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Reconstruction of the thermal conductivity coefficient by using the harmony search algorithm

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ABSTRACT

Purpose: of this paper: Aim of this paper is a presentation of the respectively new tool for solving the optimization problems, which is the Harmony Search algorithm in version slightly modified by the authors, used for identifying the thermal conductivity coefficient. Proposed approach is illustrated with an example confirming its usefulness for solving such kinds of problems.

Design/methodology/approach: For solving the considered parametric inverse heat conduction problem the approach is applied in which the essential part consists in minimization of the functional representing the differences between the measurement values of temperature and approximate values calculated with the aid of finite difference method. For minimizing the functional the Harmony Search algorithm is used.

Findings: The elaboration shows that approaches involving the algorithms of artificial intelligence for solving the inverse heat conduction problems of that kind are efficient and they ensure to receive satisfying results in shorter time in comparison with the classical procedures.

Research limitations/implications: Specific properties of the heuristic algorithms require to execute the procedure several times and to average the obtained results because each running of the algorithm can give slightly different results. Each execution of the procedure means the solution of the direct problem associated with the considered inverse problem by using the finite difference method.

Practical implications: In spite of the problem described above the approaches involving the heuristic algorithms of artificial intelligence are successful because they are respectively simple and easy to use and they give satisfying results after short time of working. Another advantage of using optimization algorithms of that kind is the fact that they do not need to satisfy any assumptions about the solved problem, oppositely to the classical optimization algorithms.

Originality/value: Proposal of the original approach involving the heuristic optimization algorithm for solving the parametric inverse heat conduction problem is discussed in the paper.

Keywords: Numerical techniques; Inverse heat conduction problem; Artificial intelligence; Harmony search algorithm

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1. Introduction

In recent time many algorithms inspired by the mechanisms functioning successfully in nature have appeared, like genetic algorithms, neural algorithms, immune algorithms or swarm intelligence algorithms. Because of their respective simplicity, short time of working and efficiency they have found an application in solving many technical problems, among others the heat conduction problems.

The inverse heat conduction problem consists in determination of the temperature distribution in considered region and, usually, in reconstruction of one of the boundary conditions, by having the incomplete information about the process [1,2]. Therefore, finding an analytical solution of inverse problem is almost impossible, except the simplest cases. This fact explains the constant need of searching for the new approximate methods. The bibliography dedicated to the inverse heat conduction problem is much smaller than the bibliography about the direct problems and it includes some analytical techniques like the Adomian method [3], variational iteration method [4,5], method using the Green function [6], method of iterative regularization and Tichonov regularization, as well as the numerical methods like, for example, the Monte Carlo method [7], mollification method introduced by Mourio [8], methods based on the wavelets theory [9], genetic algorithms [10-14] or methods based on artificial intelligence algorithms [15-18], like Harmony Search Algorithm [19], Ant Colony Optimization [20] and Artificial Bee Colony [21].

Algorithms mentioned above: the genetic algorithms, immune algorithms or swarm intelligence algorithms inspired by the techniques of searching for the food by the swarms of insects and methods of communication between particular individuals of the swarm, create a group of optimization algorithms imitating behaviours from the real world. Algorithm of that kind is also the Harmony Search (HS) algorithm, proposed by Zong Woo Geem, which bases on the process of searching for the harmony of sounds by the musicians taking part in the act of jazz improvisation [22,23]. All the mentioned algorithms are the heuristic algorithms, which means that the solution received in result of applying the procedure is the best, but in this particular moment. Another execution of algorithm can give another solution - slightly better of slightly worse. However, this fact does not diminish the effectiveness of those approaches, usually simple in application, rapid in action and not requiring to satisfy any specific assumptions about the solved problem.

Harmony Search algorithm has been used in many fields of computer science and engineering. To the group of problems solved with the aid of this algorithm belong, for instance, visual tracking, tour planning, vehicle routing, water network design, soil stability analysis and others [24,25]. In this paper we propose to apply this algorithm for minimizing the functional expressing the differences between the measurement values of temperature and values calculated by solving the direct problem associated with the considered inverse problem. In this way we desire to determine the value of thermal conductivity coefficient such that the approximate values of temperature are as closest as possible to the given measurement values. Similar approach but providing to reconstruction of the heat transfer coefficient or leading to reconstruction of the first and second boundary conditions with the aid of ABC, ACO and HS algorithms are presented in papers [19-21].

