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The supply of formal notions to synthesis of the vibrating discrete-continuous mechatronic systems

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Analysis and modelling

ABSTRACT

Purpose: of this paper is modeling by different category graphs and analysis of vibrating clamped - free mechatronic system by the approximate method called Galerkin's method. Such approach considers the frequency - modal analysis and assignment of amplitude - frequence charcteristics of the mechatronic system.
Design/methodology/approach: was to nominate the relevance or irrelevance between the characteristics obtained by exact - only for shaft - and considered method. Such formulation especially concerns the relevance the relevance of the natural frequencies-poles of characteristics both of mechanical subsystem and the discrete - continuous clamped - free vibrating mechatronic system.
Findings: this approach is a fact, that approximate solutions fulfill all conditions for vibrating mechanical and/ or mechatronic systems and can be an introduction to synthesis of these systems modeled by different category graphs.
Research limitations/implications: Research limitation is that both torsional vibrating continuous mechanical subsystem and mechatronic discrete - continuous subsystems are linear discrete - continuous are linear systems.
Practical implications: of this study is that the main point can be the introduction to synthesis of considered class mechatronic bar-systems with constant changeable cross-section.
Originality/value: Originality of such formulation rely on the use of the hypergraph methods of modelling and synthesis of torsionally vibrating bars to the synthesis of discrete-continuous mechatronic systems.
Keywords: Applied mechanics; Approximate-Galerkin's method; Graphs; Vibrating mechatronic system

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1. Introduction

The special interest of industry and the scientists during projecting the machines, is to be turned on lifting their efficiency and their reliability. Many branches of industry concentrate on problem of miniaturization of existing objects and also on decrease of waste of their energy.

Therefore it becomes necessary to search the new solutions, having on aim the reduction of movable elements as well as compiled and long kinematic chains. From here in last years it is clear that there is a huge development on the market, especially in field of new technologies basing on phenomenon of piezoelectricity, electro - and the magnetostriction (e.g. [16,18]). The piezoelectric elements are used to eliminate the oscillation [17].

The problems of analysis of vibrating beam systems, discrete and discrete-continuous mechanical systems by means of the structural numbers methods modelled by the graphs, hypergraphs, have been investigated in the research Centre in Gliwice (e.g. [4,9,19]). Other diverse problems concerned analysis and synthesis active mechanical systems [20-22,24,25], synthesis and sensitivity machine driving systems [23,26] were examined for the last several years. The problems of synthesis of electrical systems [1] and of a selected class of continuous, discrete - continuous discrete mechanical systems and active mechanical systems concerning the frequency spectrum have been dealt in [3-8]. The continuous-discrete torsionally and transverse vibrating mechatronic systems were considered in [9,10]. Transformations of hypergraps of flexibly vibrating beams were presented in [13].

The approximate method of analysis, called the orthogolization method [14] and Galerkin's method [15], has been used to obtain the frequency-modal characteristics. To compare the obtained dynamical characteristics – dynamical flexibilities only for mechanical torsionally vibrating bar and transverse vibrating beam being a parts of complex mechatronic systems, an exact method and the Galerkin's method were used [11,12,15]. To examine the influence of parameters of piezoelectric on behavior of a whole system in present paper the relationships on dynamic characteristics of the torsional vibrating continuous mechanical subsystem, joint with piezoelectric transducer into mechatronic system has been determined.

Such formulation can be an introduction to synthesis of vibrating mechatronic systems which will lead to generating the vibrations with require parameters.

2. The equation of torsional vibrating mechatronic system

The formulation of the equations concerning movement and state of mechatronic system should only begin from analysis of piezoelectric, - to be more exact from characterizing this element dependences. In linear approximation component piezoelectric equation are following:

$$\varepsilon_i = \sum_{j=1}^{6} S_{ij}\sigma_j + \sum_{k=1}^{3} d_{ki}E_k, (i = 1, \ldots, 6)\,, \tag{1}$$

$$D_l = \sum_{j=1}^{6} d_{jl}\sigma_j + \sum_{k=1}^{3} e_{kl}E_l, (l = 1,2,3)\,, \tag{2}$$

where: ε_i - components of the deformations, σ_j - the spring constants of the piezoelectric, S_{ij} - components of the tensions, d_{ki}, d_{jl} - the elements of matrix of the electromechanical couplings, E_k E_l ,- components of the electric field, - the electric induction, e_{kl} - the components of dielectric permittivity.

Considered vibrating systems has been shown in Fig. 1.

On basis of equations (1), (2), (where ,) the equation of piezoelectric transducer, was received, expressing direct the and opposite piezoelectric phenomena, in turns.

$$D_1 = d_{15}G_p\gamma_p + \frac{e_1}{l_p}U\left(1 - \frac{2d_{15}^2G_p}{e_1}\right), \tag{3}$$

$$\tau_p = G_p\gamma_p - \frac{2d_{15}G_pU}{l_p}\,, \tag{4}$$

where: G_p - the Kirchhoff's modulus.

The electric charge induced on edges of surface of transducer has the form

$$Q = D_1^* 2\pi R h_p\,, \tag{5}$$

where: D_1^* - the change of average charge along transducer,

and in this way, electric load appears in result of deformation and voltage on internal electrodes *U,* as

$$q = \frac{2\pi R h_p d_{15} G_p}{l_p}\int_{x_1}^{x_2}\gamma_p dx + 2\pi R h_p \frac{e_1}{l_p}U\left(1 - \frac{2d_{15}^2G_p}{e_1}\right). \tag{6}$$

Setting electric system is described in form of

$$R_s\frac{dQ}{dt} = -U\,. \ (7)$$

Attaching the transducer bets perfect, which means that the deformations of transducer and of shaft on the surface of the shaft are equal. Moreover, the change of deformation was skipped and connected with transducer. After presupposing such presumptions, radial distribution of tensions in shaft and the transducer is following:

$$\tau(r) = Gr\frac{\partial\varphi}{\partial x}\,, \tag{8}$$

$$\tau_p = G_p R\frac{\partial\varphi}{\partial x} - 2G_p d_{15}\frac{U}{l_p}\,. \tag{9}$$

