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Hypergraphs of simple beams-models of their analysis in synthesis of complex beam-systems

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Analysis and modelling

ABSTRACT

Purpose: of this paper is the analysis of vibrating beam by the exact and approximate methods and creating the hypergraphs of the beam concerning of two methods of analysis.

Design/methodology/approach: was to nominate the relevance or irrelevance between the characteristics obtained by considered methods - especially concerning the relevance of the natural frequencies-poles of beams characteristics. The main subject of the research is to solve the continuous free-pinned (F-P) and clamped-sliding (C-S) beams as a subsystems of vibrating beam-system.

Findings: this approach is a fact, that approximate solutions fulfill all conditions for vibrating beams and can be introduction to synthesis of these systems modeled by hypergraphs.

Research limitations/implications: is that linear continuous transverse vibrating (F-P) and (C-S) beams are considered.

Practical implications: of this study is the main point is the introduction to synthesis of transverse vibrating continuous beam-systems.

Originality/value: of this approach considers the application Galerkin's method which concerns the analysis of beams and modeling them of transformed hypergraphs.

Keywords: Applied mechanics; Exact and approximate methods; Continuous system; Vibrating beams

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1. Introduction

In the Gliwice Research Centre the different problems of different models of vibrating beam systems analyzed by the structural numbers methods modelled by means of the graphs and hypergraphs (Other diverse problems have been modeled by different kind of graphs next they were examined and analyzed in (e.g. [8-16]). The problems of synthesis of electrical systems [1] and of selected class of continuous, discrete - continuous discrete mechanical systems and active mechanical systems concerning the frequency spectrum has been made (e.g. [12-19])). have been solved (e.g.[3-7,12,15,16]). The discrete- continuous torsionally [9] and flexibly vibrating mechanical and mechatronic systems were considered [10-13]. To comparison of dynamical dynamical flexibilities only for mechanical torsionally vibrating bar and flexibly vibrating beam, as a parts of complex mechatronic systems, exact method and Galerkin's method were used [9-16].

In this paper frequency - modal analysis with the both methods has been used to obtain the frequency-modal characteristics. The problems, have been presented for the flexibly vibrating pinned - free and clamped - sliding beams.

2. Hypergraphs as models of simply beams - subsystems of flexibly vibration system

The couple

$$ {}^{k}X=\left({}_{1}X,\ {}_{2}^{k}X\right) \qquad (1)$$

is called a *hypergraph* [2], where: ${}_{1}X$ - finite set of vertices and ${}_{2}^{k}X=\left({}_{2}^{k}X^{(i)}/i\in \mathrm{N}\right)$, ($k$=2,3, ... ∈N) - a family of subsets of set ${}_{1}X$; the family ${}_{2}^{k}X$ is called a *hypergraph* over ${}_{1}X$ as well, and ${}_{2}^{k}X=\left\{{}_{2}^{k}X^{(1)},\ {}_{2}^{k}X^{(2)},\ \dots,{}_{2}^{k}X^{(m)}\right\}$ - a set of edges, called *hyperedges* or *blocks,* if

$$\begin{cases} {}_{2}^{k}X\neq\varnothing\ (i\in \mathrm{I}),\\ \bigcup\limits_{i\in \mathrm{I}}{}_{2}^{k}X^{(i)}={}_{2}X. \end{cases} \qquad (2)$$

Hypergraphs ${}^{k}X$ have been shown in their geometrical representation on plane. ${}_{2}^{k}X$ - hyperedges or blocks - have been marked as two-dimensional continuum with enhanced vertices, in the shape of circles. Hypergraphs or graphs of category k - ${}^{k}X$ (k=2,3) are used in this paper (see [2-6,12,15,16]).

For the considered boundary conditions generalized displacements - deflections ${}_{1}s_{1}$ and slopes ${}_{1}s_{3}$ correspond to its extreme points (Fig. 1). These general displacements are measured in the inertial system of reference. Moreover, the origin of the inertial system of reference has generalized coordinate ${}_{1}s_{0}=0$ assigned to it.

Making mutually one-to-one transformation imitation

$$ f\colon {}_{1}S\ \to {}_{1}X \qquad (3)$$

in this way, that

$$ f\left({}_{1}s_{j}\right)=\ {}_{1}x_{j}, \qquad (4)$$

where: ${}_{1}s_{j}\in{}_{1}S$, ${}_{1}x_{j}\in{}_{1}X$, ${}_{1}s_{j}\in{}_{1}S$, ${}_{1}x_{j}\in{}_{1}X$, j =0,1,3.

The *three-vertex hypergraphs as a models of one of end of flexibly vibrating beams* is obtained

$$ {}^{2}_{f}X=\left[{}^{2}X\ ,f\right]. \qquad (5)$$

where: ${}_{2}^{k}X$ - one-element family - five-element subset of vertices ${}_{1}X$.

Graphical representation of transformations (3) by the way of (4) for the different boundary conditions is shown in Fig. 1.

