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Analysis of probability as an impulse to synthesis of a beam-system represented by hypergraphs

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Analysis and modelling

ABSTRACT

Purpose: of this paper is to analyze vibrating beam by an exact and approximate methods and to create hypergraphs of the beam in case of two methods of analysis.

Design/methodology/approach: was to nominate the relevance or irrelevance between the characteristics obtained by the considered methods - especially concerning the relevance of the natural frequency-poles of beam characteristics. The main subject of the research is a continuous free beam as a subsystem of vibrating beam-system with constant cross sections.

Findings: this approach is that approximate solutions fulfil all conditions for vibrating beams and can be an introduction to synthesis of these systems modelled by hypergraphs.

Research limitations/implications: linear continuous flexibly vibrating free beam is considered

Practical implications: of this study is mainly the introduction to synthesis of flexibly vibrating continuous beam-systems.

Originality/value: of this approach is about application of approximate methods of analysis of a beam and modelling the one of transformed hypergraph.

Keywords: Applied mechanics, Exact and approximate methods, Continuous system, Vibrating beam

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1. Introduction

The problems of analysis of vibrating beam systems, discrete and discrete-continuous mechanical systems by means of the structural numerous methods modelled by the graphs, hypergraphs[1] have been investigated in Gliwice Research Centre (e.g. [4, 5, 8, 17]). The continuous-discrete torsionally [8] and flexibly [9, 10] vibrating mechatronic systems were considered [13]. The approximate method of analysis called a Galerkin method has been used to obtain the frequency-modal characteristics. To compare the obtained dynamical characteristics – dynamical flexibilities only for mechanical torsionally vibrating bar and flexibly vibrating beam, as a part of complex mechatronic

[1] Other diverse problems have been modeled by different kind of graphs, then they were examined and analyzed in (e.g. [14-19]). The problems of synthesis of electrical systems [1] and of selected class of continuous, discrete - continuous discrete mechanical systems and active mechanical systems concerning the frequency spectrum are investigated [3-12].

systems, exact method and the Galerkin method were used [8-10]. In this paper frequency – modal analysis has been presented for the mechanical system that means the flexibly vibrating free beam.

2. Vibration free beam as the subsystem of beam-system

2.1. Frequency - modal analysis of free beam

The beam[2] of the constant cross section, free on left end and on the right one with harmonic force excitation in a form $P(t) = P_0 \sin \omega t$ is considered.

The equation of motion of the beam goes as follows:

$$EIy(x,t)_{,xxxx} + \rho F y(x,t)_{,tt} = 0\,, \tag{1}$$

where: $y(x,t)$ - deflection at the moment t of the lining beam section within the distance x from the beginning of the system, E - Young modulus, ρ - mass density of material of the beam, I - polar inertia moment of the beam cross section, F – area of the beam cross section.

The boundary conditions on the beam ends are the following:

$$y_{,xx}(0,t) = 0,\; y_{,xxx}(0,t) = 0,\; y_{,xx}(l,t) = 0,\; EIy_{,xxx}(l,t) = 0, \tag{2}$$

where: l- length of the beam.

Internal question (close to homogeneous boundary conditions) for the beam is then the following:

$$X^{(\mathrm{IV})}(x) - k^4 X(x) = 0, \tag{3}$$

$$X''(0,t) = 0,\; X'''(0,t) = 0,\; X''(l,t) = 0,\; X'''(l,t) = 0. \tag{4}$$

The general solution of internal functions has the following form

$$X(x) = A \sin kx + B \cos kx + C \sinh kx + D \cosh kx. \tag{5}$$

After substitution of (5) into boundary conditions (4) but for homogenous ones the following was received:

$$\begin{cases} -B + D = 0, \\ -A + C = 0, \\ -A\cos kl + B \sin kl + C\cosh kl + D \sinh kl = 0, \\ -A \sin kl - B\cos kl + C \sinh kl + D\cosh kl = 0. \end{cases} \tag{6}$$

[2] The mechatronic system was considered for example in [8, 9].

