

# Deformation process of the material of mine powered roof supports in low-cycle fatigue conditions

J. Okrajni <sup>a,\*</sup>, M. Plaza <sup>a</sup>, M. Jaszczuk <sup>b</sup>

<sup>a</sup> Faculty of Materials Engineering and Metallurgy, Silesian University of Technology, ul. Krasińskiego 8, 40-019 Katowice, Poland

<sup>b</sup> Faculty of Mining and Geology, Silesian University of Technology, ul. Akademicka 2, 44-100 Gliwice, Poland

\* Corresponding e-mail address: jerzy.okrajni@polsl.pl

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## Analysis and modelling

### ABSTRACT

**Purpose:** The main purpose of the work is the description of the low-cycle fatigue process of mine powered roof supports working under mechanical high loading. The work focuses on the chosen component strain-stress characteristics. The issue of modelling the stress-strain behaviour of powered roof supports components during low-cycle fatigue has been discussed.

**Design/methodology/approach:** The FEM modelling and Neuber's method have been used to describe the local stress-strain behaviour of the chosen component.

**Findings:** In the examined devices, variable stress and strain values were calculated for a chosen characteristic load cycle. Diagrams in the form of a hysteresis loop determined using Neuber's hypothesis and FEM were compared. The values of the range of equivalent strain determined for multiaxial stress states using the finite element method proved to be close to those estimated via Neuber's method.

**Research limitations/implications:** The presented analysis is the part of the complex investigation method which main purpose is increasing the accuracy of the low-cycle fatigue process description. In such situation the investigations carried out in the work give the model approach and data for the comparison the real behaviour with the predictions. However the work is focused only on the chosen component and chosen characteristics of loading.

**Practical implications:** The method of stress-strain behaviour analysis used in the paper could be useful in the practical cases when the real components mechanical behaviour would be analysed and their fatigue life would be assessed.

**Originality/value:** The main value of this paper is the own method of the mechanical behaviour analysis of the powered roof support component. This method includes FEM modelling and Neuber's method of the stress-strain characteristics assessment. The material stress-strain behaviour has been treated as the local phenomenon that could be modelled.

**Keywords:** Applied mechanics; Numerical techniques; Low-cycle fatigue; Powered roof supports

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## 1. Introduction

Although numerous studies have been devoted to the problem of assessing fatigue life of materials in complex states of stress, this issue is still described in a variety of ways [1-3]. A number of durability criteria were formulated and then verified for some selected materials, load types and states of stress. However, there are no general and widely accepted criteria which might be applied for groups of materials in a wide range of loading types and regardless of the type of stress state. Among the existing approaches to assessing fatigue life, the procedure based on Neuber's [1-5] concept is still one of the most frequently applied. In this method, the examined processes concern crack formation in the vicinity of notches which cause stress concentration resulting from internal constraints, the effect of which is disturbance of the homogeneous state of stress. The method has been verified for simple cases of strength, such as tension or compression, bending and torsion. In each above-mentioned case, a complex state of stress is present in the vicinity of the notch. It is assumed, however, that at a certain distance from the area of stress concentration stresses are characterised by a uniform distribution of the selected component of the stress state which is referred to as the stress concentration. The problem becomes much more complex where external load induces a complex state of stress not only in the immediate vicinity of the notch, but also at a certain distance from it, for which distance a constant stress, normal for tension, compression and bending, or a constant shearing stress for torsion, were previously assumed.

## 2. Neuber's method

The method of assessing the fatigue limit or forecasting the fatigue life, which is based on Neuber's hypothesis [1, 4, 5], is grounded on two assumptions which are not explicitly formulated. The first assumption concerns the possibility of characterising the effect of the state of stress in the vicinity of the notch root using one value only, namely the maximum stress. The other assumption is connected with the adoption of a constitutive relation in the form of the following equation:

$$\sigma \varepsilon = \text{const} \quad (1)$$

For simple cases of load, the use of the above-mentioned dependence allows, among others, the determination of the maximum range of stress in the notch for the assumed nominal stress value and the stress concentration coefficient.