In the case of flexibly vibrating of the beam (i) (Fig. 2a) with combinations of boundary conditions (Fig. 1a - left end and 1c - right one) and constant cross-section and constant flexibly rigidity $(EI)^{(i)}$ (where $E^{(i)}$ - Young's modulus of the beam, $I^{(i)}$ - polar moment of inertia of cross-section of the beam) as well as length $l^{(i)}$ has been considerd. So a set of the generalized displacements of a flexibly vibrating beam can be formulated in form: ${}_{1}S^{(i)}=\left\{{}_{1}s_{0}^{(i)}={}_{1}s_{2}^{(i)}=0,\ {}_{1}s_{1}^{(i)},\ {}_{1}s_{3}^{(i)},\ {}_{1}s_{4}^{(i)}\right\}$.

Making mutually one-to-one transformation imitation in form of (3) and (4) the *five-vertex hypergraph* (Fig. 2b) as a model of flexibly vibrating beam with constant cross-section is obtained

$$ {}^{2}_{f}X^{(i)}=\left[{}^{2}X^{(i)},f\right]. \qquad (6)$$

where: ${}_{2}^{k}X^{(i)}$ - one-element family - five-element subset of vertices ${}_{1}X^{(i)}$

Graphical representation of transformations (3) by the way of (4) in case of the flexibly vibrating beam with constant cross-section is shown in Fig. 2.

The couple

$$ {}^{2}_{1}X^{(i)}=\left[{}^{2}_{f}X^{(i)},f_{1}\right] \qquad (7)$$

is called *weighted hypergraph* (The weighted hypergraphs (in this paper called also weighted block graphs or weighted graphs of category k) have been applied to modelling of the mechanical systems considered. Definitions of graphs, as mathematical objects, have been presented on the basis of the literature [2]. The bibliography of this subject is very extensive and regards the theory as well as its applications (see [1,3-10,12-17,20,21]))

where: f_{1} is function which assigns to vertices ${}_{1}x_{j}^{(i)}$ of hypergraph ${}^{2}_{f}X^{(i)}$ the generalized displacements, that means deflections: ${}_{1}s_{1}^{(i)}$ and ${}_{1}s_{2}^{(i)}$ the slopes of the beam - ${}_{1}s_{3}^{(i)}$ and ${}_{1}s_{4}^{(i)}$ as:

$$ f_{1}\left({}_{1}x_{j}^{(i)}\right)=\ {}_{1}s_{j}^{(i)},\ j=0,1,\ \dots,4.. \qquad (8)$$