Characteristic determinant of set (6) equals zero for the following equation for internal values

$$\cos z = \frac{1}{\cosh z}, \quad z = kl. \tag{7}$$

The solution of equation (7) the internal values go as follows:

$$z_n \approx \frac{2n+1}{2}\,. \tag{8}$$

Free beam has one frequency equalling zero that means

$$k = 0 \tag{9}$$

and the equal (1) goes as follows:

$$X^{(iv)}(x) = 0\,. \tag{10}$$

Solution of an equation (10) is the following:

$$X_0(x) = Ax^3 + Bx^2 + Cx + D \tag{11}$$

and considering (6)

$$X_0(x) = Cx + D\,. \tag{12}$$

The function (12) presents movement of beam as of rigid body.

Internal functions after being related between constants *A, B, C, D* are the following:

$$B = A\frac{\cos z_n - \cosh z_n}{\sin z_n + \sinh z_n}\,, C = A\,,\; D = A\frac{\cos z_n - \cosh z_n}{\sin z_n + \sinh z_n} \tag{13}$$

and therefore, internal functions have the following form:

$$X_n = A\left(\sin\frac{z_n}{l}x + \frac{\cos z_n - \cosh z_n}{\sin z_n + \sinh z_n}\cos\frac{z_n}{l}x + \right.$$
$$\left. + \sinh\frac{z_n}{l}x + \frac{\cos z_n - \cosh z_n}{\sin z_n + \sinh z_n}\cosh\frac{z_n}{l}\right),$$
$$n = 1, 2, 3, \ldots\,. \tag{14}$$

2.2. Determining the dynamical flexibility - the exact method

Solution $y(x,t)$ of an equation (1) is a harmonic function, that is

$$y(x,t) = X(x) \sin \omega t\,. \tag{15}$$

Determining suitable derivatives of (15) and substituting them into (2) the set of equations, after transformations, was obtained

$$\begin{cases} A(\cosh kl - \cos kl) + B(\sin kl + \sinh kl) = \dfrac{-P_0}{EIk^3}, \\ A(\sinh kl - \sin kl) + B(\cosh kl + \cos kl) = 0. \end{cases} \tag{16}$$

and in matrix it is formed as:

$$\mathbf{WA} = \mathbf{F}, \tag{17}$$

where:

$$\mathbf{W} = \begin{vmatrix} (\cosh kl - \cos kl), & (\sin kl + \sinh kl) \\ (\sinh kl - \sin kl), & (\cosh kl + \cos kl) \end{vmatrix}, \ \mathbf{A} = \begin{vmatrix} A \\ B \end{vmatrix}, \ \mathbf{F} = \begin{vmatrix} \dfrac{-P_0}{EIk^3} \\ 0 \end{vmatrix}.$$

The main determinant of a set of equations (17) is equal

$$|\mathbf{W}| = 2(1 - \cos kl \cosh kl) . \tag{18}$$

To qualify constants *A, B,* should count the following determinants

$$|\mathbf{W}_A| = \begin{vmatrix} \dfrac{-P_0}{EIk^3} & (\sin kl + \sinh kl) \\ 0 & \cosh kl - \cos kl \end{vmatrix} = -\frac{P_0}{EIk^3}(\cosh kl - \cos kl) , \tag{19}$$

$$|\mathbf{W}_B| = \begin{vmatrix} (\cosh kl - \cos kl) & \dfrac{-P_0}{EIk^3} \\ (\sinh kl - \sin kl) & 0 \end{vmatrix} = \frac{P_0}{EIk^3}(\sinh kl - \sin kl) . \tag{20}$$

The constants *A,..., D* on the base (16-20) are equal

$$A = C = \frac{|\mathbf{W}_A|}{|\mathbf{W}|} = -\frac{P_0(\cosh kl - \cos kl)}{2EIk^3(1 + \cos kl \cosh kl)} , \tag{21}$$

$$B = D = \frac{|\mathbf{W}_B|}{|\mathbf{W}|} = \frac{P_0(\cos kl + \cosh kl)}{2EIk^3(1 + \cos kl \cosh kl)} . \tag{22}$$