Deformation moment can be express as follows

$$M = \left\{GI_0 + \frac{2}{3}\pi G_p R\left[\left(R+h_p\right)^3 - R^3\right]\right\}\frac{\partial\varphi}{\partial x} - \frac{2}{3}\pi G_p\left[\left(R+h_p\right)^3 - R^3\right]d_{15}\frac{U}{l_p}\,. \tag{10}$$

It is skipping the small development of torsional rigidity answering the only transducer the following expression, was received on moment, important in case every $x \in <0,1>$

$$M = GI_0\frac{\partial\varphi}{\partial x} - \lambda^* U\left[H(x) - H(x - l_p)\right], \tag{11}$$

where: $\lambda^* = \frac{2}{3}\pi G_p\left[\left(R+h_p\right)^3 - R^3\right]\frac{d_{15}}{l_p}$.

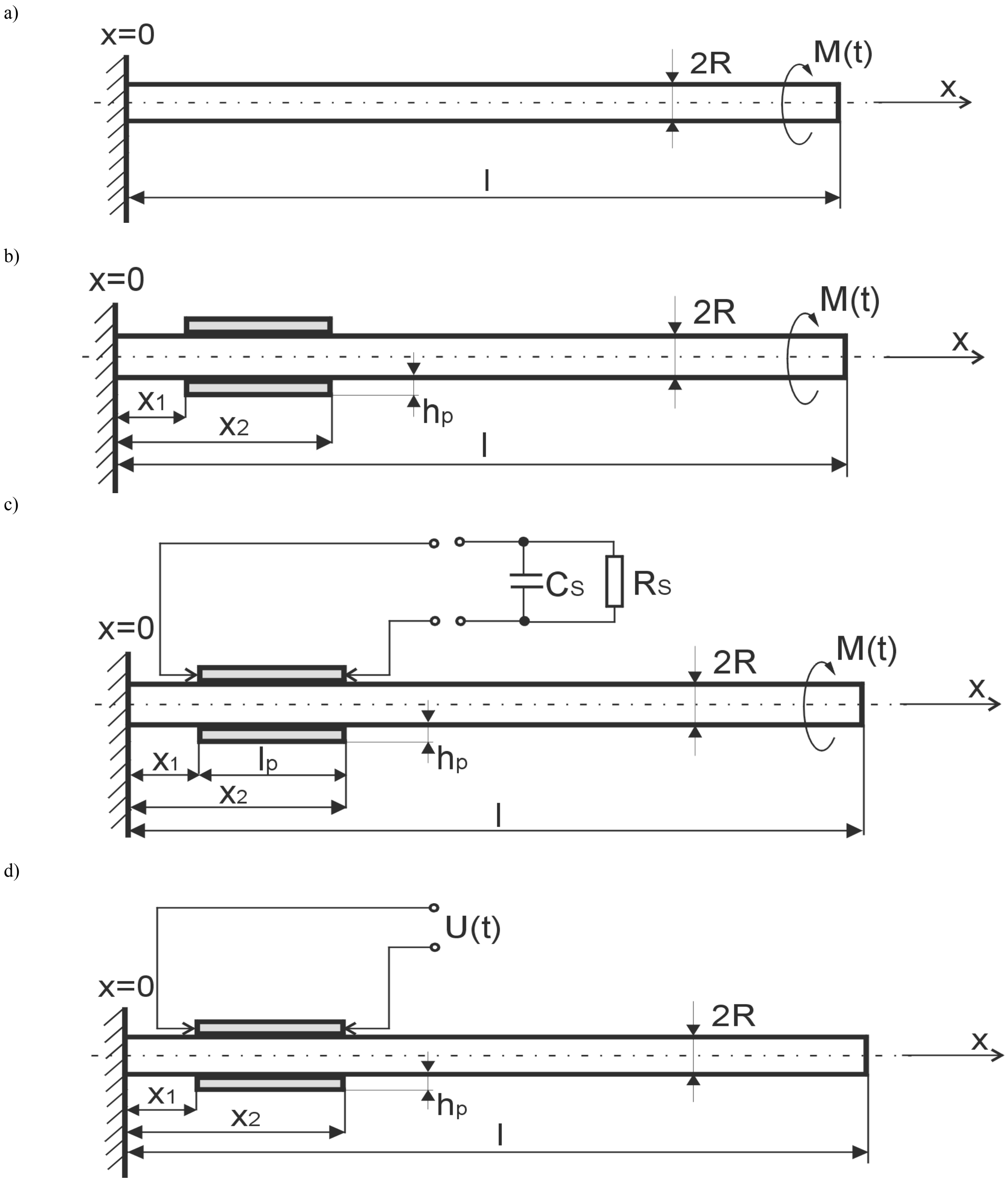

Fig. 1. The torsional vibrating mechanical (a) and mechatronic systems (b-d) with mechanical (a-c) and electrical excitation

And therefore the equation of torsional vibrating shaft with ideally attached piezotransducer is following:

$$\rho I_o \frac{\partial^2\varphi}{\partial t^2} - GI_o\frac{\partial^2\varphi}{\partial x^2} = \frac{-\lambda^*}{l}U\left[\delta(x-x_1)-\delta(x-x_2)\right] + \frac{M_o}{l}\sin\omega t\,\delta(x-l), \qquad (12)$$

where: $\delta(\cdot)$ - Dirac's function.

Dependence (12) is coupling with equation transducer, which can be written in form:

$$\frac{dU}{dt} + \frac{1}{R_sC_p}U + \frac{2\pi R^2 h_p d_{15}G_p}{l_pC_p}\dot{\varphi}(l_p,t)=0 \qquad (13)$$

where: $C_p = 2\pi R h_p \frac{e_1}{l_p}\left(1-\frac{2d_{15}G_p}{e_1}\right)+C_x$

(C_x - additional capacity in short circuit system).

