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GENERALIZATION OF FUZZY PROBABILISTIC CONTROLLER

Summary: The paper deals with an idea of generalization of control algorithm called fuzzy probabilistic controller.
Various operations on probabilistic sets and their distribution function characterization are discussed in the first part of this work.
Then the construction of generalized fuzzy probabilistic control algorithm is presented as well.

1. INTRODUCTION

The idea for implementation of control strategy of human operator of complex ill-defined industrial process was proposed for the first time by Mamdani as a form of algorithm called fuzzy controller.
The fuzzy controller consists of a set of linguistic rules tied together by two concepts i.e. fuzzy implication and compositional rule of inference.

The possibility of expressing the control strategy for ill-defined process in terms of fuzzy probabilistic sets should be taken into account because of the existence of ambiguity and subjectivity of human operator controlling such process. This approach provides a more general class of controllers called fuzzy probabilistic controllers.
Czogała and Pedrycz [4,5] have discussed the control algorithm called fuzzy probabilistic controller for the first time. Using the distribution function characterization of probabilistic sets they have taken into account only max and min operations on respective probabilistic sets.

In this paper the generalized concept of fuzzy probabilistic controller for any operations on probabilistic sets is discussed. It is also pointed out that the concept of fuzzy controller proposed by Mamdani is embedded in the concept of fuzzy probabilistic controller that results from the embedding of the fuzzy set in the concepts of fuzzy probabilistic set.

2. VARIOUS OPERATIONS ON PROBABILISTIC SETS

The notion of probabilistic set in sense of Hirota has been introduced and described in many papers [1, 2, 5, 6], so we will not introduce it here.

Similarly, as in theory of fuzzy sets we can consider various operations on probabilistic sets [2].

Generally we should solve the following problem: from the distribution function of a collection of probabilistic sets:

$$X_1 : X^1 \times \Omega \to [0,1]$$
$$\vdots \qquad (1)$$
$$X_n : X^n \times \Omega \to [0,1]$$

where $X^1, \ldots, X^n$ are for simplicity finite universes of discourse i.e.

$$X^1 = \left\{x_1^1, \ldots, x_{K_1}^1\right\} \quad (\mathrm{card}\, X^1 = K_1)$$
$$\vdots$$
$$X^n = \left\{x_1^n, \ldots, x_{K_n}^n\right\} \quad (\mathrm{card}\, X^n = K_n)$$

determine the distribution function of the following collection of Borel functions of above given probabilistic sets for each point

$(x_{i_1}^1, \ldots, x_{i_n}^n) \in X^1 \ \ldots \ X^n \quad (1 \leqslant i_j \leqslant K_j, \quad 1 \leqslant j \leqslant n)$

$$Y_1 = g_1(X_1, \ldots, X_n)$$
$$\vdots \qquad (2)$$
$$Y_k = g_k(X_1, \ldots, X_n)$$

For general solution of this problem we will present here two methods well-known from probability theory.

(i) Let us consider the case when n-dimensional vector $(X_1, \ldots, X_n)$ has a probability density function $f_{X_1 \ldots X_n}(x_1, \ldots, x_n)$.
It is shown in probability theory that the desired distribution function may be defined by the formula

$$F_{Y_1 \ldots Y_k}(y_1, \ldots, y_k) = \int_{G(y_1, \ldots, y_k)} \cdots \int f_{X_1 \ldots X_n}(x_1, \ldots, x_n) dx_1 \ldots dx_n \qquad (3)$$

where $G(y_1, \ldots, y_k)$ is the region of integration being determined as

$$G(y_1, \ldots, y_k) = \left\{(x_1, \ldots, x_n) \mid g_i(x_1, \ldots, x_n) < y_i \ (i = 1, 2, \ldots, k)\right\} \qquad (4)$$

In the case of discrete random variables the solution is obviously given by means of an n-fold sum which is also extended over the domain $G(y_1, \ldots, y_k)$

