Journal of Achievements in Materials and Manufacturing Engineering
VOLUME 43 ISSUE 2 December 2010

Graphs different category of subsystem as models to synthesis of transverse vibrating beam - system

A. Buchacz*
Institute of Engineering Processes Automation and Integrated Manufacturing Systems, Silesian University of Technology,
ul. Konarskiego 18a, 44-100 Gliwice, Poland
* Corresponding author: E-mail address: andrzej.buchacz@polsl.pl

Received 20.10.2010; published in revised form 01.12.2010

Analysis and modelling

ABSTRACT

Purpose: of this paper is modeling by different categorygraphs and analysis of vibrating clamped – free beam as subsystem of transverse vibrating beam-system by the exact and approximate methods and creating the hypergraphs of the beams in case of presented methods of analysis.
Design/methodology/approach: was to nominate the relevance or irrelevance between the characteristics obtained by considered methods - especially concerning the relevance of the natural frequencies-poles of characteristics of considered beam. The main subject of the research is the continuous clamped – free beam with constant cross sections as a subsystem of transverse vibrating beam - system.
Findings: this approach is fact, that approximate solutions fulfill all conditions for vibrating beams and can be introduction to synthesis of these systems modeled by different category graphs.
Research limitations/implications: is that linear continuous transverse vibrating clamped-free beam is considered.
Practical implications: of this study is the main point is the introduction to synthesis of transverse vibrating continuous beam-systems with constant changeable cross-section.
Originality/value: of this approach relies on application approximate methods of analysis of clamped – free beam and modeling the one of transformed hypergraph.
Keywords: Applied mechanics; Exact and Galerkin's methods; Graphs; Vibrating clamped – free beam

Reference to this paper should be given in the following way:
A. Buchacz, Graphs different category of subsystem as models to synthesis of transverse vibrating beam - system, Journal of Achievements in Materials and Manufacturing Engineering 43/2 (2010) 644-650.

1. Introduction

In the Gliwice research Centre the problems of analysis of vibrating beam systems, discrete and discrete-continuous mechanical systems by means the structural numbers methods modelled by the graphs, hypergraphs has been made (e.g.[4, 17]).

The problems of synthesis of electrical systems [1] and of selected class of continuous, discrete - continuous discrete mechanical systems and active mechanical systems concerning the frequency spectrum has been made [4-8]. The continuous-discrete torsionally and flexibly vibrating mechatronic systems were considered in [9-12,15,16]. The approximate method of analysis called Galerkin's method has been used to obtain the frequency-modal characteristics. To comparison of obtaining

© Copyright by International OCSCO World Press. All rights reserved. 2010

dynamical characteristics – dynamical flexibilities only for mechanical torsionally vibrating bar and flexibly vibrating beam, as a parts of complex mechatronic systems, exact method and Galerkin's method were used [9-12, 15]. In this paper frequency – modal analysis have been presented for the mechanical system, that means vibrating clamped-free beam. The model of the free-free beam were presented in the five-vertex hypergraph, which in case of approximate frequency-modal analysis we can imitate in three-vertex ones [13]. Such formulation maybe the introduction to synthesis of flexibly vibration complex beam systems with changeable constant cross-section.

2. Graphs different category as models of vibrating clamped - free beam

The couple

$$ {}^{k}X=\left({}_{1}X,\ {}^{k}_{2}X\right) \tag{1}$$

is called a *hypergraph,* where: ${}_{1}X$ is the set as in (1), and ${}^{k}_{2}X=\left({}^{k}_{2}X^{(i)}/i\in \mathrm{N}\right)$, ($k$=2,3, ... ∈N) is a family of subsets of set ${}_{1}X$; the family ${}^{k}_{2}X$ is called a *hypergraph* over ${}_{1}X$ as well, and ${}^{k}_{2}X=\left\{{}^{k}_{2}X^{(1)},\ {}^{k}_{2}X^{(2)},\ \dots,{}^{k}_{2}X^{(m)}\right\}$ is a set of edges [2], called *hyperedges* or *blocks,* if

$$\begin{cases} {}^{k}_{2}X\neq\varnothing\ (i\in \mathrm{I}),\\ \bigcup\limits_{i\in \mathrm{I}}{}^{k}_{2}X^{(i)}={}_{2}X. \end{cases} \tag{2}$$

If a subset from the family of subsets of vertices with $n_z \leq n$, is distinguished from hypergraph ${}^{k}X$ with n vertices, then the *complete graph* of hypergraph ${}^{k}X$ is the graph X_Z. In this graph each pair of vertices is incident, and graph X_Z has $m=\binom{n}{2}$ edges.