The expressions (12) and (13) take the form of:

$$\begin{cases}\ddot{\varphi}-a^2\varphi_{xx}-bU\left[\delta(x-x_1)-\delta(x-x_2)\right]=cM\delta(x-l),\\ \dot{U}+\alpha_1U+\alpha_2\dot{\varphi}(l_p,t)=0,\end{cases} \qquad (14)$$

where: $a=\sqrt{\frac{G}{\rho}},\ b=\frac{-\lambda^*}{I_o l\rho},\ c=\frac{1}{I_o l\rho},\ \alpha_1=\frac{1}{R_sC_p},\ \alpha_2=\frac{2\pi R^2 h_p d_{15}G_p}{l_pC_p}$.

The set of equations (14) is solving with approximate – Galerkin's method, accepting the solution as

$$\varphi(x,t)=A\sum_{n=1}^{\infty}\sin\left[(2n-1)\frac{\pi}{2l}x\right]\cos\omega t \qquad (15)$$

It is accepted that the considered system is exited by harmonic moment

$$M=M_0\cos\omega t\,. \qquad (16)$$

If extortion has harmonic character, then tension, generated on clamps, piezotransducer will have the same character:

$$U=B\sin\omega t\,. \qquad (17)$$

Solution (15) has to fulfill boundary conditions as:

$$\begin{aligned}&\varphi(0,t)=0,\ \ X(0)T(t)=0\Rightarrow X(0)=0,\\ &\left.\frac{\partial\varphi}{\partial x}\right|_{x=1}=0,\ \ X'(l)T(t)=0\Rightarrow X'(l)=0.\end{aligned} \qquad (18)$$

After calculation of suitable derivatives and their substitution to the equations describing vibration and the state of mechatronic system the set of equations (14) takes the form

$$\begin{cases}A\sin kx\cos\omega t\left[a^2\left(\frac{\pi}{2l}\right)^2-\omega^2\right]-Bb\delta(x)\sin\omega t=cM_0\cos\omega t\,\delta(x-l),\\ B\omega\cos\omega t+\alpha_1B\sin\omega t-\alpha_2A\omega\sin\left((2n-1)\frac{\pi}{2l}l_p\right)\sin\omega t=0,\end{cases} \qquad (19)$$

where: $k=(2n-1)\frac{\pi}{2l}x$, $\delta(x)=\delta(x-x_1)-\delta(x-x_2)$.

or after transformations in matrix form and using the Euler'' theorem

$$\begin{bmatrix}\sin kx\left[a^2\left(\frac{\pi}{2l}\right)^2-\omega^2\right] & -\frac{1}{e^{i\frac{\pi}{2}}}b\delta(x)\\ -\alpha_2\frac{1}{e^{i\frac{\pi}{2}}}\omega\sin kl_p & \omega+\frac{\alpha_1}{e^{i\frac{\pi}{2}}}\end{bmatrix}\cdot\begin{bmatrix}A\\ B\end{bmatrix}=\begin{bmatrix}cM_0\delta(x-l)\\ 0\end{bmatrix} \qquad (20)$$

that is

$$\mathbf{WA}=\mathbf{F}\,. \qquad (21)$$

The main determinant of equation set (21) is equal

$$|\mathbf{W}|=\sin kx\left[a^2\left(\frac{\pi}{2l}\right)^2-\omega^2\right]\left(\omega+\frac{\alpha_1}{e^{i\frac{\pi}{2}}}\right)-\frac{b}{\left(e^{i\frac{\pi}{2}}\right)^2}\delta(x)\alpha_2\omega\sin kl_p\,. \qquad (22)$$

Substituting in (20) first column, by column of free words, the determinant $\mathbf{W}_A$ was received

$$|\mathbf{W}_A|=cM_0\delta(x-l)\left(\omega+\frac{\alpha_1}{e^{i\frac{\pi}{2}}}\right). \qquad (23)$$

The amplitude A is determined as

$$A_n=\frac{|\mathbf{W}_{A_n}|}{|\mathbf{W}|} \qquad (24)$$

and after substitution of it to solution (15), receives after transformations dynamic flexibility as

$$Y_{xl}^{(n)}=\frac{c\delta(x-l)\left(\omega+\frac{\alpha_1}{e^{i\frac{\pi}{2}}}\right)}{\sin kx\left[a^2\left(\frac{\pi}{2l}\right)^2-\omega^2\right]\left(\omega+\frac{\alpha_1}{e^{i\frac{\pi}{2}}}\right)-\frac{b}{\left(e^{i\frac{\pi}{2}}\right)^2}\delta(x)\alpha_2\omega\sin kl_p} \qquad (25)$$

and from here

$$Y_{xl} = \sum_{n=1}^{\infty} Y_{xl}^{(n)} . \tag{26}$$

The transient of absolute value of flexibility (26) (the red line), for three first vibration modes - after further formal transformations and after putting of the numerical values of parameters and when $x=l$, that is $\alpha_Y = |Y_{ll}|$ - it was showed in Fig. 2.

3. Graphs different category as supply formal nations to modeling the considered systems

To fix the meaning of necessary terms and symbols, the review of essential concepts of graph theory have been presented before modelling the torsionally vibrating continuous bar systems and problems connected with it. Weighted hypergraphs (in this paper called also weighted block graphs or weighted graphs of category *k*) have been applied to modelling of the considered mechanical or/and mechatronic systems Definitions of graphs, as mathematical objects, have been presented on the basis of the literature. The bibliography of this subject is very extensive and regards the theory as well as its applications (see [1,2,4]).

$$X = \left({}_1X,\ {}_2X\right) \tag{27}$$

Using the symbols introduced in papers [4,5,19], a following couple is called a *graph,* where: ${}_1X = \{{}_1x_0, {}_1x_1, {}_1x_2, \ldots, {}_1x_n\}$ - finite set of vertices, ${}_2X = \{{}_2x_1, {}_2x_2, \ldots, {}_2x_m\}$ -family of edges, being two-element subsets of vertices, in the form of ${}_2x_k = \left({}_1x_i, {}_1x_j\right)$ $(i,j = 0,1, \ldots, n)$.