Neuber's hypothesis was modified in Glinka-Molski method [6-8], in which the expression:

$$\sigma_e \varepsilon_e = \frac{(K_t \sigma_{nom})^2}{E} = \sigma \varepsilon = \text{const}, \quad (2)$$

resulting from Neuber's hypothesis was replaced with the equation:

$$\frac{(K_t \sigma_{nom})^2}{2E} = W_\sigma = \int_0^\varepsilon \sigma d\varepsilon \quad (3)$$

where  $W_\sigma$  is the specific energy of strain in the vicinity of the notch root.

The expression on the left side of the equations represents the strain energy calculated in the elastic range for stresses and strains in the root of the notch.

Assuming that the value of the product  $\sigma_e \varepsilon_e = \sigma \varepsilon$  remains constant also in the case of a complex state of stress, as seen from a global perspective, an attempt to generalise the Neuber's hypothesis can be made for complex stress states by applying this dependence to reduced stresses. Consequently, we receive:

$$\sigma_{rede} \varepsilon_{ie} = \sigma_{red} \varepsilon_i = \text{const} \quad (4)$$

The approach thus formulated allows determining the reduced stress and equivalent strain in the root of a notch in a complex stress state, assuming that the values  $\sigma_{rede}, \varepsilon_{ie}$  for the elastic range and the curve of cyclic deformation in the  $\sigma_{red}(\varepsilon_i)$  system, where  $\varepsilon_i$  is the strain intensity, are known. This method of determining the equivalent values of  $\sigma_{red}$  and  $\varepsilon_i$  has been verified in the paper by using a mine powered roof support component - sill piece model for which stresses were determined by adopting models of an elastic and elastic-plastic material.

## 3. The model of the device

Figure 1 shows the sill piece model for which the variable in time stresses and strains were determined.

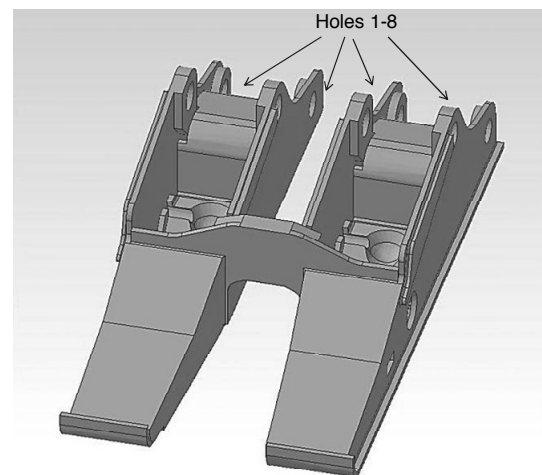


Fig. 1. Sill piece model

Data received from the manufacturer of mine powered roof support were used in the analysis. The data covered geometric properties of the sill piece and the type of material used in the examined component. The force acting in the prop of a mine powered roof support was assumed to amount to 6078 kN and the inclination angle of the props to the horizontal plane was 56°. A case which was marked with number 3 in the fatigue strength tests of the components of a mine powered roof support was selected from among numerous possible techniques of propping mine powered roof support. While concentrating on an analysis of the behaviour of one component of a mine powered roof support section, namely the sill piece, the action of the props in the form

of forces applied to the surface as well as, in which lemniscate connectors are fixed, were assumed in the calculations.

The values of the strains were determined for the entire section of the mine powered roof support on the basis of a previously conducted analysis, which concerned the elastic model of the material. The characteristics of variable in time load in the form shown in Figure 2a were adopted. An elastic-plastic material model with linear reinforcement (Fig. 2b) was assumed. It was also assumed that the material was kinematically reinforced, which was corroborated by the previous low-cycle fatigue tests.