Substituting expression (21) and (22) to (11) and taking (10) deflection beam into account is the following:

$$y(x,t) = -P_0 \sin \omega t \left[\frac{(\cosh kl - \cos kl)(\sin x + \sinh kx)}{2EIk^3(1 - \cos kl \cosh kl)} + \right. \\ \left. - \frac{(\sinh kl - \sin kl)(\cos kx + \cosh kx)}{2EIk^3(1 - \cos kl \cosh kl)} \right] . \tag{23}$$

According to definition of dynamic flexibility, on the basis of (23), it goes as follows:

$$Y = -\frac{(\cosh kl - \cos kl)(\sin x + \sinh kx) - (\sinh kl - \sin kl)(\cos kx + \cosh kx)}{2EIk^3(1 - \cos kl \cosh kl)}. \tag{24}$$

The transient of absolute value of dynamical flexibility (24) for $x=l$, that is $\alpha_Y = |Y|$ is drawn in Fig. 1.

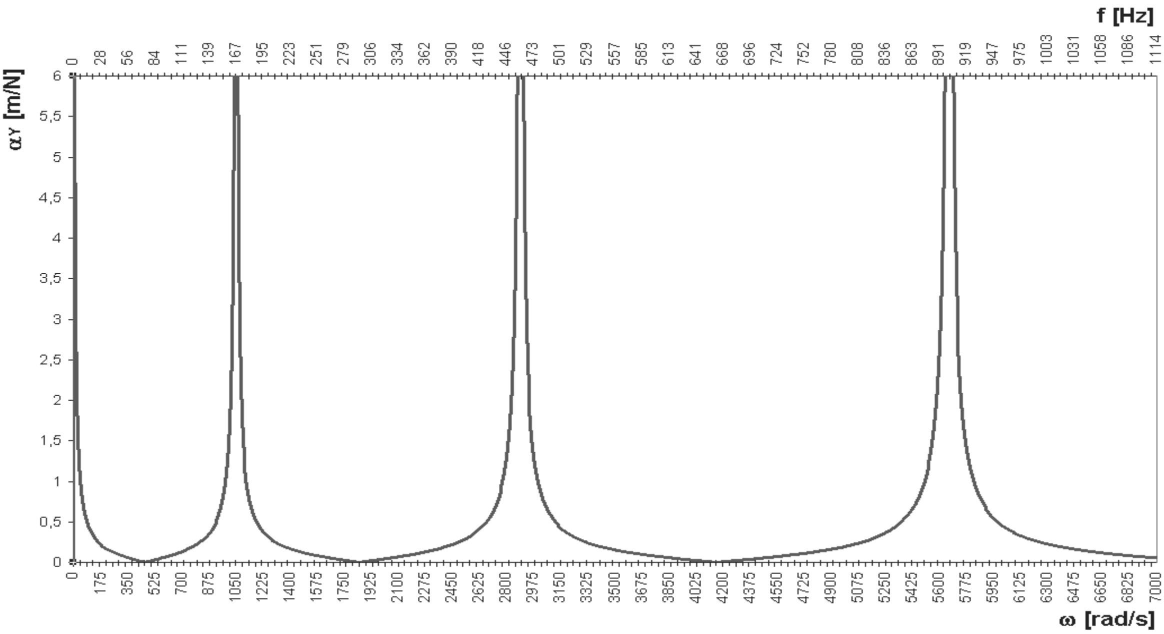

Fig. 1. The plot of absolute value of dynamical flexibility of flexibly vibrating free beam

2.3. Calculation of the dynamical flexibility of the beam - Galerkin method

The equation of excitated vibrations of beam can be described as

$$EIy(x,t)_{,xxxx} + \rho F y(x,t)_{,tt} = P_0 \sin \omega t \, . \tag{25}$$

The solution of beam (25) by means of the Galerkin method is given in a shape of:

$$y(x,t) = \sum_{n=1}^{\infty} y_n(x,t) = \sum_{n=1}^{\infty} A^{(n)} \sin kx \sin \omega t \, , \tag{26}$$

where: $k = (2n+1)\dfrac{\pi}{2l}$.