2. Harmony Search Algorithm

Improvisation of the jazz music consists in finding the best state of harmony, similarly as the optimization algorithm consists in finding the argument realizing minimum of the function. This similarity is a ground of the idea of Harmony Search algorithm. When one of the musicians plays a note, the other members of the jazz group must remember its sound and select their notes such that a harmonic music can be composed from the chaos of sounds. In the course of practicing, the musicians remember the notes played before, add the new notes and improve them such that the most beautiful music will arise. Described process can be defined as the optimization of jazz composition and gives a base for the idea of the considered optimization algorithm.

Similarity between the process of jazz improvisation and the problem of optimizing the function is used as fallows. Arguments of optimized function can be considered as the notes and the values for these arguments as the tones of instruments caused by these notes. Search for the argument in which minimum of the function is taken corresponds with the process of searching by the musicians for the combination of notes giving the best harmony of the music.

We begin the algorithm by selecting the random set of arguments (notes) and ordering them with regard to the values of minimized function in the harmony memory vector (HM). In the next step we try to improve randomly the harmony given by the combination of selected notes. We can choose the note already collected in the harmony memory vector - we can test such note one more time or we can change it slightly in hope of improving the general harmony. We can also try to find the completely new notes from the available range. Each note is put in the right order in the HM vector. After the assumed number of iterations the first element of HM vector is taken as the solution. In the presented approach we propose a slight modification, in comparison with the classical HS algorithm [23], consisted in the way of determining the values of parameter Δ regulating the sound of a note. Real interpretation of this parameter are the frets on the neck of a guitar, representing the semitones. In our approach we propose to reduce the regulating parameter if in few successive iterations the result is not improving.

In details, the algorithm is of the following form.

1. Initial data:
   - minimized function $f(x)$;
   - range of the variables $a \le x \le b$;
   - size of the harmony memory vector HMS (1-100);
   - harmony memory considering rate coefficient HMCR (0.7-0.99);
   - pitch adjusting rate coefficient PAR (0.1-0.5);
   - number of iterations IT.
2. Preparation of the harmony memory vector HM - we randomly select HMS number of elements x and we order them in vector HM according to the increasing values $f(x)$:

$$HM = \begin{bmatrix} x^1 & | \, f(x^1) \\ \vdots & | \, \vdots \\ x^{HMS} & | \, f(x^{HMS}) \end{bmatrix}.$$

3. Selection of the new harmony x'.

Element x' is selected:

- with the probability equal to HMCR, from among numbers $(x^1,\ldots,x^{HMS})$ collected in the harmony memory vector HM;
- with the probability equal to 1-HMCR, randomly from the assumed range $a \le x \le b$.

If in the previous step the element x' is selected from the harmony memory vector HM then:

- with the probability equal to PAR, we modify the element x' in the following way: $x' \to x' + \Delta$ (we regulate the sound of the note) for $\Delta = bw \cdot p$, where bw denotes the bandwidth - part of range of the variables and p is the randomly selected number from interval $[-1,1]$;
- with the probability equal to 1-PAR we do nothing.

4. If $f(x') < f(x^{HMS})$ then we put the element x' into the harmony memory vector HM in place of the element x^{HMS} and we order vector HM according to the increasing values of minimized function.
5. If the successive 5 iterations do not bring the improvement of the result we upgrade the bandwidth $bw \to 0.1\,bw$.
6. Steps 2-5 are repeated IT number of times. The first element of vector HM determines the solution.

3. Formulation of the problem

We consider the problem in which distribution of temperature is described with the aid of the following heat conduction equation

$$c\,\rho\,\frac{\partial u}{\partial t}(x,t) = \lambda\,\frac{\partial^2 u}{\partial x^2}(x,t), \quad x \in [0,d], \quad t \in [0,T], \tag{1}$$

with the given initial and boundary conditions of the form

$$u(x,0) = u_0, \quad x \in [0,d] \tag{2}$$

$$\frac{\partial u}{\partial x}(0,t) = 0, \quad t \in [0,T], \tag{3}$$

where c, ρ and λ denote, respectively, the specific heat, mass density and thermal conductivity, t and x refer to the time and spatial location, whereas u defines the distribution of temperature. On the boundary for $x = d$ the boundary condition of the third kind is assumed

$$-\lambda\,\frac{\partial u}{\partial t}(d,t) = \alpha(t)\,(u(d,t) - u_\infty), \quad t \in [0,T], \tag{4}$$

where u_∞ denotes the ambient temperature and $\alpha(t)$ describes the heat transfer coefficient.

Solving of the considered problem consists in identifying the value of thermal conductivity coefficient λ and in reconstructing the distribution of temperature $u(x,t)$ in domain of the problem.