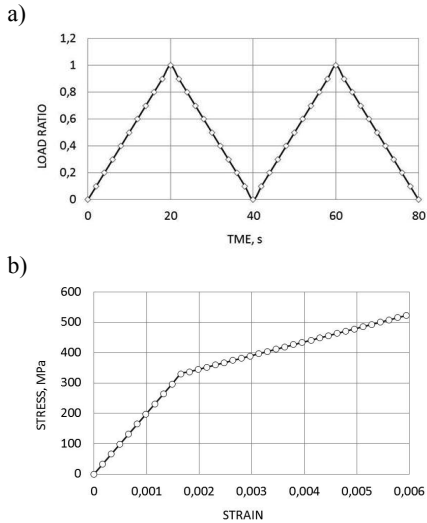


Fig. 2. The assumed course of changes in the load whose value is designated by the coefficient being a ratio of the current, variable in time, load or displacement to its maximum value –a) and the characteristics of the material assumed in the calculations and determined based on the cyclic stress-strain curve –b)

The course of load changes was adopted in the form of stages of loading and unloading which repeated in cycles. In that case, time should be treated conventionally. The effects of rheological nature were omitted in the calculations. Consequently, the subsequent instants of time correspond to particular levels of the increasing or decreasing load.

#### 4. Stress-strain characteristics determined by means of FEM

The FEM method has been used, which allow to determine the local characteristics of the deformation process and is commonly applied in material science and mechanics [9-13]. The variable in time distributions of stress (Fig. 3), strain and their equivalent values were determined for the conditions characterised above. Characteristics of the variable in time components of the stress state, reduced stresses and strain intensity in selected points of the sill piece marked in Figure 3 were also determined. Examples of characteristics determined using the method described above are presented in Figure 4.

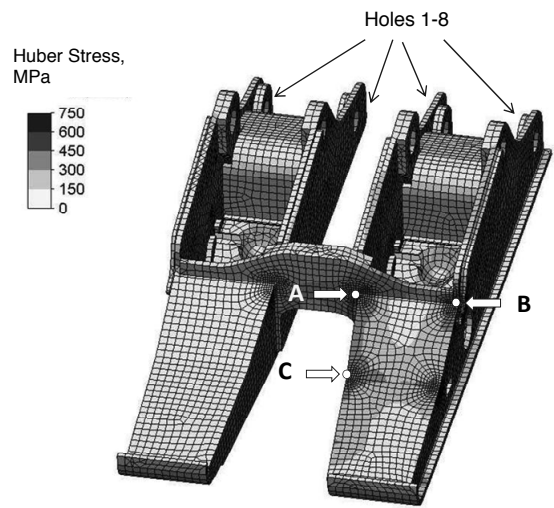


Fig. 3. Distribution of the reduced stress determined in accordance with Huber’s hypothesis for the load in the time instant  $t=20$  s (Fig. 2)

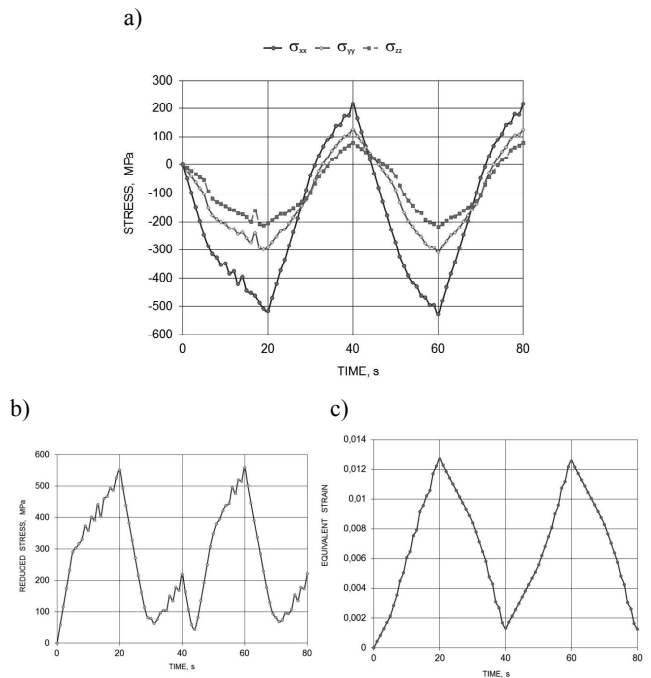


Fig. 4. Changes of the components of the stress state  $\sigma_x$ ,  $\sigma_y$  and  $\sigma_z$  –a) of reduced stress determined in accordance with Huber’s hypothesis –b) and strain intensity –c) in the selected point A marked in Fig. 2