The couple

$${}^kX = \left({}_1X,\ {}_2^kX\right) \tag{28}$$

is called a *hypergraph,* where: ${}_1X$ is the set as in (28), and ${}_2^kX = \left({}_2^kX^{(i)} / i \in \mathrm{N}\right)$, ($k$=2,3, ... $\in$N) is a family of subsets of set ${}_1X$; the family ${}_2^kX$ is called a *hypergraph* over ${}_1X$ as well, and ${}_2^kX = \left\{{}_2^kX^{(1)}, {}_2^kX^{(2)}, \ldots, {}_2^kX^{(m)}\right\}$ is a set of edges [2], called *hyperedges* or *blocks,* if

$$\begin{cases} {}_2^kX \neq \varnothing \ (i \in \mathrm{I}), \\ \bigcup\limits_{i\in \mathrm{I}} {}_2^kX^{(i)} = {}_2X. \end{cases} \tag{29}$$

If a subset from the family of subsets of vertices with $n_z \le n$, is distinguished from hypergraph ${}^kX$ with *n* vertices, then the *complete graph* of hypergraph ${}^kX$ is the graph X_Z. In this graph each pair of vertices is incident, and graph X_Z has $m = \binom{n}{2}$ edges.

Skeleton ${}^kX_0$ of hypergraph ${}^kX$ is a graph obtained as the result of substitution of each subset of vertices by tree $\underset{0}{X}$, composed of one-dimensional edges and stretched on all vertices of hypergraph ${}^kX$. The tree $\underset{0}{X}$ of graph *X* with *n* vertices and *m* edges is a connected subgraph with the same number of vertices and with $\underset{0}{m} = n-1$ edges, in which there are no circuits and loops. So every skeleton of subsets of vertices is a tree of substitute-complete graph.

A tree in which every vertex ${}_1x_i$ $(i = 1, \ldots, n)$ is incident with vertex ${}_1x_0$ by edge ${}_2x_k = \left({}_1x_i, {}_1x_0\right)$, $(k = 1, \ldots, \underset{0}{m})$, (see e.g. [4,5]) is called the *Lagrange skeleton*.

Graphs X and hypergraphs ${}^kX$ have been shown in their geometrical representation on plane. Sets of edges ${}_2X$ have been marked by lines, subsets of family ${}_2^kX$ (hyperedges or blocks) - two-dimensional continuum with enhanced vertices, in the shape of circles.

In this paper hypergraphs - graphs of category k - ${}^kX$ (k=2,3) are used, which will be clearly mentioned each time, as well as graphs *X*, called also graphs of the first category - ${}^1X$ (see [4,5]).

The basic notions which have been written in italics are shown in Fig. 3.

Using notion of graph and hypergraph and their connections with structural numbers [1] and system of notation [4,5,19], methods of modification of transforming vibration system as task of the synthesis of dynamical characteristic - mobility has been presented.

A characteristic - dynamical flexibility is given in form

$$Y(s) = \frac{\sum\limits_{i=0}^{k} c_i th^j \Gamma s}{s \sum\limits_{j=0}^{l} d_j th^j \Gamma s} . \tag{30}$$

After transformations [4,514,15]

$$V(s) = sY(s) , \tag{31}$$

$$r = \mathrm{th}\Gamma s , \tag{32}$$

the mobility has been obtained as

$$V(r) = \frac{\sum\limits_{i=0}^{k} c_i r^i}{s \sum\limits_{j=0}^{l} d_j r^j} . \tag{33}$$

where: $c_k, c_{k-1}, \dots, c_0, d_l, d_{l-1}, \dots, d_0$ are any real numbers,

$$\Gamma = \sqrt{\frac{\rho}{G}}L = \sqrt{\frac{\rho^{(i)}}{G^{(i)}}}L^{(i)}$$, ρ -mass density, $G^{(i)}$

Kirchhoff's modulus , $L=L^{(i)}$ - length of basic element, $s=j\omega$, $j=\sqrt{-1}$, $c_k, c_{k-1}, \dots, c_0, d_l, d_{l-1}, \dots, d_0$ -real numbers, i,j,k,l natural numbers, k-l=1.

4. Modelling of torsionally vibrating subsystems of mechatronic systems by hypergraphs

The problem consists in modelling of torsionally vibrating multiple-segment with mechanical bar systems as subsystems of mechatronic systems in the form of models with uniformly distributed parameters and constant section in the segment.

In the modelling of the considered class of continuous systems, the dependence between the amplitudes of *generalized forces* ${}_2s_k \in {}_2S$ and *generalized displacements* ${}_1s_i \in {}_1S$ can by described by *dynamical flexibility* Y_{ik} [4,5]. In other words, dynamical flexibility is the assigned amplitude of generalized displacement in the direction of i-th generalized coordinate caused by generalized force in the form of harmonic function with unitary amplitude, in relation with k-th generalized coordinate, so

$$ {}_1s_i = Y_{ij}\ {}_2s_j, \tag{34}$$

where: ${}_2s_j = Q_j \sin \omega t = 1\, e^{j\omega t}$, $j=\sqrt{-1}$, ω - frequency.

In the example of torsional vibration of the subsystem (i) with constant cross-section and constant torsional rigidity $(GJ_0)^{(i)}$ (where $G^{(i)}$ - Kirchhoff's modulus of a bar structure, $J_0^{(i)}$ - polar moment of inertia of bar cross-section as well as length $l^{(i)}$, the model in the form of a determined and continuous system is introduced.

In this model, generalized displacements ${}_1s_1^{(i)}$ and ${}_1s_2^{(i)}$ - angles of rotation correspond to its extreme points. These displacements are measured in the inertial system of reference. Moreover, the origin of the inertial system of reference has generalized coordinate ${}_1s_0^{(i)} = 0$ assigned to it (see e.g. [4,5]).