After substituting the following derivatives of (26) to (25) the following is obtained:

$$\begin{aligned} &-EIA^{(n)} k^4 \sin kx \sin \omega t + \\ &+\rho F \omega^2 A^{(n)} \sin kx \sin \omega t = P_0 \sin \omega t \end{aligned} \tag{27}$$

The amplitude value $A^{(n)}$ after transformations of the vibrations goes as follows:

$$A^{(n)} = \frac{P_0}{\rho F \omega^2 - EIk^4} \, . \tag{28}$$

Using the equation (28) and putting it to (26) the dynamical flexibility, it equals:

$$Y_{xl}^{(n)} = \frac{\sin kx}{\rho A \omega^2 - EIk^4} \, . \tag{29}$$

When $x=l$, then (29) is given as

$$Y_{ll}^{(n)} = \frac{1}{\omega^2 - a^2 k^4} \, , \tag{30}$$

where: $a = \sqrt{\dfrac{EI}{\rho F}}$.

The absolute value of dynamical flexibility at the end of the beam, i.e. when $x=l$ takes the following form:

$$\alpha_Y^{(l)} = \left| Y_{ll}^{(n)} \right| = \left| \frac{1}{\omega^2 - a^2 k^4} \right| . \tag{31}$$

In a global case of the dynamical flexibility at the end of the beam, it gets a shape of:

$$Y_{xl} = \sum_{n=1}^{\infty} Y_{xl}^{(n)} = \sum_{n=1}^{\infty} \frac{\sin\left[(2n+1)\dfrac{\pi}{2l}x\right]}{\omega^2 - \dfrac{EI}{\rho F}\left[(2n+1)\dfrac{\pi}{2l}\right]^4} \, . \tag{32}$$

For sum k=0, 1, 2, 3 the plot of absolute value of dynamical flexibility defined by expression (32) is shown in Fig.2.

3. Hypergraphs as models of vibration free beam

To fix the meaning of necessary terms and symbols, the review of essential concepts of graph theory was presented before modelling the flexibly vibrating continuous beam systems and problems connected with it.

Weighted hypergraphs (in this paper called also weighted block graphs or weighted graphs of category *k*) have been applied for modelling the considered mechanical systems. Definitions of graphs, as mathematical objects, have been presented on the basis of the literature [2]. The bibliography of this subject is very extensive and regards the theory as well as its applications (see [1, 4-11, 13-19]).

The couple

$$^{k}X = \left({}_1X, \, {}_2^kX\right). \tag{33}$$

is called a *hypergraph,* where: ${}_1X$ is the set as in (33), and ${}_2^kX = \left({}_2^kX^{(i)} / i \in \mathrm{N}\right)$, ($k$=2,3, ... $\in$N) is a family of subsets of set ${}_1X$; the family ${}_2^kX$ is called a *hypergraph* over ${}_1X$ as well, and ${}_2^kX = \left\{{}_2^kX^{(1)}, \, {}_2^kX^{(2)}, \, \ldots, {}_2^kX^{(m)}\right\}$ is a set of edges [2], called *hyperedges* or *blocks,* if

$$\begin{cases} {}_2^kX \neq \varnothing \; (i \in \mathrm{I}), \\ \bigcup_{i \in \mathrm{I}} {}_2^kX^{(i)} = {}_2X. \end{cases} \tag{34}$$

Hypergraphs ^kX have been shown in their geometrical representation on plane. Sets of edges ${}_2X$ have been marked by lines, subsets of family ${}_2^kX$ (hyperedges or blocks) - two-dimensional continuum with enhanced vertices, in the shape of circles. In this paper hypergraphs - graphs of category k - ^kX (k=2, 3) are used (see [2, 4, 5, 17]).

In the case of flexibly vibrating beam (i) of constant cross-section and constant flexible rigidity $(EI)^{(i)}$ (where $E^{(i)}$ - Young's modulus of the beam, $I^{(i)}$ - polar moment of inertia of cross-section of the beam) as well as length $l^{(i)}$ is considered. The model in the form of a determined and continuous system is introduced.

The model in this way takes the root also and - this beam as well as every studied beam in farther draught in a figure of limited arrangement. In this model, generalized displacements – deflections ${}_1s_1^{(i)}$ and ${}_1s_2^{(i)}$ correspond to its extreme points.