(ii) Considering singular n-ary operation on collection of probabilistic sets $X_1,\dots,X_n$ we may also use a particular case of equation (2) i.e.

$$\begin{aligned} Y_1 &= g(X_1,\dots,X_n) \\ Y_2 &= X_2 \\ &\vdots \\ Y_n &= X_n \end{aligned} \tag{5}$$

Assuming that functions

$$\begin{aligned} y_1 &= g(x_1,\dots,x_n) \\ y_2 &= x_2 \\ &\vdots \\ y_n &= x_n \end{aligned} \tag{6}$$

are continuous and one-to-one and the partial derivatives of g i.e. $\partial g/\partial x_i$ are continuous and the Jacobian of the transformation (6) defined as

$$J = \begin{vmatrix} \frac{\partial g}{\partial x_1} & \dots & \frac{\partial g}{\partial x_n} \\ & 1 & & \\ & & 1 & \\ & & & \ddots \\ & & & & 1 \end{vmatrix} \neq 0 \tag{7}$$

exists in considered n-dimentional domain $x_i \in [0,1]$ the inverse transformation

$$\begin{aligned} x_1 &= h(y_1,\dots,y_n) \\ x_2 &= y_2 \\ &\vdots \\ x_n &= y_n \end{aligned} \tag{8}$$

exists as well.

The n-dimentional density function of $Y_1,\dots,Y_n$ by using the above defined Jacobian takes a form

$$f_{Y_1\dots Y_n}(y_1,\dots,y_n) = \frac{1}{|J|} f_{X_1\dots X_n}(h(y_1,\dots,y_n),\, y_2,\dots,y_n) \tag{9}$$

where
$$|J| = \left|\frac{\partial g}{\partial x_n}\right| \; x_1 = h(y_1,\dots,y_n), x_2 = y_2,\dots,x_n = y_n \tag{10}$$

Taking into account the last formula we get the probability density function for $Y_1 = V$, i.e. for $y_1 = v$

$$f_V(v) = \int_{-\infty}^{\infty} \dots \int_{-\infty}^{\infty} \frac{1}{|J|} f_{X_1 \dots X_n}(h(v,y_2,\dots,y_n),y_2,\dots,y_n)dy_2 \dots dy_n \qquad (11)$$

Specifying these above written results for binary operations we get from equation (3)

$$F_Z(z) = \iint_{G(z)} f_{XY}(x,y)dx\,dy \qquad (12)$$

where $G(z) = \{(x,y) \mid g(x,y) < z\}$
and from equation (11) we get

$$f_V(v) = \int_{-\infty}^{\infty} \frac{1}{|J|} f_{XY}(h(v,u),u)\,du \qquad (13)$$

Of course, while the first method may be used for continuous and non-continuous operations, the second method suits fine only for continuous operations on probabilistic sets but sometimes for many operations it is useful to combine both these methods.

Now let us introduce operations on probabilistic sets induced in the interval $[0,1]$ which are special cases of general notion of so-called "t-norm" in $[0,1]$ interval [3]. All these operations on probabilistic sets may be described by means of distribution or density functions. Let X and Y are probabilistic sets defined on the cartesian products $X \times \Omega$ and $Y \times \Omega$ respectively.

We can introduce for example [3]

a) lattice operations

$$X \wedge Y = \min(X,Y), \quad X \vee Y = \max(X,Y) \qquad (14)$$

b) probabilistic (or algebraic) operations

$$X.Y = XY, \quad X \dotplus Y = X + Y - XY \qquad (15)$$

c) bounded (or logical) operations

$$X \odot Y = \max(0, X + Y - 1), \quad X \oplus Y = \min(1, X + X) \qquad (16)$$

d) drastic operations

$$X \mathbin{\dot\wedge} Y = \begin{cases} X \wedge Y & \text{if } X + Y = 1 \\ 0 & \text{if } X + Y < 1 \end{cases},$$
$$X \mathbin{\dot\vee} Y = \begin{cases} 1 & \text{if } X \cdot Y > 0 \\ X \vee Y & \text{if } X \cdot Y = 0 \end{cases}. \qquad (17)$$