In addition, characteristics in the form of a dependence between the components of the stress state and the deformation state were determined. Examples of such characteristics for selected components are presented in Figure 5. The dependencies

between stresses and strains show that during cyclic loading and unloading, there are areas in the sill piece where the process of cyclic deformation is the elastic-plastic nature (Figs. 5a and 5b). As the load increases, plastic deformations occur in the examined points during local compression. While the sill piece is unloaded, plastic strains occur during tension. In this case, the course of local cyclic deformation may be characterised using hysteresis loops in a stress/strain system, as determined for the individual components of the stress and strain state.

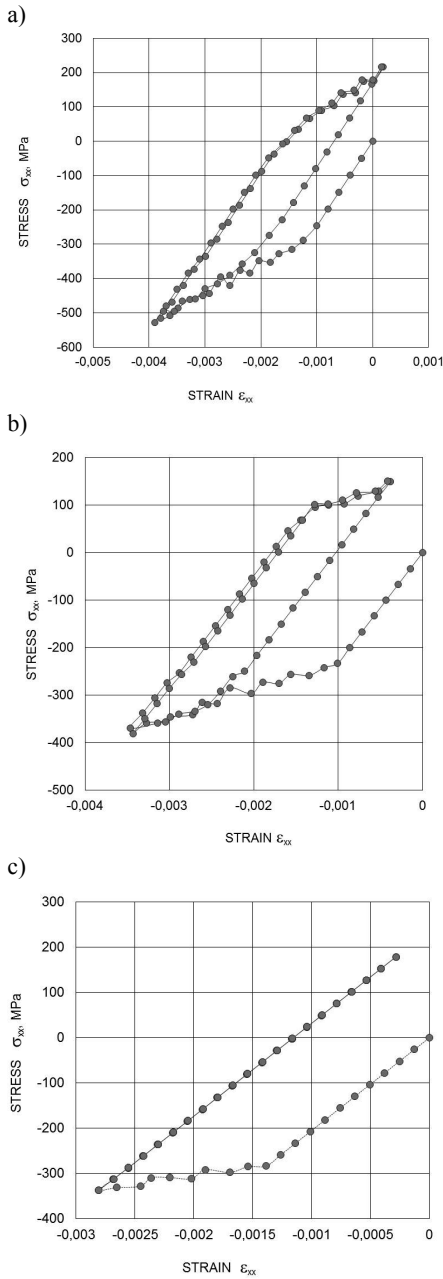


Fig. 5. Dependencies between strains  $\epsilon_{xx}$  and stresses  $\sigma_{xx}$  determined in points A –a), B –b) and C –c)