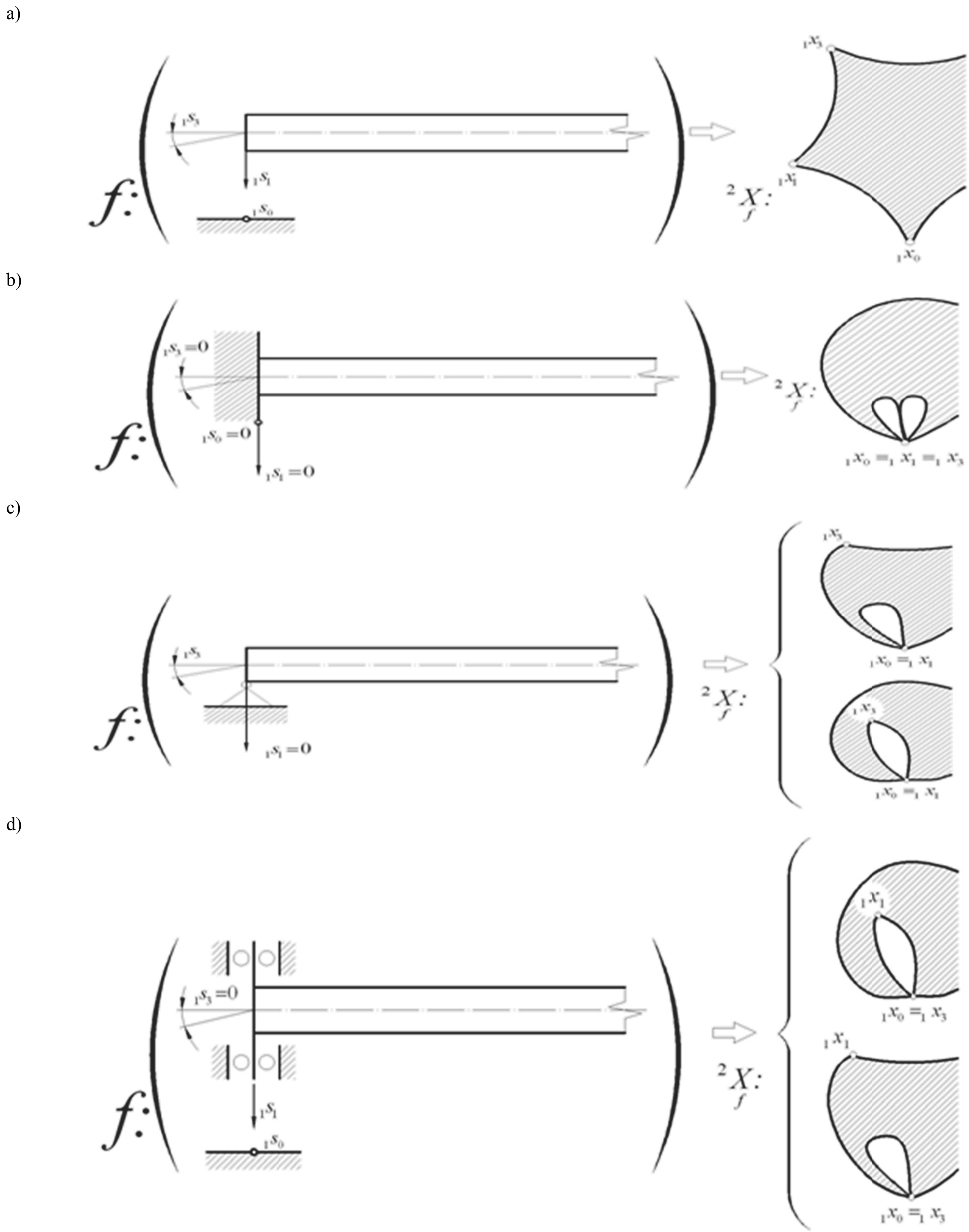

Fig. 1. Different boundary conditions of subsystems of beam system and their hypergraphs

The graph ${}^{2}_{1}X^{(i)}$ as graphical representation of sentence (8) is shown in Fig. 3.

In the example of beam with boundary conditions presented in Fig. 1b and 1d the set of generalized displacements is following: ${}_1S^{(i)} = \left\{ {}_1s_0^{(i)} = {}_1s_2^{(i)} = 0,\ {}_1s_1^{(i)},\ {}_1s_3^{(i)},\ {}_1s_4^{(i)} \right\}$. Using (3) in the way (4) the hypergraph ${}^{2}_{f}X^{(i)}$ is presented in Fig. 4. Next after transformation (8) *weighted hypergraph* ${}^{2}_{1}X^{(i)}$ is obtained (Fig. 5).

3. Models and characteristics of Simply vibration beams - subsystems of beam-system

3.1. Frequency - modal analysis of the subsystem of beam system

The subsystem of mechatronic system, that is the beam (The mechatronic system was considered for example in [8-10]), extorted with harmonic force in form $P(t) = P_0 \sin \omega t$ was considered in (e.g. [11]).

At first in the global case the equation of motion of the beam is considered

$$EIy(x,t)_{,xxxx} + \rho F y(x,t)_{,tt} = 0, \tag{9}$$

where: $y(x,t)$ - deflection at the time moment t of the lining beam section within the distance x from the beginning of the system, E - Young modulus, ρ - mass density of material of the beam, I - polar inertia moment of the beam cross section, F - area of the beam cross section.

The boundary conditions subsystem of beam system are known and suitably equal in case for:

- free end (Fig. 1a)

$$y_{,xx}(0,t) = 0,\ y_{,xxx}(0,t) = 0, \tag{10}$$

- clamped end (Fig. 1b)

$$y(0,t) = 0,\ y_{,x}(0,t) = 0, \tag{11}$$

- pinned end (Fig. 1c)

$$y(0,t) = 0,\ y_{,xx}(0,t) = 0, \tag{12}$$

- sliding end

$$y_{,x}(0,t) = 0,\ y_{,xxx}(0,t) = 0. \tag{13}$$

$$X(x) = A_1 \sin kx + A_2 \cos kx + A_3 \sinh kx + A_4 \cosh kx. \tag{14}$$

After substitution of (9) into combinations of boundary conditions (10-13) was received

$$\mathbf{WA} = \mathbf{0}, \tag{15}$$

where $|\mathbf{W}|$ - main determinant of set of equation (for example [8-14]), $\mathbf{A} = [A_1, \ldots, A_4]^T$.

After comparing the characteristic determinant of set (15) to zero

$$|\mathbf{W}| = 0 \tag{16}$$

own values are obtain.

Own functions after relationships between constants A_1, ..., A_4 have form

$$X_n = f\left(\frac{z_n}{l} x\right),\quad n = 1, 2, 3, \ldots. \tag{17}$$

Own question (ear homogeneous boundary conditions) for every boundary conditions of beam is however following

$$X^{(\mathrm{IV})}(x) - k^4 X(x) = 0, \tag{18}$$

$$X''(0,t) = 0,\ X'''(0,t) = 0, \tag{19}$$

$$X''(0,t) = 0,\ X'(0,t) = 0, \tag{20}$$

$$X(0,t) = 0,\ X''(0,t) = 0, \tag{21}$$

$$X'(0,t) = 0,\ X'''(0,t) = 0. \tag{22}$$

The conditions (19-22) we should write for the second end of beam that then, when *x=l*. After this operations the general solution of own functions has the form.