Several examples of connectives of probabilistic sets similar to these introduced by Yager [7] can be also presented here.

e) generalized minimum (intersection) and maximum (union)

$$X \cap_p Y = 1 - \min(1, \sqrt[p]{(1 - X)^p + (1 - Y)^p})$$
$$X \cup_p Y = \min(1, \sqrt[p]{X^p + Y^p}) \quad (18)$$

$p \to \infty$ and $p = 1$ involve results

$$X \cap_\infty Y = \min(X, Y)$$
$$X \cup_\infty Y = \max(X, Y)$$

and

$$X \cap_1 Y = \max(0, X + Y - 1)$$
$$X \cup_1 Y = \min(1, X + Y) \quad (20)$$

Let us illustrate our considerations up to now by means of some examples. Using the first method it may be easily proved for max and min operations on probabilistic sets the following [1]:

If $X_1, X_2, \ldots, X_n$ are probabilistic sets on cartesian products $X^1 \times \Omega, X^2 \times \Omega, \ldots, X^n \times \Omega$ respectively characterized by the respective distribution functions then the distribution functions of $\max(X_1, X_2, \ldots, X_n)$ and $\min(X_1, X_2, \ldots, X_n)$ take then forms

$$F_{\max(X_1, X_2, \ldots, X_n)}(w) = F_{X_1, X_2 \ldots X_n}(w, w, \ldots, w) \quad (21)$$

$$F_{\min(X_1, X_2, \ldots, X_n)}(w) = \sum_{j=1}^{n} F_{X_j}(w) - \sum_{1 \leq j < k \leq n} F_{X_j X_k}(w, w) +$$
$$\ldots + (-1)^{n+1} F_{X_1, X_2 \ldots X_n}(w, w, \ldots, w) \quad (22)$$

for each $x^1_{i_1} \in X^1$ and $w \in [0, 1]$.

Assuming additionally the independency of the whole collection of X_1 for each $x^1_{i_1} \in X^1$ the distribution function of max and min operations can be rewritten as follows

$$F_{\max(X_1, X_2, \ldots, X_n)}(w) = \prod_{j=1}^{n} F_{X_j}(w) \quad (23)$$

$$F_{\min(X_1, X_2, \ldots, X_n)}(w) = 1 - \prod_{j=1}^{n} (1 - F_{X_j}(w)) \quad (24)$$

The above written formulae are useful for the constructions of a kind decision making algorithm called fuzzy probabilistic controller [4,5].
Let us illustrate the second method by looking for the distribution function of probabilistic operations on two probabilistic sets i.e.

1. for probabilistic sum

$$W = X + Y - X \cdot Y$$

$$U = Y$$

$$J = \begin{vmatrix} 1-y & 1-x \\ 0 & 1 \end{vmatrix} = 1-y \tag{25}$$

$$f_{X+Y-X\cdot Y}\, w = \int_0^w \frac{1}{|1-y|} f_{XY}\left(\frac{w-y}{1-y},\ y\right)dy$$

$$F_{X+Y-X\cdot Y}(z) = \int_0^z f_{X+Y-X\cdot Y}(w)dw$$

2. for probabilistic product

$$W = X \cdot Y$$

$$U = Y$$

$$J = \begin{vmatrix} y & x \\ 0 & 1 \end{vmatrix} = y \tag{26}$$

$$f_{X\cdot Y}(w) = \int_w^1 \frac{1}{|y|} f_{XY}\left(\frac{w}{y},\ y\right)dy$$

$$F_{X\cdot Y}(z) = \int_0^z f_{XY}(w)dw$$

3. THE CONSTRUCTION OF A GENERALIZED FUZZY PROBABILISTIC CONTROL ALGORITHM

Sometimes by a human being controlling the process we can obtain the kind of fuzzy probabilistic information which enables us to form a special type of decision making algorithm called control algorithm. We will assume that in the case of single input - single output the control algorithm is based on the following collection of heuristic control rules

$$\underline{\text{if}}\ X_i\ \underline{\text{then}}\ U_i = (X_i \rightarrow U_i) \qquad (i = 1,2,\dots,N) \tag{27}$$

where X_i and U_i are treated as probabilistic sets of the state and control variables in the spaces $X = \{x_1, \dots, x_n\}$ and $U = \{u_1, \dots, u_m\}$ respectively i.e.