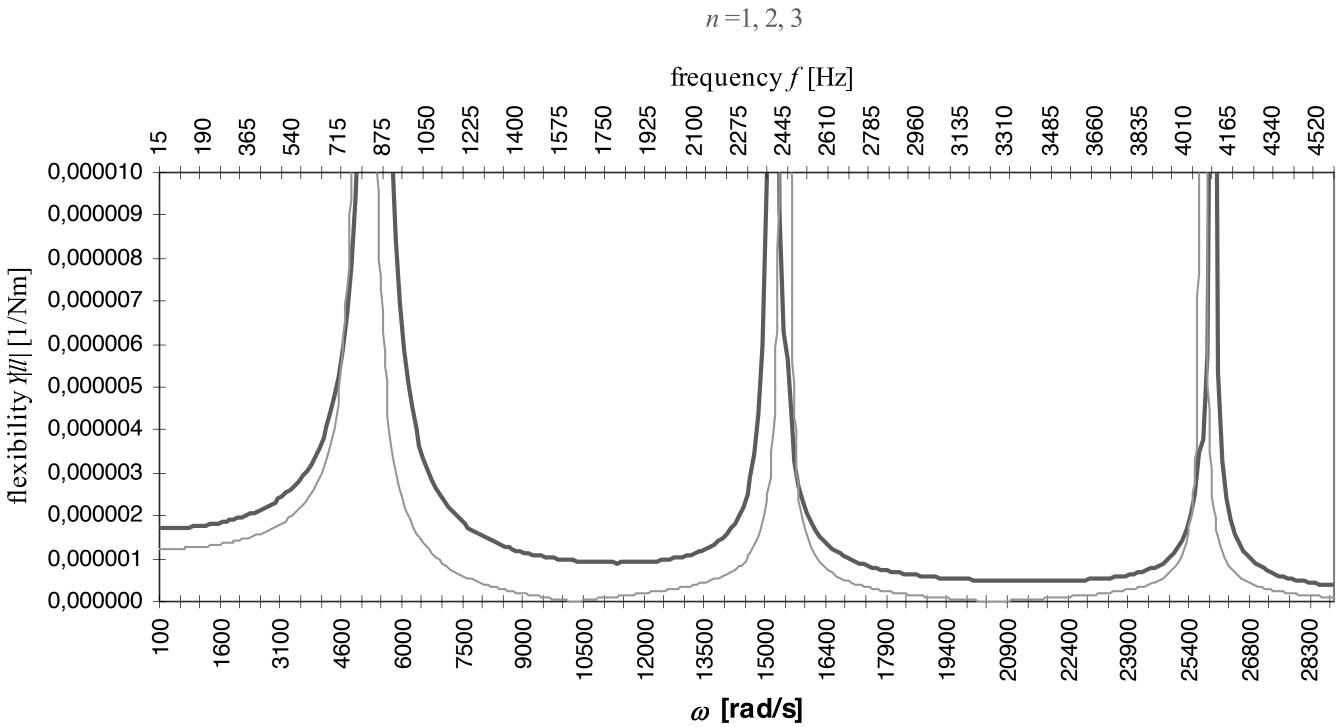

Fig. 2. Transient of the sum for n=1, 2, 3 vibration mode

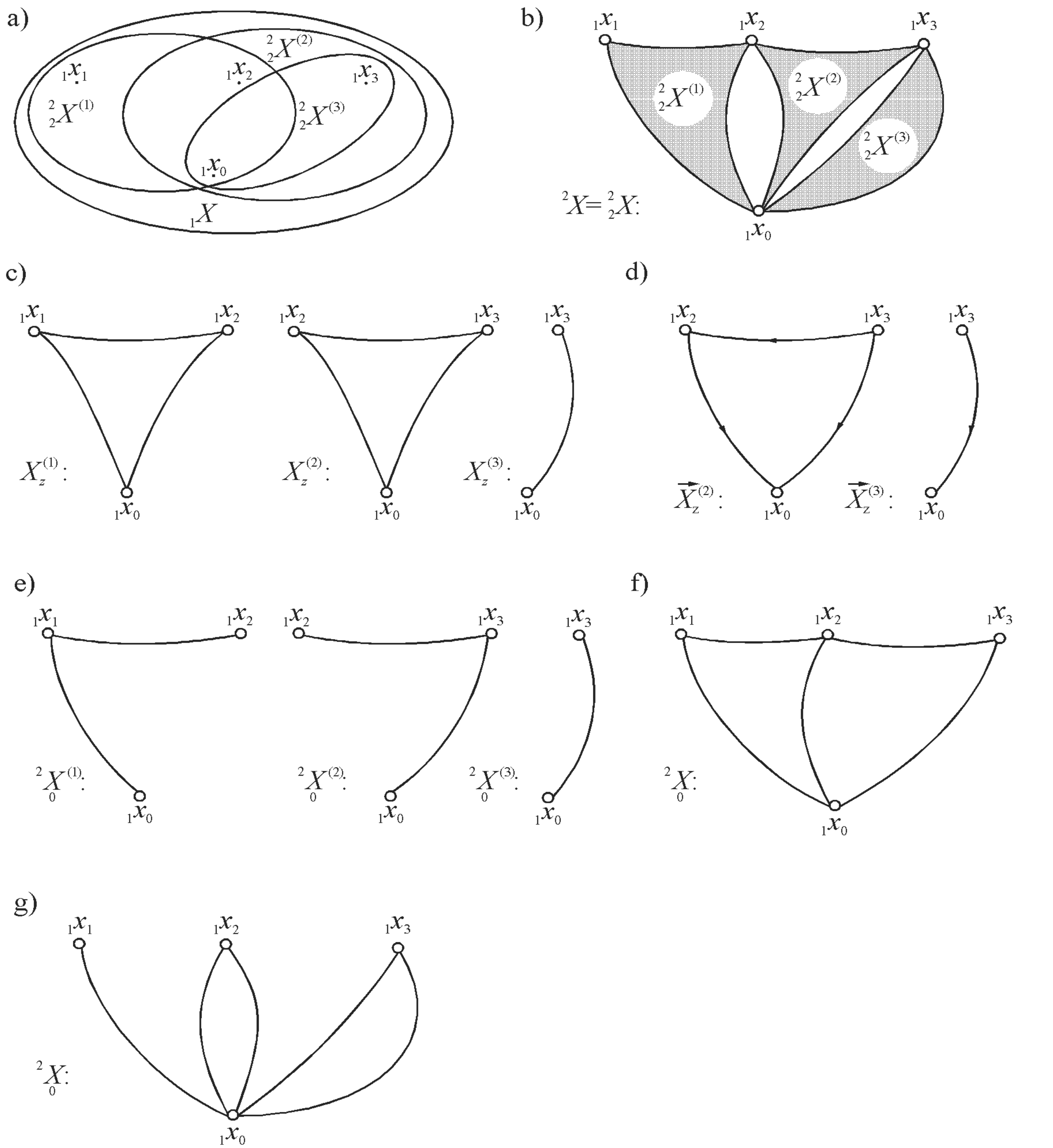

Fig. 3. Basic notions concerning the class of graphs, which are used throughout this paper: a) set of vertices of a hypergraph, b) graphical representation of a three-block graph, c) complete graphs of hypergraph blocks, d) complete oriented graphs of three-vertex blocks and of a two-vertex block, e) optionally selected tree-skeletons of hypergraph blocks, f) skeleton of hypergraph, g) Lagrange skeleton of hypergraph

Therefore a set of generalized displacements of a torsional vibrating bar can be formulated as follows: ${}_1S^{(i)} = \left\{{}_1s_0^{(i)},\ {}_1s_1^{(1)},\ {}_1s_2^{(2)}\right\}$, while its dynamical flexibilities set can be denoted as $Y^{(i)} = \left\{Y_{11}^{(i)},\ Y_{22}^{(i)},\ Y_{12}^{(i)}\right\}\ \left(Y_{12}^{(i)} = Y_{21}^{(i)}\right)$.

In the course such one-to-one transformation, that:

$$f\colon {}_1S^{(i)} \to {}_1X^{(i)},\ {}_1s_j^{(i)} \in {}_1S^{(i)},\ {}_1x_j^{(i)} \in {}_1X^{(i)},\quad j = 0,1,2), \tag{35}$$

the *hypergraph of bar model* is obtained

$$\underset{f}{{}^2X^{(i)}} = \left[{}^2X^{(i)}, f\right], \tag{36}$$

where: ${}^2X^{(i)} = \left({}_1X^{(i)},\ {}_2^kX^{(i)}\right)$, ${}_1X^{(i)} = \left\{{}_1x_0^{(i)},\ {}_1x_1^{(i)},\ {}_1x_2^{(i)}\right\}$, ${}_2^kX^{(i)}$ - one-element family - three-element subset of vertices ${}_1X^{(i)}$.

Investigating *i*-th *segment* in a *n-segment* mechanical bar as a subsystem of mechatronic system with constant section, the hypergraph model $\underset{f}{{}^kX^{(i)}}$ is introduced.

All the elements of hypergraph and all the generalized displacements, material coefficients and geometric coefficients should be denoted by subscript (i) placed to the right of the stem symbol, whereas subscript k=2 should be placed to the left of the stem symbol.