Moreover, the extreme points of the beam were subordinated, generalized displacements also - the slopes of the beam - ${}_1s_3^{(i)}$ and ${}_1s_4^{(i)}$. These general displacements are measured in the inertial system of reference. Furthermore, the origin of the inertial system of reference has generalized a coordinate ${}_1s_0^{(i)} = 0$ assigned to it. So, a set of the generalized displacements of a flexibly vibrating beam can be formulated as follows: ${}_1S^{(i)} = \left\{ {}_1s_0^{(i)}, {}_1s_1^{(i)}, {}_1s_2^{(i)}, {}_1s_3^{(i)}, {}_1s_4^{(i)} \right\}$, while its dynamical flexibilities set can be denoted as $Y^{(i)} = \left\{ Y_{ij}^{(i)} \right\} \left(Y_{ij}^{(i)} = Y_{ji}^{(i)}, i, j = 1,...,4 \right)$.

Making mutually, one-to-one transformation imitation

$$f\colon {}_1S^{(i)} \rightarrow {}_1X^{(i)}. \tag{35}$$

in this way that

$$f\left({}_1s_j^{(i)}\right) = {}_1x_j^{(i)},. \tag{36}$$

where: ${}_1s_j^{(i)} \in {}_1S^{(i)}$, ${}_1x_j^{(i)} \in {}_1X^{(i)}$, ${}_1s_j^{(i)} \in {}_1S^{(i)}$, ${}_1x_j^{(i)} \in {}_1X^{(i)}$, $j = 0,1,2,3,4$

the *five-vertex hypergraph as a model of flexibly vibrating beam* of constant cross-section is obtained

$$\underset{f}{{}^2X^{(i)}} = \left[{}^2X^{(i)}, f \right]. \tag{37}$$

where: ${}_2^kX^{(i)}$ - one-element family - five-element subset of vertices ${}_1X^{(i)}$

Graphical representation of transformations (35) by the way of (36) in case of the flexibly vibrating beam of constant cross-section is shown in Fig. 3.

The couple

$$\underset{1}{{}^2X^{(i)}} = \left[\underset{f}{{}^2X^{(i)}}, f_1 \right]. \tag{38}$$

is called *weighted hypergraph,* where: f_1 is function which is assigned to vertices ${}_1x_j^{(i)}$ of hypergraph $\underset{f}{{}^2X^{(i)}}$ the generalized displacements, that are deflections: ${}_1s_1^{(i)}$ and ${}_1s_2^{(i)}$ the slopes of the beam - ${}_1s_3^{(i)}$ and ${}_1s_4^{(i)}$ as:

$$f_1\left({}_1x_j^{(i)}\right) = {}_1s_j^{(i)},\ j = 0,1,\ldots,4.. \tag{39}$$

The graph $\underset{1}{{}^2X^{(i)}}$ as graphical representation of sentence (38) is shown in Fig. 4.