3.2. The exact method of determining of dynamical flexibility

The solution of equation (9) is the harmonic function in form of

$$y(x,t) = X(x) \sin \omega t. \tag{23}$$

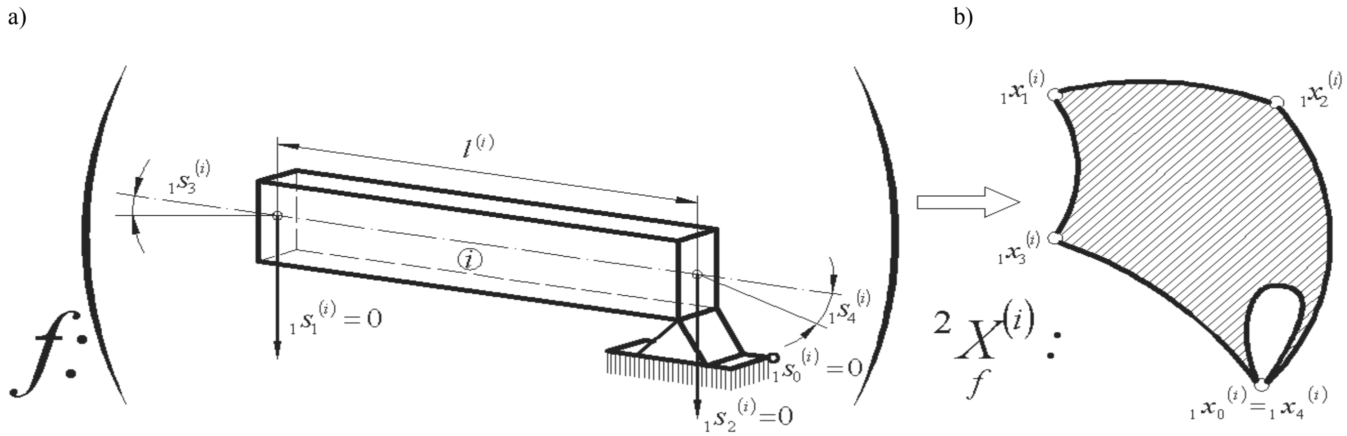

Fig. 2. Hypergraph (b) of model of flexibly vibrating beam (a) as graphical representation of transformations (3) and (4)

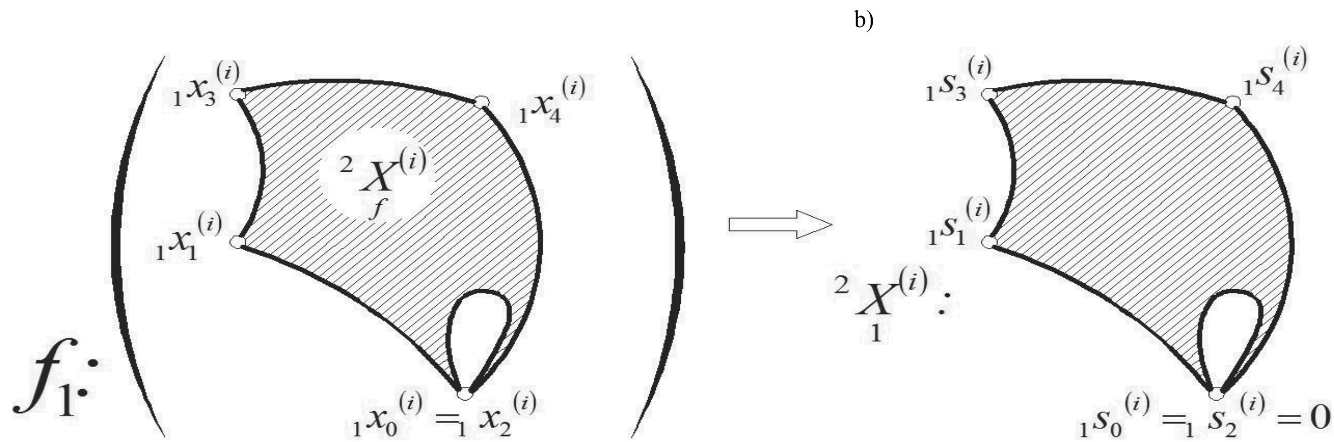

Fig. 3. Hypergraphs of flexibly vibrating beam as representation of transformation (8)