$$X_i : X \times \Omega \to [0,1]$$
$$U_i : U \times \Omega \to [0,1] \quad (28)$$

This reflects the situation when the ambiguity and subjectivity of human operators may not be determined uniquely in $[0,1]$-interval. For the implication we assume the binary operation d_1 on probabilistic sets as follows

$$V_i = V_i(x_j, u_k, \omega) = (X_i \to U_i)(x_i, u_k, \omega) = d_1(X_i(x_j, \omega), U_i(u_k, \omega)) = d_1(X_i U_i) \quad (29)$$

Characterizing this operation by distribution function we have

$$F_{V_i}(v_i) = \int_{G_1} \int_{(v_i)} f_{X_i U_i}(s_i, t_i) ds_i \, dt_i \quad (30)$$

where $f_{X_i U_i}(s_i, t_i)$ is two-dimensional density function of probabilistic sets X_i and U_i and $G_1(v_i)$ denotes the domain being determined by the inequality i.e.

$$G_1(v_i) = \{(s_i, t_i) \mid d_1(s_i, t_i) < v_i\} \quad (31)$$

The whole collectiou of control rules we will describe by N-ary operation on all probabilistic sets $V_i (i=1,2,\dots,N)$ denoting respective relation

$$R = R(x_j, u_k, \omega) = d_2(V_1, \dots, V_N) \quad (32)$$

for which the distribution function has a form

$$F_R(w) = \int \dots \int_{G_2(w)} f_{V_1 \dots V_N}(v_1, \dots, v_N) dv_1 \dots dv_N \quad (33)$$

where

$$G_2(w) = \{(v_1, \dots, v_N) \mid d_2(v_1, \dots, v_N) < w\} \quad (34)$$

Assuming that for each state of the process X'(nonfuzzy, fuzzy or probabilistic set) control variable U' can be computed from the equality

$$U' = d(X', R) \quad (35)$$

Operation d in the case of probabilistic sets should be read as a composition of binary operation d_3 and n-ary operation d_4 i.e.

$$U' u_{k'} = d_4 \left(d_3(X'(x_j,\omega),\ R(x_j,u_k,\omega))\right) \tag{36}$$

For operation d_3 denoted as

$$Z_j' = Z_j'(x_j,u_k,\omega) = d_3(X'(x_j,\omega),\ R(x_j,u_k,\omega)) \tag{37}$$

the distribution function takes a form

$$F_{Z_j'}(z_j') = \int\int_{G_3(z_j')} f_{X'R}(x',r)dx'dr \tag{38}$$

where

$$G_3(z_j') = \left\{(x',r) \mid d_3(x',r) < z_j'\right\}$$

Then for operation d_4 we put

$$U' = U'(u_k,\omega) = d_4(Z_1',\dots,Z_n') \tag{39}$$

and for distribution function $F_{U'}(z)$ we have

$$F_{U'}(z) = \int\dots\int_{G_4(z)} f_{Z_1'\dots Z_n'}(z_1',\dots,z_n')dz_1'\dots dz_n' \tag{40}$$

where

$$G_4(z) = \left\{(z_1',\dots,z_n') \mid d_4(z_1',\dots,z_n') < z\right\} \tag{41}$$

Assuming that the appropriate probability distribution functions for variables X_i, U_i $(i=1,2,\dots,N)$ are given and X_i, U_i are independent, taking for operations d_1, d_3 - min and for d_2, d_4 - max, we can describe R by distribution function

$$F_{R(x_j,u_k)}(w) = \prod_{i=1}^{N} \left(F_{X_i(x_j)}(w) + F_{U_i(u_k)}(w) - F_{X_i(x_j)}(w)F_{U_i(u_k)}(w)\right) \tag{42}$$