By assuming the value of sought parameter λ as given, the problem defined by equations (1)-(4) can be solved by using one of the known method for direct problems, for instance the finite difference method or finite element method (in example presented in the current paper we use the finite difference method). In this way, the values of temperature $\tilde{u}(x_P,t_j)$ in selected point of the domain can be received.

By using the calculated values of temperature $\tilde{u}(x_P,t_j)$, $j=1,\ldots,m$, in control point x_P and the exact temperatures $u(x_P,t_j)$, given or determined for the known exact value of parameter λ, we can construct the following functional:

$$P(\lambda) = \sqrt{\sum_{j=1}^{m}\left(u(x_P,t_j) - \tilde{u}(x_P,t_j)\right)^2}, \tag{5}$$

representing the error of approximate solution $\tilde{u}$ in control point x_P for measurements taken in successive moments t_j, $j=1,\ldots,m$. By this means, the value of parameter λ. will be determined in such a way that the approximate distribution of temperature will be as close as possible to the known values of temperature in control point. For minimizing the functional (5) we use the Harmony Search algorithm, paying attention to the fact that each execution of the procedure means the necessity of solving the appropriate direct heat conduction problem.

4. Numerical example

Presented approach applying the Harmony Search algorithm will be investigated for the following values of parameters: $c = 1000$ [J/(kg K)], $\rho = 2679$ [kg/m^3], $T = 1000$ [s], $d = 1$[m], $u_0 = 980$ [K] and $u_\infty = 298$ [K] and the following values of $\alpha(t)$ [W/(m^2 K)]:

$$\alpha(t) = \begin{cases} 250 & for \quad t \in [0,90], \\ 150 & for \quad t \in (90,250], \\ 28 & for \quad t \in (250,1000]. \end{cases}$$

We know the exact value of the sought parameter λ which is equal to 240 [W/(m K)]. For constructing the functional (5) we use the control values of temperature, determined for the known exact value of λ and noised by the random error of 1, 2, 5 and 10%. The measurement point is located on the boundary for $x = 1$ and in the 5% and 10% distance away from this boundary and the measurements are taken at every 1 s. We will verify how much the shift of measurement point away from the boundary affects the quality of results.

Harmony Search algorithm is executed for the following values of parameters: HMS=20, HMCR=0.85, PAR=0.3, IT=1-100. Elements of the initial harmony memory vector HM are randomly selected from the range [0,500] and the initial value of the bandwidth parameter bw corresponds with 10% of the range of variables. The approximate values of reconstructed parameter is received by running the algorithm 30 times and by averaging the obtained results. We examine efficiency of the Harmony Search algorithm in regard to the number of iterations (parameter IT), because each execution of the procedure means the necessity of solving the direct heat conduction problem corresponding with the solved inverse problem.

Calculations have been performed for successive perturbations of input data (0,1,2,5 and 10%) and for various locations of control point (on the boundary for x=1, in point x=0.95 and in point x=0,90). We investigated how many iterations is necessary to receive satisfying reconstruction of the sought parameter and temperature distribution in particular cases of input data.

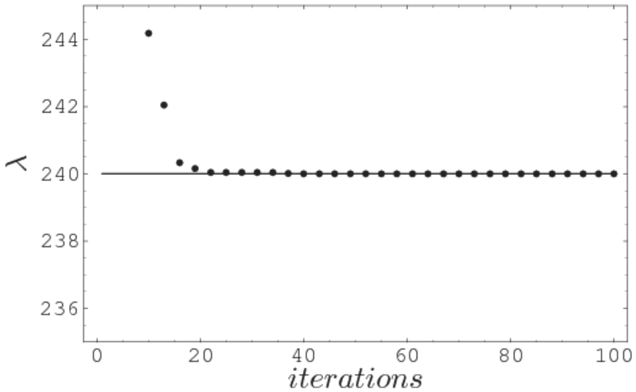

Fig. 1. Reconstructed values of parameter λ (dashed line) depending on the number of iterations, calculated for unnoised input data and control point located 10% away from the boundary