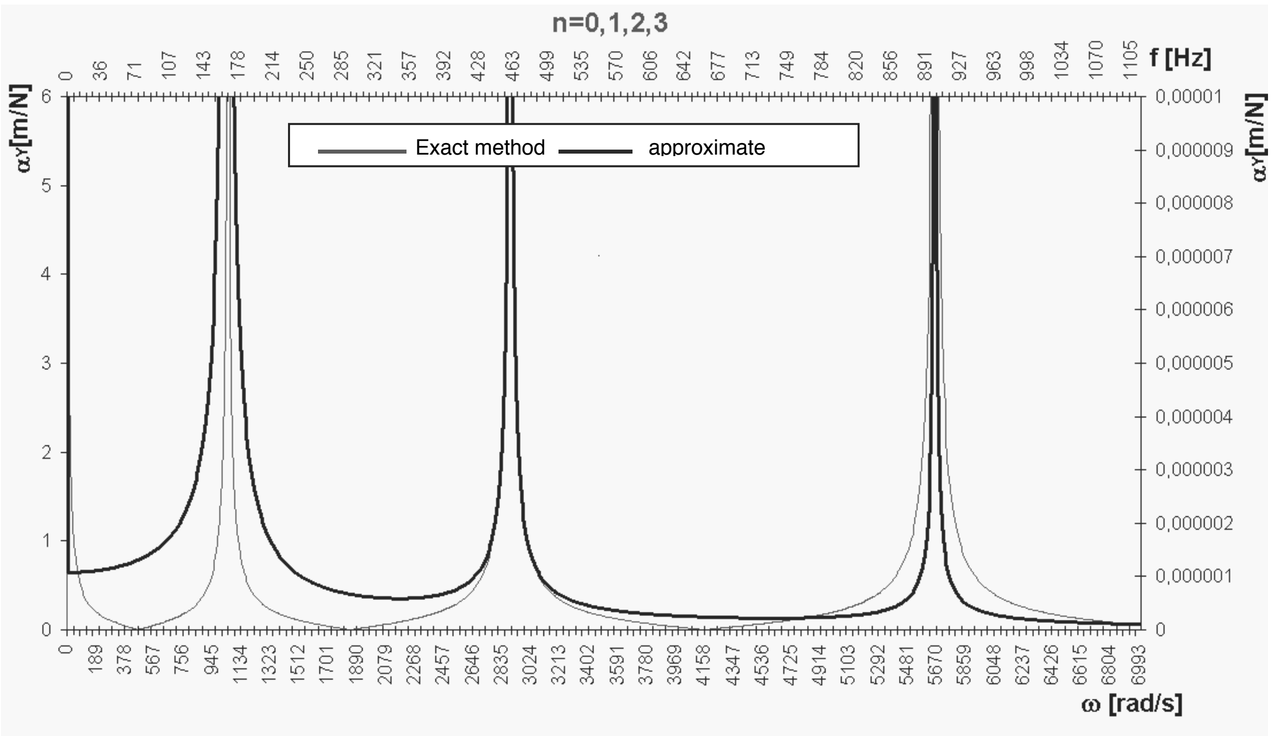

Fig. 2. The plot of absolute value for dynamical flexibility of the sum for n=1, 2, 3 mode vibration

On the base of, for example, Galerkin transformation the five-vertex hypergraph (Fig. 5a) into three-vertex block graph (Fig. 5b, c) will be applied.
Complete weighted graph – substitute graph

$$f_1\left({}_1x_j^{(i)}\right)={}_1s_j^{(i)},\ j=0,1,\ \ldots\ ,4.. \tag{40}$$

is received:

- after transformation [according to (7)] which is assigned to vertices of a complete graph ${}^{2}\underset{f}{X}{}_Z^{(i)}$ of the hypergraph ${}^{3}\underset{f}{X}{}^{(i)}$ (that is the hypergraph after Galerkin transformation) of values of generalized co-ordinates and
- after transformation f_2 which is assigned to edges of the complete graph of dynamical flexibilities that were appointed in following way:

$$f_2\left(\left\{{}_1x_0^{(i)},\ {}_1x_0^{(i)}\right\},\left\{{}_1x_0^{(i)},\ {}_1x_2^{(i)}\right\},\left\{{}_1x_1^{(i)},\ {}_1x_2^{(i)}\right\}\right)=$$
$$=\left[\left\{Y_{11}^{(i)}\right\},\left\{Y_{22}^{(i)}\right\},\left\{Y_{12}^{(i)}\right\}\right], \tag{41}$$

where: $Y_{11}^{(i)}, Y_{22}^{(i)}, Y_{12}^{(i)}$ are dynamical flexibilities obtained by the Galerkin method.

Weighted Lagrang skeleton of hypergraph ${}^{3}\underset{f}{X}{}^{(i)}$

$$=\left[\left\{Y_{11}^{(i)}\right\},\left\{Y_{22}^{(i)}\right\},\left\{Y_{12}^{(i)}\right\}\right], \tag{42}$$

is *weighted subgraph* of weighted complete graph – substitute one ${}^{2}\underset{12}{X}{}_Z^{(i)}$.

Graphical representation of these subgraphs is shown in [4-6, 17].