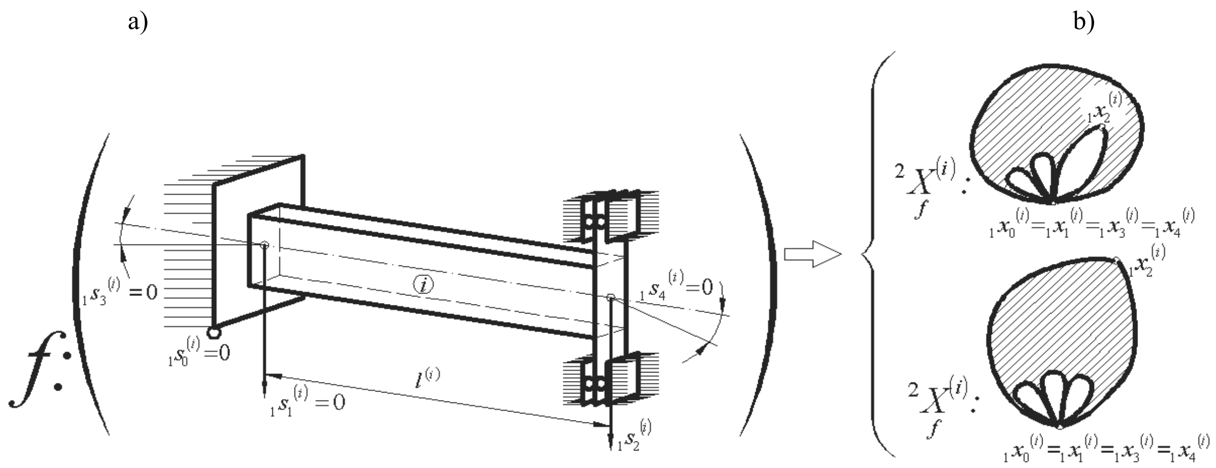

Fig. 4. Hypergraph (b) of model of flexibly vibrating beam (a) as graphical representation of transformations (3) and (4)

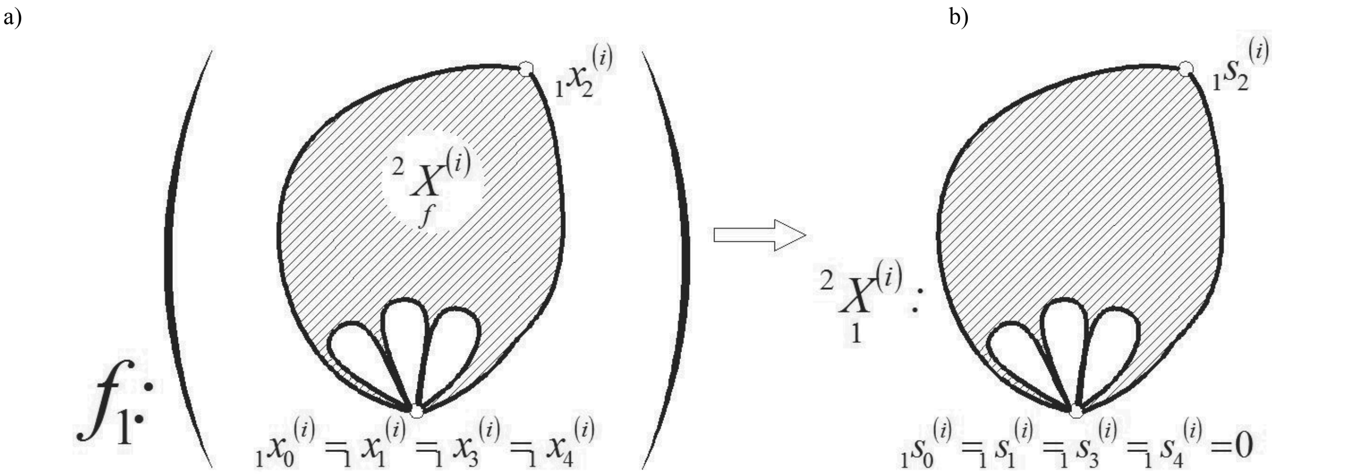

Fig. 5. Hypergraphs of flexibly vibrating beam as representation of transformation (8)

Determining suitable derivatives of (23) and substituting them into boundary conditions the set of equations, after transformations, was obtained in matrix form as

$$\mathbf{WA} = \mathbf{F}, \tag{24}$$

where: $\mathbf{W}$, $\mathbf{A}$, $\mathbf{F}$ - the matrices depended from one of the boundary conditions of elementary beam.

To qualify constants A_1, ..., A_4, we should count combinations of following determinants (e.g. [8-14]).

$$|\mathbf{W}|,\ |\mathbf{W}_1|,\ \dots,\ |\mathbf{W}_4|. \tag{25}$$

The constants A_1, ... , A_4 on the base of combination of (10)-(13) and [for $x=l$ and $x=l$], and regard

$$A_i = \frac{|\mathbf{W}_i|}{|\mathbf{W}|},\quad i=1, \dots, 4. \tag{26}$$

Substituting expression (25) and (26) to (14) and taking into account (10) deflection beam is

$$y(x,t) = f(E, I, k, l) P_0 \sin \omega t. \tag{27}$$

According to definition of dynamic flexibility, on the basis of (24), it takes form

$$Y = f(E, I, k, l). \tag{28}$$

The transients of absolute value of dynamical flexibility (28) where $x=l$, for combinations of boundary conditions, and that is suitable for F-P (Fig. 2a) and C-S (Fig. 4a) in Fig. 6.