On the basis of this assumption, geometrical representation of mapping (36) has been shown in [4,5].

Introducing f_1 that assigns the generalized displacements to vertices ${}_1x_j^{(i)}$ of hypergraph $\underset{f}{{}^2X^{(i)}}$ as:

$$f_1\left({}_1x_j^{(i)}\right) = \left|{}_1s_j^{(i)}\right|\ (j = 0,1,2), \tag{37}$$

weighted hypergraph is obtained

$$\underset{1}{{}^2X^{(i)}} = \left[\underset{f}{{}^2X^{(i)}}, f_1\right]. \tag{38}$$

According to (37), generalized coordinates to the vertices of complete graph $\underset{f}{{}^2X_Z^{(i)}}$ of hypergraph $\underset{f}{{}^2X^{(i)}}$ assigned by f_2 dynamical flexibilities to edges of this graph:

$$f_2\left(\left\{{}_1x_0^{(i)},\ {}_1x_1^{(i)}\right\},\ \left\{{}_1x_0^{(i)},\ {}_1x_2^{(i)}\right\},\ \left\{{}_1x_0^{(i)},\ {}_1x_2^{(i)}\right\}\right) = \left[\left\{Y_{11}^{(i)}\right\},\ \left\{Y_{22}^{(i)}\right\},\ \left\{Y_{12}^{(i)}\right\}\right], \tag{39}$$

a complete-substitute weighted graph is obtained:

$$\underset{12}{{}^2X_Z^{(i)}} = \left[\underset{f}{{}^2X_Z^{(i)}}, f_1, f_2\right]. \tag{40}$$

Weighted Lagrange skeleton

$$\underset{12}{{}^2X_0^{(i)}} = \left[\underset{f}{{}^2X_Z^{(i)}}, f_1, f_2\right] \tag{41}$$

is a weighted subgraph of complete weighted graph $\underset{12}{{}^2X_Z^{(i)}}$. Geometrical representation of graph $\underset{12}{{}^2X_0^{(i)}}$ is shown in [4,5].

For a weighted Lagrange skeleton, taken into consideration in (37), it can be noted that (see [4,5]):

$${}_1s_j^{(i)} - {}_1s_0^{(i)} = {}_1s_j^{(i)} - 0 = {}_1s_j^{(i)},\ (j = 1,2). \tag{42}$$

Considering (39), (40) and (35), weighted Lagrange skeleton may be treated as oriented polar graph $\underset{00}{\vec{X}^{(i)}}$.