In the case of synthesis of n-segment model of the system, composed of subsystems of constant section, vibrating flexibly, it is modelled by the loaded graph of the third category – after Galerkin transformation - with n three-vertices-blocks, connected to those vertices to which the corresponding generalized coordinates are assigned to (see i.e. [4,6]).

The use of a weighted hypergraph and its weighted subgraphs (as a model of flexibly vibrating system) in this way may provide the basis for the formalization which is the necessary condition of discretization of the considered class of continuous mechanical systems.

4. Last remark

On the base of the obtained formulas it is possible to make the analysis of the considered class vibrating mechanical and mechatronic systems as introduction to a synthesis. In case of other boundary conditions of the beams, it is necessary to achieve the offered researches in this paper. In the future research, these problems shall be discussed.

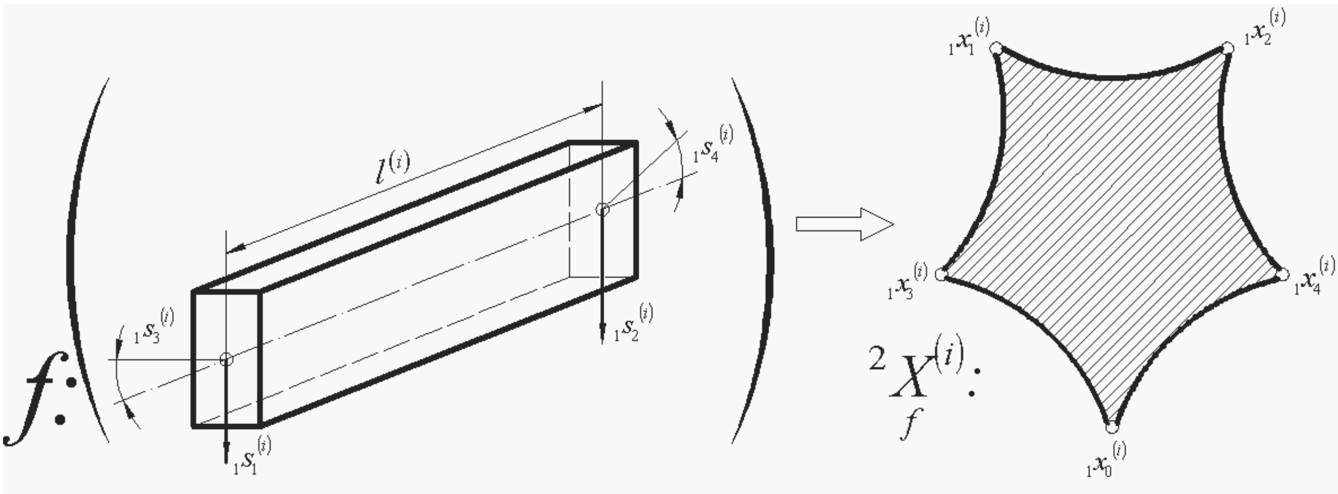

Fig. 3. Hypergraph of model of flexibly vibrating free beam of constant cross-section as graphical representation of transformations (36) and (37)