3.3. Galerkin's method of calculation of the dynamical flexibility of the beam

It has to be considered that if the shaft is under the action of moment with continuous factorization threw the beam length with the value $F(x)\sin\omega t$ on the length unit - then the equation of motion of the element with length dx lining in the point x is:

$$EIy_{,xxxx}dx + \rho A y_{,tt}dx = F(x)\sin\omega t dx. \tag{29}$$

To determine the dynamic flexibility of the factors, which are compatible to concentrate loading $F\sin\omega t$, which works in point z have to be found. The loading can be considered as a limit of concentrate loading threw the length- as follows:

$$F(x) = \begin{cases} \frac{F}{h} & \text{when } z-h \le x \le z, \\ 0 & \text{in other section,} \end{cases} \tag{30}$$

and the equation (29) takes form of

$$EIy_{,xxxx} + \rho A y_{,tt} = P_0 \sin\omega t, \tag{31}$$

and the equation (29) takes form of

$$EIy_{,xxxx} + \rho A y_{,tt} = P_0 \sin\omega t, \tag{32}$$

where: $P_0 = \frac{F}{h}$.

Using approximate method - Galerkin's one the solution of equation (32) is given in form

$$y(x,t) = \sum_{n=1}^{\infty} y_n(x,t) = \sum_{n=1}^{\infty} A^{(n)} f(k,x) \sin\omega t. \tag{33}$$

Substituting the following derivatives of (33) to (32), the amplitude value $A^{(n)}$ - after transformations - of the vibrations takes form of

$$A^{(n)} = P_0 f(a,k,\omega) \quad (34)$$

where: $a = \sqrt{\frac{EI}{\rho F}}$.

Using the equation (34) and putting it to (33) the dynamical flexibility equals

$$Y_{xl}^{(n)} = f(a,k,x,l,\omega) \quad (35)$$

The absolute value of dynamical flexibility at the end of the beam, i.e. when $x=l$ takes the following form

$$\alpha_Y^{(l)} = \left|Y_{ll}^{(n)}\right| = f(a,k,l,\omega). \quad (36)$$

In general case the dynamical flexibility at the arbitrary point of the beam gets shape of

$$Y_{xl} = \sum_{n=1}^{\infty} Y_{xl}^{(n)} \quad (37)$$

For sum $k=0,1,2,3$ the plot of absolute value of dynamical flexibility defined by expression (37) for combinations of boundary conditions, that means suitably F-P (Fig. 6) and C-S in (Fig. 7).

On the base of, for presented Galerkin's transformation the five-vertex hypergraphs (Figs. 8a and 9a) into three-vertex block graph (Figs. 8b,c and 9b,c) will be applied.

In the case of synthesis of n-segment model of the system, composed of subsystems with constant section, vibrating flexibly, it is modelled by the loaded graph of the third category - after Galerkin's transformation - with n three-vertices-blocks, connected to those vertices to which the corresponding generalized coordinates are assigned (see i.e. [4,12,15,16]).

In the problem of synthesis are used subgraphs of k category graphs according to (8), generalized coordinates to the vertices of complete graph ${}^2_f X_Z^{(i)}$ of hypergraph ${}^2_f X^{(i)}$ and assigning by f_2 dynamical flexibilities to edges of this graph:

$$f_2\left(\{{}_1x_0^{(i)}, {}_1x_1^{(i)}\}, \{{}_1x_0^{(i)}, {}_1x_2^{(i)}\}, \{{}_1x_0^{(i)}, {}_1x_2^{(i)}\}\right) = \left[\{Y_{11}^{(i)}\}, \{Y_{22}^{(i)}\}, \{Y_{12}^{(i)}\}\right], \quad (38)$$

a complete-substitute weighted graph is obtained:

$$\underset{12}{{}^2X_Z^{(i)}} = \left[{}^2_f X_Z^{(i)}, f_1, f_2\right]. \quad (39)$$

The weighted subgraph of complete weighted graph $\underset{12}{{}^2X_Z^{(i)}}$

$$\underset{12}{{}^2X_0^{(i)}} = \left[{}^2_f X_Z^{(i)}, f_1, f_2\right]$$

is a *weighted Lagrange's skeleton*.

Geometrical representation of graph $\underset{12}{{}^2X_0^{(i)}}$ is shown in [4,5].

For a weighted Lagrange skeleton, taken into consideration in (8), it can be noted that (see [4,5]):

$${}_1s_j^{(i)} - {}_1s_0^{(i)} = {}_1s_j^{(i)} - 0 = {}_1s_j^{(i)}, \quad (j = 1,2). \quad (40)$$