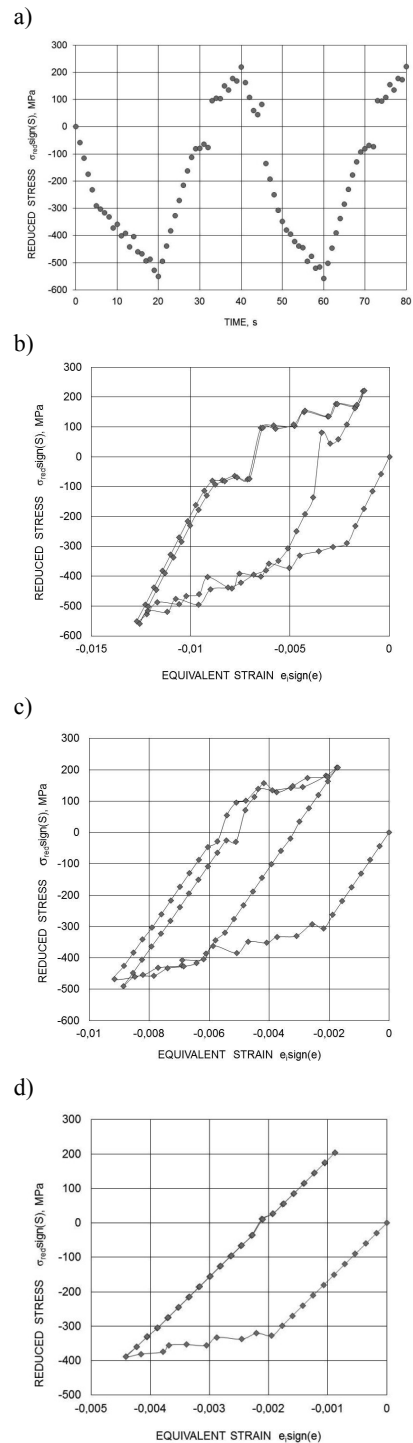


Fig. 6. The course of changes in time of the reduced stress state multiplied by the sign of the first invariant of the stress state, determined in point A (Fig. 3) –a) and hysteresis loops determined for points A- b), B- c), C- d) in the system: strain intensity - reduced stress multiplied by the sign of the first invariant of the stress state

Similar characteristics of dependencies between the strain intensity and reduced stresses, to which a sign depending on the first invariant of the stress state  $S = \sigma_x + \sigma_y + \sigma_z$  was assigned, were also determined as part of the study. A positive reduced stress value was adopted in the case of a positive value of the first invariant of the stress state. The minus sign was assigned to negative values of the first invariant. On this basis, equivalent characteristics of  $\sigma_{red} \operatorname{sgn}(\sigma_x + \sigma_y + \sigma_z)$  were determined as a function of  $\mathcal{E}_i$ . Thus, hysteresis loops characterising the course of cyclic deformation in a complex stress state (Fig. 6) were obtained.

### 5. Stress-strain characteristics estimated on the basis of Neuber's hypothesis

Elastic-plastic models of a material are very rarely used in engineering calculations. This applies in particular to the construction

stage at which it is necessary to check the strength and durability of many different construction solutions. In such cases, calculations in the elastic range are used most often. A similar approach is applied when assessing the current strength and durability of objects after a specified operation time. Therefore, there is a need to compare the solutions obtained for the elastic model with more precise solutions which adopt the elastic-plastic material model. In order to compare both methods of determining the characteristics of the cyclic deformation process, the calculations performed for an elastic-plastic material were repeated for an elastic material, while determining the components of the stress and strain state, variable in time. The values of reduced stresses and strain intensity were determined in previously defined points. Based on them, the ranges of equivalent strain were then determined. Figures 7-9 show the method of determining an equivalent hysteresis loop. The equation adopted for the Neuber's hyperbola has a form of a dependence (4). Values of  $\sigma_{rede} \mathcal{E}_{ie}$  were determined in points A, B and C for the instant of time  $t=20$  s for the elastic material model. The values  $\sigma_{rede} \mathcal{E}_{ie}$ , Neuber's hyperbolae and hysteresis loops which characterise the course of cyclic deformation are shown in Figures 7-9.

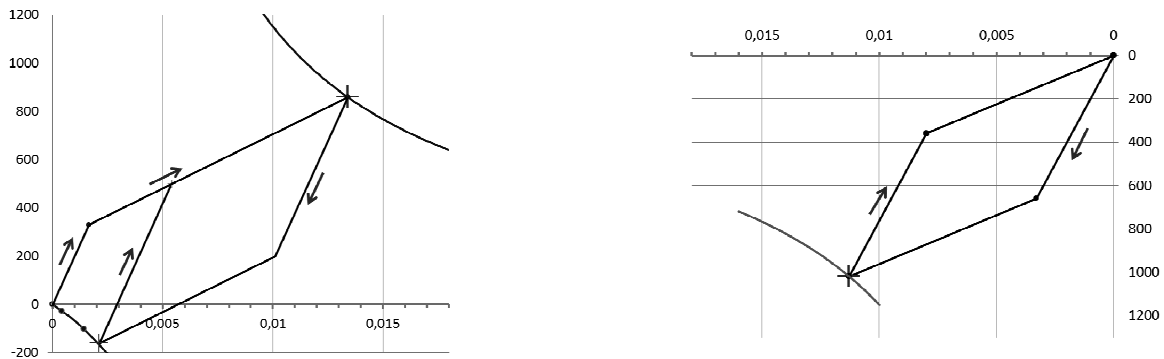


Fig. 7. Dependencies between the strain intensity and reduced stress determined for point A;  $\sigma_{rede} = 1161$  MPa and  $\mathcal{E}_{ie} = 0.00991$