It is not difficult to notice that making the assignment f_3 to the edges of weighted Lagrange skeleton $\underset{12}{{}^2\vec{X}_0^{(i)}}$ of hypergraph for the model of i-th bar - $\underset{f}{{}^2X^{(i)}}$, couples of numbers - respectively - generalized coordinates and generalized forces

${}_2S = \left[\left|{}_2s_1^{(i)}\right|,\ \left|{}_2s_2^{(i)}\right|\right]$ so that:

$$f_3\left(\left\{{}_1x_0^{(i)},\ {}_1x_1^{(i)}\right\},\ \left\{{}_1x_0^{(i)},\ {}_1x_2^{(i)}\right\}\right) = \left[\left\{\left|{}_1s_1^{(i)}\right|,\ \left|{}_2s_1^{(i)}\right|\right\},\ \left\{\left|{}_1s_2^{(i)}\right|,\left|{}_2s_2^{(i)}\right|\right\}\right], \tag{43}$$

a *polar graph* is obtained

$$\underset{00}{\vec{X}^{(i)}} = \underset{3}{\vec{X}_0^{(i)}} = \left[\underset{f}{{}^2\vec{X}_0^{(i)}}, f_3\right]. \tag{44}$$

Moreover in the case of oriented polar graph $\underset{00}{\vec{X}^{(i)}}$, polar equation [4,5] can be formulated as:

$$\begin{bmatrix} {}_1s_1^{(i)} \\ {}_1s_2^{(i)} \end{bmatrix} = \begin{bmatrix} Y_{11}^{(i)} & 0 \\ 0 & Y_{22}^{(i)} \end{bmatrix} \begin{bmatrix} {}_2s_1^{(i)} \\ {}_2s_2^{(i)} \end{bmatrix}. \tag{45}$$

Matrix equation (45) is a particular case of equation (34).

In the case of analysis of n-segment model of the system, composed of subsystems with constant section, vibrating torsionally, it is modelled by the loaded graph of the second category with n three-vertices-blocks, connected to those vertices to which the corresponding generalized coordinates are assigned (see i.e. [4,5]).

The use of a weighted hypergraph (as a model of torsionally vibrating mechanical and/or mechatronic system) in this way may provide to the basis for the formalization which is the necessary condition of numerical discretization of the considered class of continuous mechanical systems.

5. Formulation of the synthesis of both the mechanical and mechatronic systems

The synthesis consists in investigating the structure of a system with the continuous distribution of parameters and specific requirements set for the realization of the desired mechanical phenomena, because the discrete model is too remote from the real object. Moreover, the problem of synthesizing discrete physical, mechanical and, first of all, electrical and electronic systems is widely analyzed in scientific research. However, it is considerably difficult to find examples of methods of synthesizing continuous mechanical systems. The authors of the papers were approaching these problems emphasize that the synthesis of systems with continuous distribution of parameters is only beginning to be developed, so the exact formulation and solution are still to be considered. So far, this task has been approached by attempts to determine a rectangular bar or a sleeve (depending on longitudinal or torsional vibrations) from the frequency equation and only on the basis of the first natural frequency.

The first attempt of the solution to this problem concerning the frequency spectrum has been made in the Gliwice research centre in [4,5].

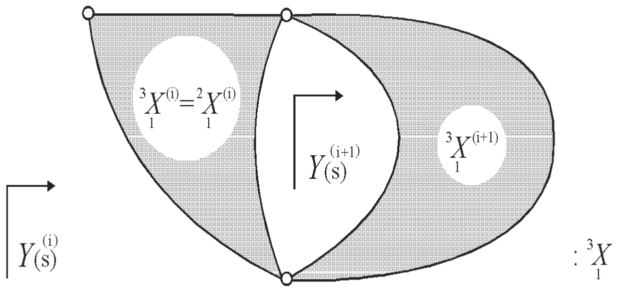

Fig. 4. Illustration of transformation of the dynamical characteristic and of the graph of a free bar and a clamped bar

In this paper the method was applied in order to synthesize the dynamical characteristic of the torsionally vibrating mechanical or/and mechatronic system with cascade structure. This is the method of decomposition of characteristics into partial fractions presented by graphs.

6. Cascade method of synthesis of torsionally vibrating bar subsystems of mechatronic systems modeling by graphs

In this paper the equation enabling the synthesis of the mobility function and, at the same time, its inversion is introduced. In order to obtain these function, the third category graph ${}^3_1X = \left\{ {}^3_1X^{(i)}, {}^3_1X^{(i+1)} \right\}$ [4,5], as a model of longitudinally or torsionally transforming vibration mechnical or mechatronic system, is considered in Fig. 4 (hypergraph ${}^3_1X^{(i)} = {}^2_1X^{(i)}$ is a model of the "i" basic element, whereas hypergraph ${}^3_1X^{(i+1)}$ is a model of the other part the system contains elements from (i+1) to n.

On the base of a skeleton ${}^3X_0$ of the hypergraph ${}^3_1X$ (Fig. 4) structural complementary number A_0^d, third category structural number ${}^3A$ and its algebraical derivative ${}^3A_{a_i}$ are equal [4,5]

$$\begin{cases} A_0^d = \begin{bmatrix} b_i & a_{i+1} \end{bmatrix},\ {}^3A = [A]\downarrow \begin{bmatrix} {}^2A_i \\ {}^2A_{i+1} \end{bmatrix} = \\ = \begin{bmatrix} b_i & a_{i+1} \end{bmatrix}\downarrow \begin{bmatrix} {}^2A_i \\ {}^2A_{i+1} \end{bmatrix} == \begin{bmatrix} {}^2A_{ib} & {}^2A_i \\ {}^2A_{i+1} & {}^2A_{i+1,a} \end{bmatrix}, \\ {}^3A_{a_i} = [a_i]\downarrow \left[{}^3A\right] = \begin{bmatrix} {}^2A_{iab} & {}^2A_{ia} \\ {}^2A_{i+1} & {}^2A_{i+1,a} \end{bmatrix}. \end{cases} \tag{46}$$