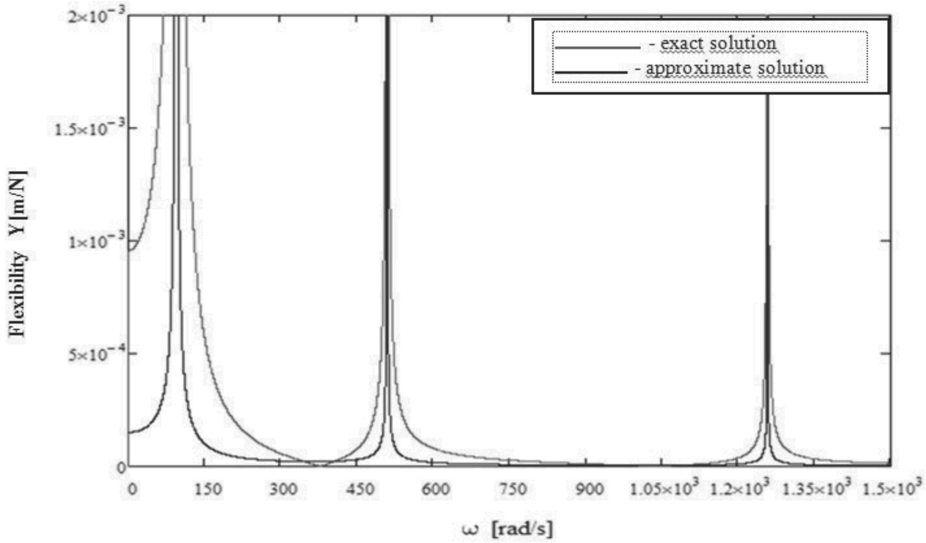

Fig. 6. The plot of absolute value of dynamical flexibility of the sum for n=1, 2, 3 mode vibration for the system from Fig. 2a

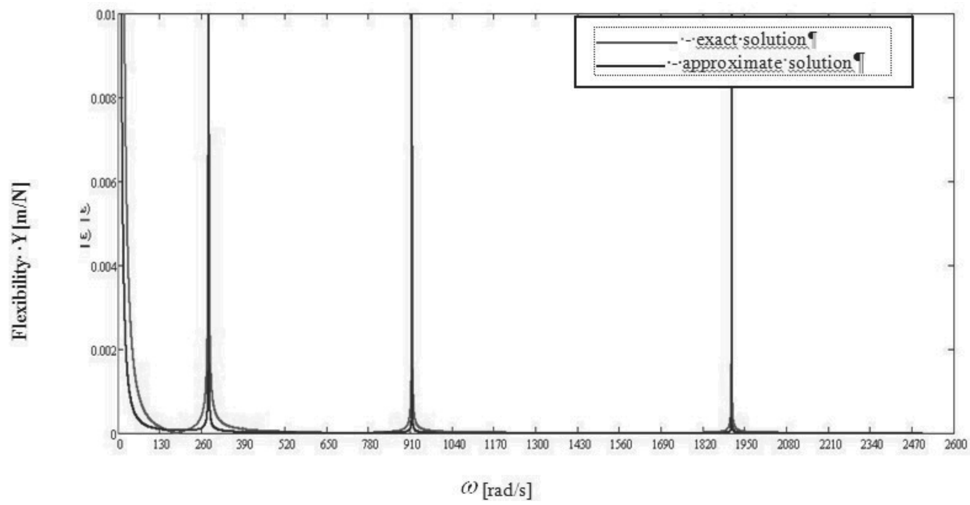

Fig. 7. The plot of absolute value of dynamical flexibility of the sum for n=1, 2, 3 mode vibration for the system from Fig. 4a

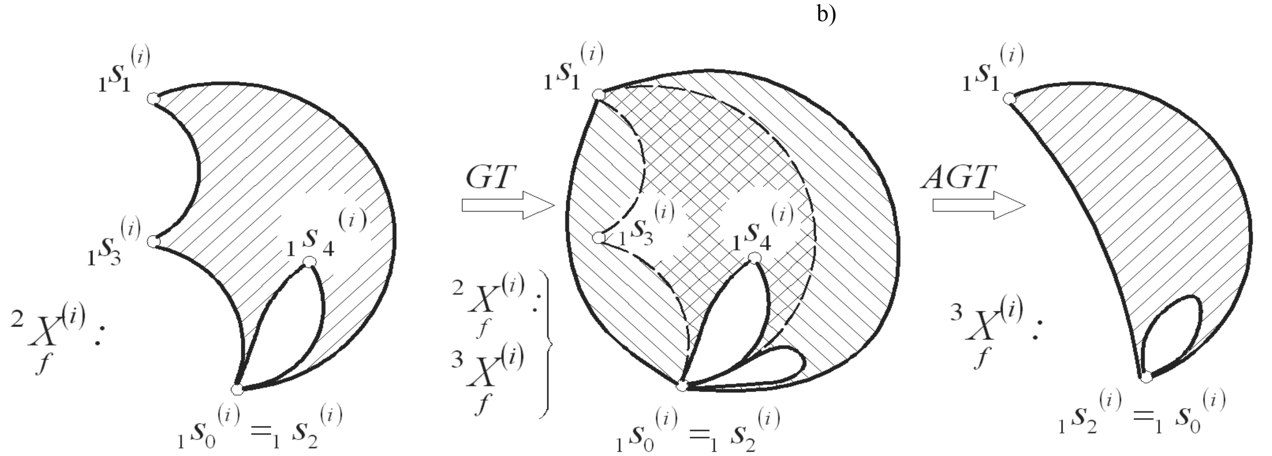

Fig. 8. The illustration of transformation of the five-vertex hypergraph into three-vertex one as effect of use of Galerkin's method