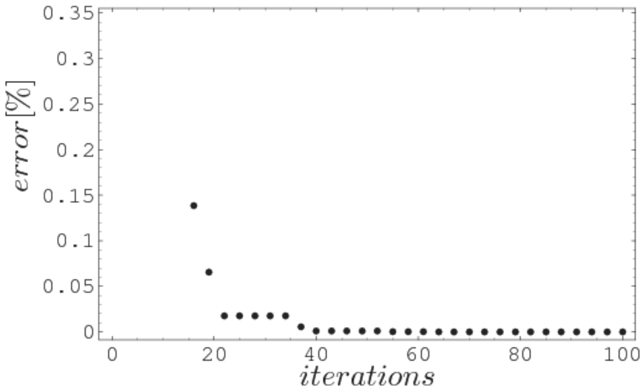

Fig. 2. Error of parameter λ reconstruction depending on the number of iterations, calculated for unnoised input data and control point located 10% away from the boundary

Figure 1 presents the reconstructed values of thermal conductivity coefficient depending on the number of iterations received for unnoised input data, but measured in control point located 10% away from the boundary. We can see that the sought value has been reconstructed in about 20 iterations. Figure 2 shows error of this reconstruction for each number of iterations.

Distribution of temperature $u(x,t)$ on the boundary for x=1 (where boundary condition of the third kind is assumed) calculated in 20 iterations for unnoised input data and control point located in the 10% distance away from the considered boundary, is presented in Figure 3. Error of this approximation is displayed in Figure 4.

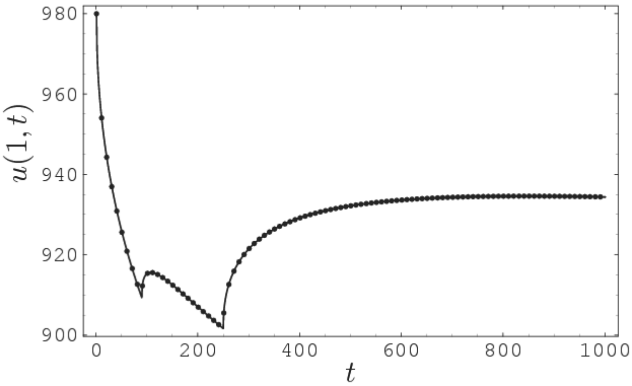

Fig. 3. Distribution of temperature $u(x,t)$ on the boundary for x=1 reconstructed for unnoised input data and control point located in the 10% distance away from this boundary (solid line - exact solution, dashed line - approximated values)

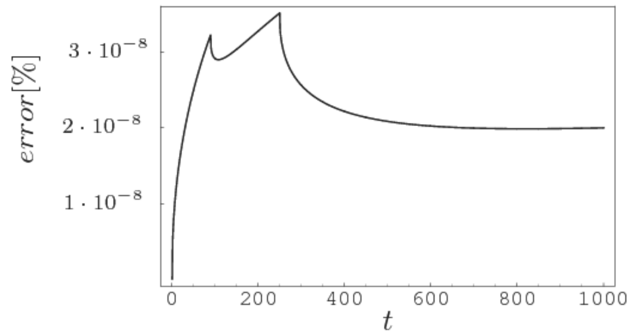

Fig. 4. Error of approximation of the temperature distribution reconstructed on the boundary for x=1, for unnoised input data and control point located in the 10% distance away from this boundary

Perturbation of the input data exerts certainly an influence on the approximate solution of considered problem. Figure 5 shows the error of parameter λ reconstruction obtained in successive iterations of the procedure, calculated for input data burdened by the error of 5% and control point located in 10% distance away from x=1 boundary. Figures 6 presents the error of approximation of temperature distribution on the x=1 boundary, calculated for this case in 20 iterations. Furthermore, Figures 7 and 8 display the error distributions of parameter λ reconstruction and temperature approximation for another case of input data - for measurements noised by the error of 10% and control point located on the x=1 boundary.

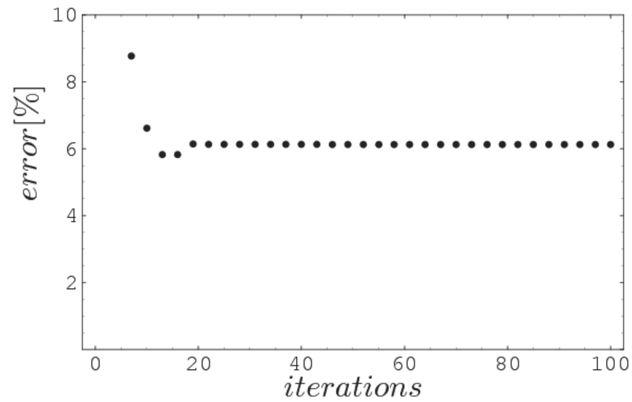

Fig. 5. Error of parameter λ reconstruction depending on the number of iterations, calculated for input data noised by the error of 5% and control point located in the 10% distance away from x=1 boundary