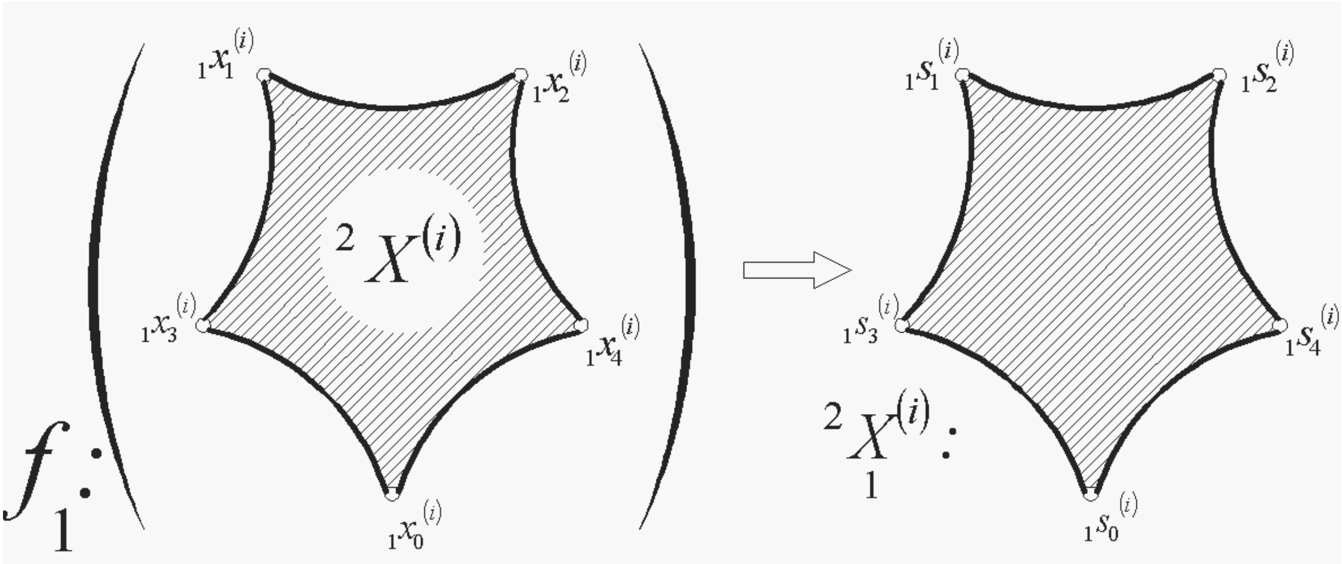

Fig. 4. Hypergraph of flexibly vibrating beam as representation of transformation (38)

a) b) c)

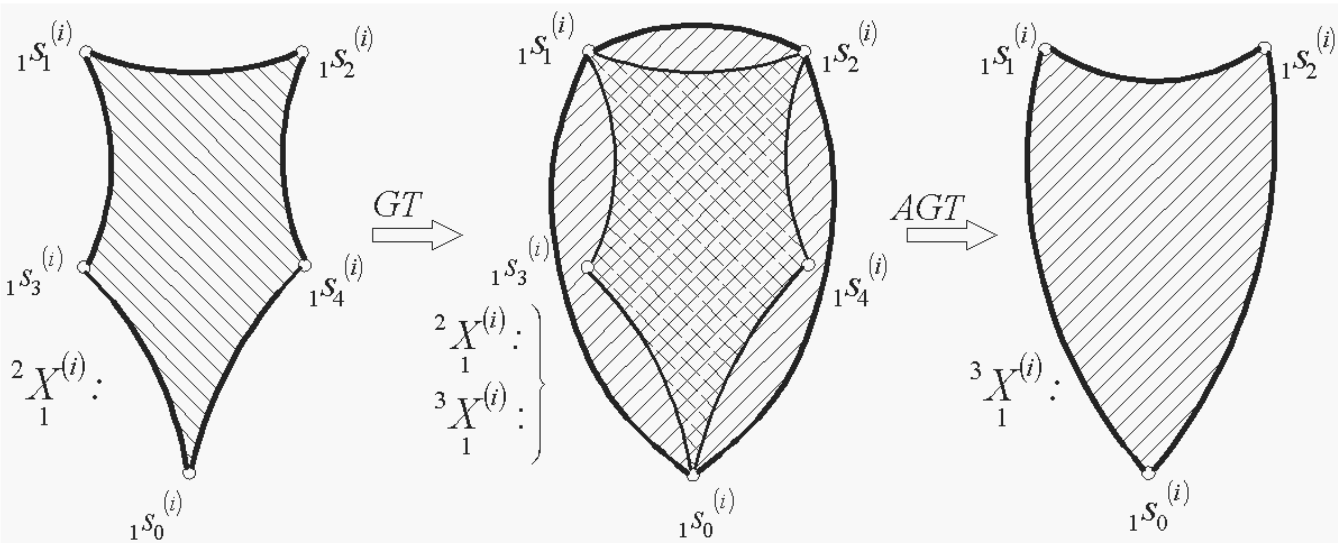

Fig. 5. The illustration of transformation of the five-vertex hypergraph into three-vertex one as an effect of use of Galerkin method

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