In the tables of algebraic derivatives of the structural numbers ${}^3A$ and ${}^3A_{a_i}$

$$\boldsymbol{P} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix},\quad \boldsymbol{P}_{a_i} = \begin{pmatrix} 2 & 1 \\ 0 & 1 \end{pmatrix} \tag{47}$$

there are not identical columns, so structural completed numbers are respectively equal

$$\mathsf{A} = \mathsf{A}_{ib}\mathsf{A}_{i+1} + \mathsf{A}_i\mathsf{A}_{i+1,a},\ \mathsf{A}_{a_i} = \mathsf{A}_{iab}\mathsf{A}_{i+1} + \mathsf{A}_{ia}\mathsf{A}_{i+1,a}. \tag{48}$$

The dynamical flexibility after transformations Y_{a_i} takes form

$$Y_{a_i} = Y^{(i)}(s) = \frac{Y_{iab}Y_{ib} + Y_{ia}Y_{i+1,a}}{Y_{ib} + Y_{i+1,a}} = \\ = \left(\frac{l}{GJ}\right)^{(i)} \frac{1}{\gamma s} \frac{th\ \gamma s + Y^{(i+1)}(s)\left(\frac{GJ}{l}\right)^{(i)} \gamma s}{1 + Y^{(i+1)}(s)\left(\frac{GJ}{l}\right)^{(i)} \gamma s\ \ th\ \ \gamma s}, \tag{49}$$

where: $Y^{(i+1)}(s)$ - dynamical flexibility of the other part of the system contains the elements from (i+1) to n), $\gamma = \sqrt{\frac{\rho}{G}}\ \ l$.

Using the transformation $V(s)$=$Y(s)$ [4,5] the mobility $V^{(i)}(s)$ is obtained in form

$$V^{(i)}(s)=\left(\frac{l}{GJ}\right)^{(i)}\frac{1}{\gamma}\frac{th\ \gamma s+V^{(i+1)}(s)\left(\frac{GJ}{l}\right)^{(i)}\gamma}{1+V^{(i+1)}(s)\left(\frac{GJ}{l}\right)^{(i)}\gamma\ \ th\ \ \gamma s}. \tag{50}$$

After the next transformation $p=th\ \gamma s$ called Wyndrum's [4,5] transformation, the mobility $V^{(i)}(p)$ takes form

$$V^{(i)}(p)=\left(\frac{l}{GJ}\right)^{(i)}\frac{1}{\gamma}\frac{p+V^{(i+1)}(p)\left(\frac{GJ}{l}\right)^{(i)}\gamma}{1+V^{(i+1)}(p)\left(\frac{GJ}{l}\right)^{(i)}\gamma\ \ p}, \tag{51}$$

where: $V^{(i+1)}(s)=sY^{(i+1)}(s)$.

From (51) mobility $V^{(i+1)}(p)$ is obtained

$$V^{(i+1)}(p)=\left(\frac{l}{GJ}\right)^{(i)}\frac{1}{\gamma}\frac{V^{(i)}(p)+\left(\frac{l}{GJ}\right)^{(i)}\frac{1}{\gamma}p}{\left(\frac{l}{GJ}\right)^{(i)}\frac{1}{\gamma}-V^{(i)}(p)\ \ p}, \tag{52}$$

where: $V^{(i+1)}(p)$ is the mobility of the transforming vibration system after its basic element with mobility $V^{(i)}(p)$ is removed, J-polar inertial moment of the bar cross section.

Equation (52) makes it possible to determine the next (i+1) (i=1,2, ... ,n) dynamical characteristic of the transforming vibration system synthesized in this way.

7. Algorithm of synthesis of mechanical subsystem by the recurrent cascade method as necessary condition of mechatronic one

To carry out the synthesis of the mobility $V(p)$ the form of (52) or its inversions $U(p)$ (comp. with [4,5]) by the cascade method it necessary to:

1^0 Assume that

$$V(p)=V^{(1)}(p) \tag{53}$$

2^0 Determine values of parameters $(GJ)^{(i)}$ and $(\rho J)^{(i)}$ from equations

$$(GJ)^{(i)}=\frac{1}{\beta V^{(i)}(1)},\quad (\rho J)^{(i)}=\frac{(GJ)^{(i)}}{G}\rho, \tag{54}$$

assuming p=1 and i=1, $\beta=\sqrt{\frac{\rho}{G}}$.

3^0 Determine $V^{(2)}(p)$ of other part of system containing segments from i=2 to n, from Richards' theorem

$$V^{(i+1)}(p)=V^{(i)}(1)\quad\frac{V^{(i)}(p)-pV^{(i)}(1)}{V^{(i)}(1)-pV^{(i)}(p)}. \tag{55}$$

4^0 Devide the numerator and denominator of mobility $V^{(2)}(p)$ by (p^2-1); this is the condition of the physical realization of calculated mobility $V^{(2)}(p)$.

5^0 Repeat step 2, assuming i=2.

6^0 Carry out step 3^0 in order to calculate $V^{(3)}(p)$.

7^0 Check step 4^0 by dividing the numerator and denominator of $V^{(3)}(p)$ by (p^2-1).

8^0 Repeat steps 2^0, 3^0, 4^0, ... successively to determine formulas $V^{(4)}(p)$, $V^{(5)}(p)$, ..., $V^{(n)}(p)$.

The algorithm described above is to be continued until type p or $\frac{1}{p}$ of mobility $V^{(n)}(p)$ is achieved - multiplied by real constant H - and it is not possible to carry out step 3^0 after step 2^0 in order to determine $(GJ)^{(n)}$ and $(\rho J)^{(n)}$. This is the end of the synthesizing process.

8. Remarks

Applied method and received results can make up the introduction to the synthesis of torsionaly vibrating mechatronic systems with constant changeable cross-section. The problems will be presented in future works.

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