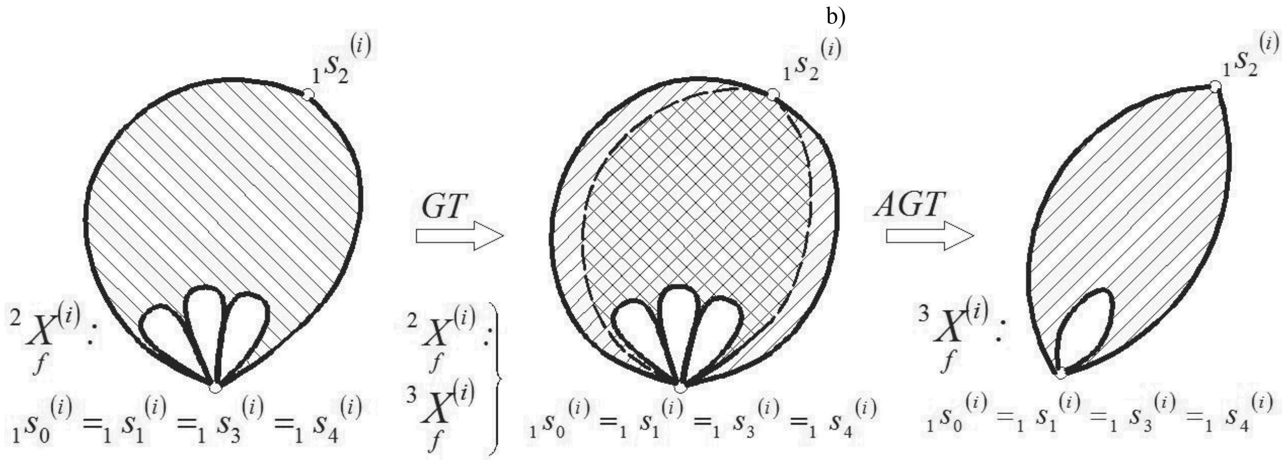

Fig. 9. The illustration of transformation of the five-vertex hypergraph into three-vertex one as effect of use of Galerkin's method

Considering (38), (39) and the definition of dynamical flexibility [4,5], weighted Lagrange skeleton may be treated as oriented polar graph $\overrightarrow{X}^{(i)}_{00}$.

It is not difficult to notice that making the assignment f_3 to the edges of weighted Lagrange skeleton ${}^2\overrightarrow{X}^{(i)}_{0\,12}$ of hypergraph for the model of i-th bar - ${}^2X^{(i)}_{f}$, couples of numbers - respectively - generalized coordinates and generalized forces ${}_2S = \left\| \left| {}_2s_1^{(i)} \right|, \left| {}_2s_2^{(i)} \right| \right\|$ so that:

$$f_3\left(\left\{{}_1x_0^{(i)}, {}_1x_1^{(i)}\right\}, \left\{{}_1x_0^{(i)}, {}_1x_2^{(i)}\right\}\right) = \left[\left\{\left|{}_1s_1^{(i)}\right|, \left|{}_2s_1^{(i)}\right|\right\}, \left\{\left|{}_1s_2^{(i)}\right|, \left|{}_2s_2^{(i)}\right|\right\}\right], \tag{41}$$

a *polar graph* is obtained

$$\overrightarrow{X}^{(i)}_{00} = \overrightarrow{X}^{(i)}_{0\,3} = \left[{}^2\overrightarrow{X}^{(i)}_{0\,f}, f_3\right]. \tag{42}$$

Moreover in the case of oriented polar graph $\overrightarrow{X}^{(i)}_{00}$, polar equation [4,5] can be formulated as:

$$\begin{bmatrix} {}_1s_1^{(i)} \\ {}_1s_2^{(i)} \end{bmatrix} = \begin{bmatrix} Y_{11}^{(i)} & 0 \\ 0 & Y_{22}^{(i)} \end{bmatrix} \begin{bmatrix} {}_2s_1^{(i)} \\ {}_2s_2^{(i)} \end{bmatrix}. \tag{43}$$

Graphical representation of these subgraphs are shown in [3-8,12,15,16].

In the case of synthesis of n-segment model of the system, composed of subsystems with constant section, transverse vibrating, it is modeled by the loaded graph of the second category with n three-vertices-blocks, connected to those vertices to which the corresponding generalized coordinates are assigned (see i.e. [5]).

The use of a weighted hypergraph and its subgraphs (as a model of flexibly vibrating system) in this way may provide the basis for the formalization which is the necessary condition of discretization of the considered class of continuous mechanical systems.

4. Last remark

On the base of the obtained formulas, which were determined by the exact method and approximate one, it is possible to make the synthesis of the considered class vibrating mechanical systems. Moreover the others of boundary conditions of mechanical subsystems of complex mechanical or mechatronic systems that means the beam it is necessary to achieve offered researches in this paper. These problems shall be discussed in future research works.

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