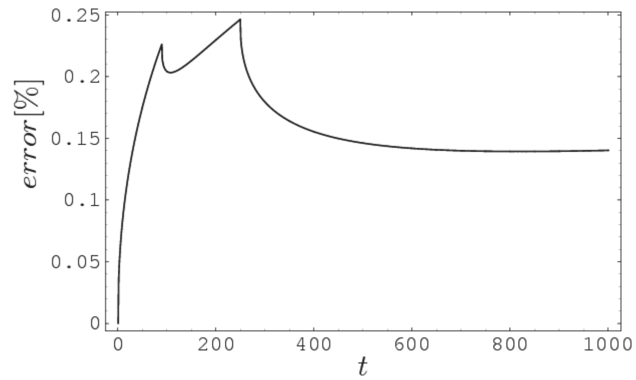

Fig. 6. Error of approximation of the temperature distribution reconstructed on the boundary for x=1, for input data noised by the error of 5% and control point located in the 10% distance away from x=1 boundary

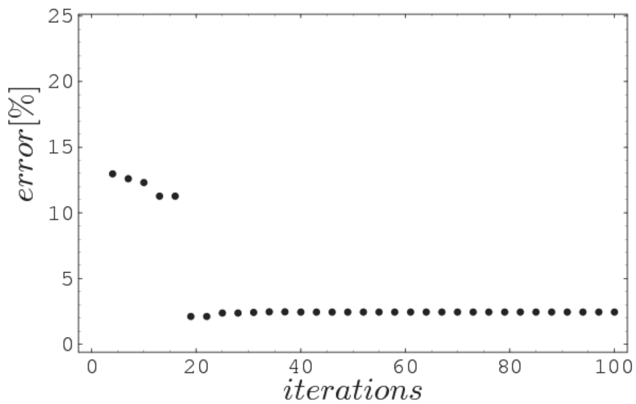

Fig. 7. Error of parameter λ reconstruction depending on the number of iterations, calculated for input data noised by the error of 10% and control point located on the x=1 boundary

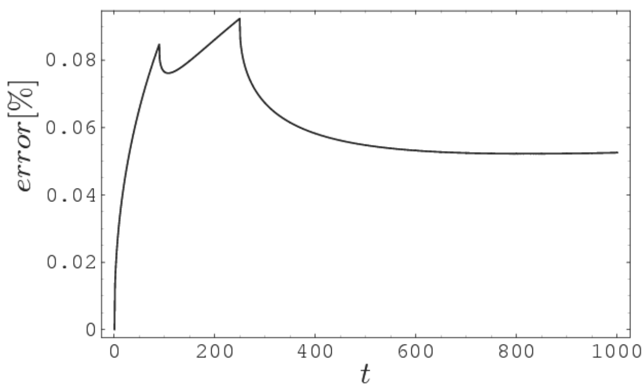

Fig. 8. Error of approximation of the temperature distribution reconstructed on the boundary for x=1, for input data noised by the error of 10% and control point located on the x=1 boundary

Executed experiments indicated that for considered perturbations of input data and various locations of control point the number of iterations sufficient to obtain satisfying results is equal to 20. In all investigated cases the errors of thermal conductivity coefficient reconstruction received in 20 iterations of the algorithm are comparable with the input data noise and further iterations do not improve the results any more. Whereas temperature calculated for 20 iterations is burdened by the approximation error much smaller than the error of input data.

5. Conclusions

Goal of the current paper was the study of a method used for solving the inverse heat conduction problem with boundary condition of the third kind. Solution of the problem consisted in identification of the thermal conductivity coefficient and reconstruction of the temperature distribution in considered region. Specificity of the proposed approach lies in applying the Harmony Search algorithm for minimizing the appropriate functional representing the essential part of the procedure.

Results presented in the paper indicate that receiving the satisfying approximations of the sought elements is possible in 20 iterations of the procedure. From this moment algorithm does not improve the results. In most of presented cases the approximation errors are comparable or much smaller than errors of input data. Just the perturbation at level of 10% causes the significant increase of approximation error, but only in cases of control point moved away from the considered boundary. According to the presented research we may conclude that Harmony Search algorithm is useful tool in solving optimization problems and an indisputable advantage of the HS algorithm is its simplicity and universality. The only assumption needed by this algorithm is the existence of solution. Moreover, the proposed modification of the Harmony Search algorithm positively affected the convergence of procedure in comparison with the results obtained by using the approach presented in paper [23].

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