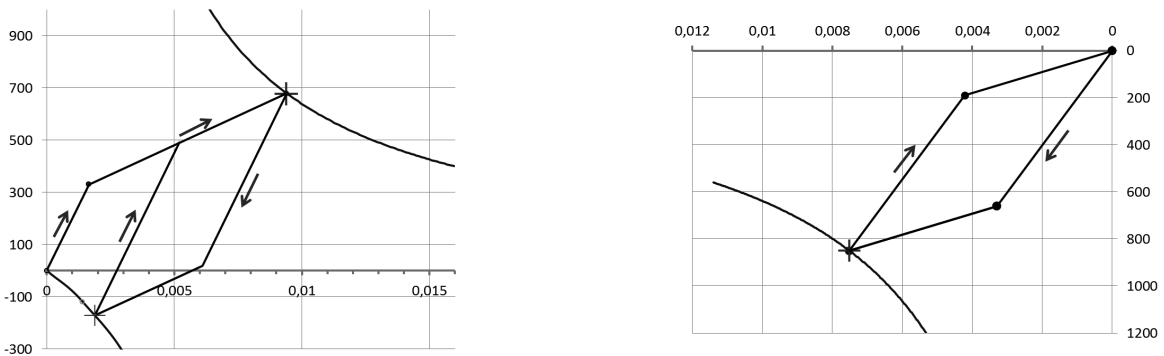


Fig. 8. Dependencies between the strain intensity and reduced stress determined for point B;  $\sigma_{rede} = 878$  MPa and  $\mathcal{E}_{ie} = 0.00727$

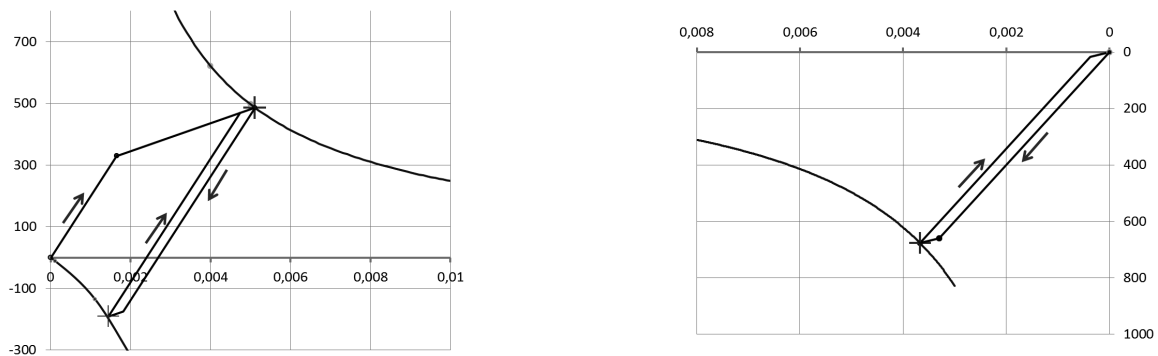


Fig. 9. Dependencies between the strain intensity and reduced stress determined for point C;  $\sigma_{rede} = 600$  MPa and  $\epsilon_{ie} = 0.00414$