For a given distribution function of any input X′ the distribution function of control variable U′ has a form

$$F_{U'(u_k)}(z) = \prod_{i=1}^{N}\left(F_{X'(x_j)}(z) + F_{R(x_j,u_k)}(z) - F_{X'(x_j)}(z)F_{R(x_j,u_k)}(z)\right) \tag{43}$$

for X′,R independent

It is well-known fact that fuzzy sets form a particular class of probabilistic sets. This implies that the concept of fuzzy controller is embedded in the concept of generalized probabilistic controller discussed above. Let be given X' and R as fuzzy input and fuzzy relation by means of the following membership functions

$$R(x_j,u_k) = r_{jk} \in [0,1]$$

$$X'(x_j) = x'_j \in [0,1]$$

$$\text{for } j=1,2,\dots,n \text{ and } k=1,2,\dots,m$$

Expressing the above written membership functions by the respective distribution function we have

$$F_{R(x_j,u_k)}(z) = \begin{cases} 0 & \text{if } z < r_{jk} \\ 1 & \text{otherwise} \end{cases}$$

$$F_{X'(x_j)}(z) = \begin{cases} 0 & \text{if } z < x'_j \\ 1 & \text{otherwise} \end{cases}$$

Determining the distribution function of U' for the above given equalities we have

$$F_{U'(u_k)}(z) = \begin{cases} 0 & \text{if } z < \max(\min(x'_1,r_{1k}),\min(x'_2,r_{2k}),\dots,\min(x'_n,r_{nk})) \\ 1 & \text{otherwise} \end{cases} \tag{44}$$

what implies that U' is a fuzzy set with the membership function

$$U'(u_k) = \max_{1\leqslant j\leqslant n} (\min(x'_j,r_{jk})) =$$

$$= \max(\min(x'_1,r_{1k}),\ \min(x'_2,r_{2k}),\dots,\ \min(x'_n,r_{nk})) \tag{45}$$

for $k = 1,2,\dots,m$ and the equality

$$U' = X' \circ R$$

(where symbol $\circ$ denotes maxmin composition) holds.
So the embedding of fuzzy controller in the concept of fuzzy probabilistic controller is obvious.

4. FINAL CONCLUSIONS

The concept presented in this paper provides an extension of the theory of fuzzy controller proposed by Mamdani in which the notion of probabilistic set in sense of Hirota is used.
The generalisation relies on the possibility of using any operations (not only max and min) on the respective probabilistic sets.
The distribution function of probabilistic set allows to carry out a moment analysis. This forms important feature distinguishing the concept of fuzzy controller from this one of fuzzy probabilistic controller.
However, it should be noted that for the applications of the presented concept there is needed a great amount of information (the distribution functions or density functions of respective probabilistic sets should be known), So it is necessary to look for resonable methods leading to the achievement of these functions.

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ОБОБЩЕНИЕ РАСПЛЫВЧАТОГО-ВЕРОЯТНОСТНОГО РЕГУЛЯТОРА

Р е з ю м е

В работе представлено идею обобщения алгоритма управления называемого расплывчато-вероятностным регулятором. Оговорено различные операции выполняемые на вероятностных множествах а также их описание функциями вероятности. Предложенс конструкцию обобщенного расплывчато-вероятностного алгоритма управления.

UOGÓLNIENIE ROZMYTO-PROBABILISTYCZNEGO REGULATORA

S t r e s z c z e n i e

W pracy przedstawiono ideę uogólnienia algorytmu sterowania zwanego rozmyto-probabilistycznym regulatorem.
W pierwszej części niniejszej pracy przedyskutowano różne operacje na zbiorach probabilistycznych oraz ich dystrybuantowe opisy.
Następnie przedstawiono konstrukcję uogólnionego rozmyto-probabilistycznego algorytmu sterowania.