.2	.4	.4	.8	.9	.9	.7	.5	.3	.3

Taking concrete numerical values $\lambda_1 = 3.10^{-6}\left[\frac{1}{h}\right]$ $\lambda_2 = .25.10^{-6}\left[\frac{1}{h}\right]$ we get:

$\lambda = 3.25\ 10^{-6}\left[\frac{1}{h}\right]$, $\varphi_{\underset{\sim}{C}} = 6.6$ so $R_f(t) = 6.6\ \exp\left(-3.25.10^{-6}\ t\right)$

Using the same way we could compute reliability of parallel structure:

$$\frac{1}{\underset{\sim}{C}} = \frac{1}{\underset{\sim}{C}_1} + \frac{1}{\underset{\sim}{C}_2} \tag{33}$$

which is equivalent to the following equation:

$$\underset{\sim}{C} = \frac{\underset{\sim}{C}_1 \cdot \underset{\sim}{C}_2}{\underset{\sim}{C}_1 + \underset{\sim}{C}_2} \tag{34}$$

i.e.

$$\mu_{\underset{\sim}{C}}(z) = \sup_{(x,y)\,|\,z = \frac{xy}{x+}} \left(\min\left(\mu_{\underset{\sim}{C}_1}(x), \mu_{\underset{\sim}{C}_2}(y)\right)\right) \tag{35}$$

and the reliability function $R(t)$ is expressed as

$$R(t) = (1- (1-R_1(t))\,(1-R_2(t)) \tag{36}$$

4. Concluding remarks

The idea presented here is an attempt to overcome and formalize some difficult problems of a heuristic nature, using a concept of fuzzy set theory which is natural in these cases.
The well-known difficulties of estimation of reliability characteristics are strong enough, so the subjective way of thinking in order to choose the tolerance intervals leads to the necessity on constructing a model based on probability-fuzzy information.

The obtained results are in agreement with designer's intuition. The presented method could be useful in many areas of designing, especially facilitating the sensitivity analysis, although it should be emphasized here, that further research in this field as well theoretical as aplicational one is needed.

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List of symbols

$\underset{\sim}{X}$ - fuzzy set $\underset{\sim}{X}$

$\mu_{\underset{\sim}{X}}$ - membership function of fuzzy set $\underset{\sim}{X}$

$\mathscr{X}$ - universe of discourse

$\mathcal{F}(\mathscr{X})$ - family of fuzzy sets defined in $\mathscr{X}$

$\mathcal{F}_t(\mathscr{X})$ - space of fuzzy tolerance

$\mathcal{F}_z(\mathscr{X})$ - family of membership functions of fuzzy sets defined in $\mathscr{X}$

χ_x - characteristic function of set X

$$\chi_x : \mathscr{X} \to \{0,1\}$$

X_α - α - cut of fuzzy set $\underset{\sim}{X}$ with the characteristic function:

$$\chi_{x_\alpha} = \begin{cases} 1, \text{if } \mu_{\underset{\sim}{x}}(x) \geq \alpha \\ 0, \text{otherwise} \end{cases}$$

$[\underline{c}_{mi}, \bar{c}_{mi}]$ - interval of tolerance of the i-th feature

supp $\underset{\sim}{X}$ - support of fuzzy set $\underset{\sim}{X}$

$$\text{supp } \underset{\sim}{X} = \{x \in \mathscr{X} \mid \mu_{\underset{\sim}{x}}(x) > 0\}$$

КОНЦЕПЦИЯ РАСПЛЫВЧАТОЙ НАДЕЖНОСТИ СИСТЕМЫ

Р е з ю м е

Теория налёжности, имеющая все большее значение, основывается главным образом на теорию вероятностей. Однако возникают проблемы, которые можна решать на основе субъективной и объективной информации.

Работа посвящена концепции расплывчатой надежности, формулирует ее основные понятия на основе теории расплывчатых множеств (таких как: расплывчатый отказ, надёжность, среднее время до отказа) и дает идею определения надёжности сложных структур.

Предлагаемые концепции сравниваются с соотношенными концепциями из области теории вероятностей.

KONCEPCJA ROZMYTEJ NIEZAWODNOŚCI SYSTEMÓW

S t r e s z c z e n i e

Teoria niezawodności, mająca coraz większe znaczenie, bazuje głównie na teorii prawdopodobieństwa. Pojawiają się problemy, które mogą być dyskutowane w oparciu o subiektywny i obiektywny rodzaj informacji.

Praca dotyczy koncepcji rozmytej niezawodności, formułuje jej podstawowe pojęcia w sensie teorii zbiorów rozmytych (tj. rozmyte uszkodzenie, niezawodność, średni czas między uszkodzeniami) i prezentuje ideę wyznaczania niezawodności złożonych struktur.

Proponowane koncepcje są porównywane z odpowiednimi koncepcjami bazującymi na gruncie teorii prawdopodobieństwa.