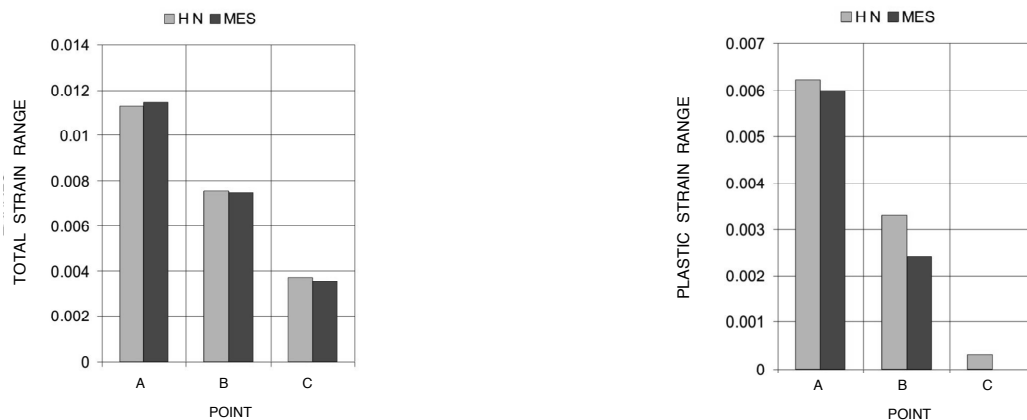


Fig. 10. Comparison of the results of calculations of strain ranges using a method based on Neuber's idea and the finite element method (FEM)

Table 1.

Comparison of the results of calculations using a method based on Neuber's idea and the finite element method (FEM)

	POINT A		POINT B		POINT C	
	Neuber's method	FEM	Neuber's method	FEM	Neuber's method	FEM
$\Delta\sigma, MPa$	1020	772	850	677	677	592
$\Delta\epsilon_{ie}$	0.0113	0.01145	0.0075	0.00744	0.0037	0.00353
$\Delta\epsilon_{ipl}$	0.0062	0.006	0.0033	0.00242	0.0003	0

## 6. Comparison of calculation results

The diagrams shown in Figures 7-9 allow the determination of the ranges of stress and strain intensity based on a model which uses Neuber's method. These values were compared with those determined by means of the finite element method, while assuming the elastic-plastic material model and using the courses shown in Figures 6b-6d. They are juxtaposed in Table 1 and in the Fig. 10.

The values of the range of strain intensity determined for complex stress states using the finite element method proved to be close to those estimated via Neuber's method. In a majority of cases, the estimation conducted using a method based on Neuber's hypothesis is of a conservative nature. By using the determined

values of deformation ranges for the assumed, variable in time, course of loading for an element made of steel S690Q, the number of cycles until failure was determined, which amounted to:

- a) point A – 435 cycles,
- b) point B – 2046 cycles,
- c) point C – 31270 cycles.

Hence, a fracture may appear first in point A. As soon as after 435 load cycles with an assumed constant course, the accumulated damage reaches a critical value in this place. However, thus calculated number of cycles should be treated as a value estimated for the assumed load cycle. The number of cycles calculated in this way may constitute a basis for comparing the life determined for different materials or different courses of load cycles. For a load cycle which changes during operation, one should calculate

the degree of damage caused by each type of the cycle and then sum up the damage for the particular types of cycles [13-16]. The method of determining the hysteresis loop as presented in the paper may also be used in this case in order to estimate local fatigue conditions for the prescribed load cycles. Such an approach may be applied for gradual loading which could constitute an approximation of the real operating conditions of a section of a mine powered roof support.

The results presented in the paper are an attempt to approximate the characteristics of material behaviour in the stress concentration areas in components of complex shapes. This issue has been illustrated based on an example of a selected device, for which the ability to estimate durability in operating conditions is of special importance on account of work safety in mines. Such a possibility depends on the availability of data concerning the material properties, while taking into account the inhomogeneity of their distribution throughout the volume of the components and the problem of determining the load history. The presented method and selected results of calculations may in this context constitute only a fragment of a complex methodology of forecasting the life of analysed objects. A particular attention was paid in the study to the phenomenon of fatigue damage formation in the areas of stress concentration and to the possibility of a quantitative approach to the deformation process in these areas, which characterises the phenomenon of the formation of damage of a fatigue nature. The results obtained should be considered in a close connection with the material testing methodology [16, 17] in order to determine the type and range of the test parameters.

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