Skeleton ${}^{k}X_0$ of hypergraph ${}^{k}X$ is a graph obtained as the result of substitution of each subset of vertices by tree $\underset{0}{X}$, composed of one-dimensional edges and stretched on all vertices of hypergraph ${}^{k}X$. The tree $\underset{0}{X}$ of graph X with n vertices and m edges is a connected subgraph with the same number of vertices and with $\underset{0}{m}=n$-1 edges, in which there are no circuits and loops. So every skeleton of subsets of vertices is a tree of substitute-complete graph.

A tree in which every vertex ${}_{1}x_i$ $(i=1,\dots,n)$ is incident with vertex ${}_{1}x_0$ by edge ${}_{2}x_k=\left({}_{1}x_i,{}_{1}x_0\right)$, ($k$ = 1, ... , $\underset{0}{m}$), (see e.g. [4, 5]) is called the *Lagrange skeleton*.

Graphs X and hypergraphs ${}^{k}X$ have been shown in their geometrical representation on plane. Sets of edges ${}_{2}X$ have been marked by lines, subsets of family ${}^{k}_{2}X$ (hyperedges or blocks) - two-dimensional continuum with enhanced vertices, in the shape of circles.

In this paper hypergraphs - graphs of category k - ${}^{k}X$ (k=2,3) are used, which will be clearly mentioned each time, as well as graphs X, called also graphs of the first category - ${}^{1}X$ (see [4, 5]).

The basic notions which have been written in italics are shown in Fig. 1.

In the case of flexibly vibrating of the free-free beam (i) with constant cross-section and constant flexibly rigidity $(EJ_z)^{(i)}$ (where $E^{(i)}$ - Young's modulus of the beam, $J_z^{(i)}$ - polar moment of inertia of cross-section of the beam) as well as length $l^{(i)}$ has been considered. The model in the form of a determined and continuous system is introduced.

The model in this way takes the root also and - this beam as well as every studied in more far draught of beam in figure of arrangement limited. In this model, generalized displacements – deflections ${}_{1}s_1^{(i)}$ and ${}_{1}s_2^{(i)}$ correspond to its extreme points. Moreover the extreme points of beam were subordinated generalized displacements also - the slopes of the beam - ${}_{1}s_3^{(i)}$ and ${}_{1}s_4^{(i)}$. These general displacements are measured in the inertial system of reference. Moreover, the origin of the inertial system of reference has generalized coordinate ${}_{1}s_0^{(i)}=0$ assigned to it. So a set of the generalized displacements of a flexibly vibrating beam can be formulated as follows: ${}_{1}S^{(i)}=\left\{{}_{1}s_0^{(i)},\ {}_{1}s_1^{(i)},\ {}_{1}s_2^{(i)},{}_{1}s_3^{(i)},\ {}_{1}s_4^{(i)}\right\}$.

Making mutually one-to-one transformation imitation

$$ f\colon {}_{1}S^{(i)} \rightarrow {}_{1}X^{(i)} \tag{3}$$

in this way, that

$$ f\left({}_{1}s_j^{(i)}\right)=\ {}_{1}x_j^{(i)}, \tag{4}$$

where: ${}_{1}s_j^{(i)}\in{}_{1}S^{(i)},\ {}_{1}x_j^{(i)}\in{}_{1}X^{(i)},\ \ {}_{1}s_j^{(i)}\in\ {}_{1}S^{(i)},\ {}_{1}x_j^{(i)}\in\ {}_{1}X^{(i)}$, j=0,…,4,

the *five-vertex hypergraph as a model of flexibly vibrating beam* with constant cross-section is obtained

$$ {}^{2}_{f}X^{(i)}=\left[{}^{2}X^{(i)},f\right] \tag{5}$$

where: ${}^{k}_{2}X^{(i)}$ - one-element family - five-element subset of vertices ${}_{1}X^{(i)}$

In the case of transverse vibrating beam, clamped on the left end and free on the right one, the set of the generalized displacements of the beam can be described in form: ${}_{1}S^{(i)}=\left\{{}_{1}s_0^{(i)},\ {}_{1}s_1^{(i)}=0,\ {}_{1}s_2^{(i)},\ {}_{1}s_3^{(i)}=0,\ {}_{1}s_4^{(i)}\right\}$. After transformations of (3) and (4) in case of the flexibly vibrating clamped –free beam with constant cross-section graphical representation is shown in Fig. 2.

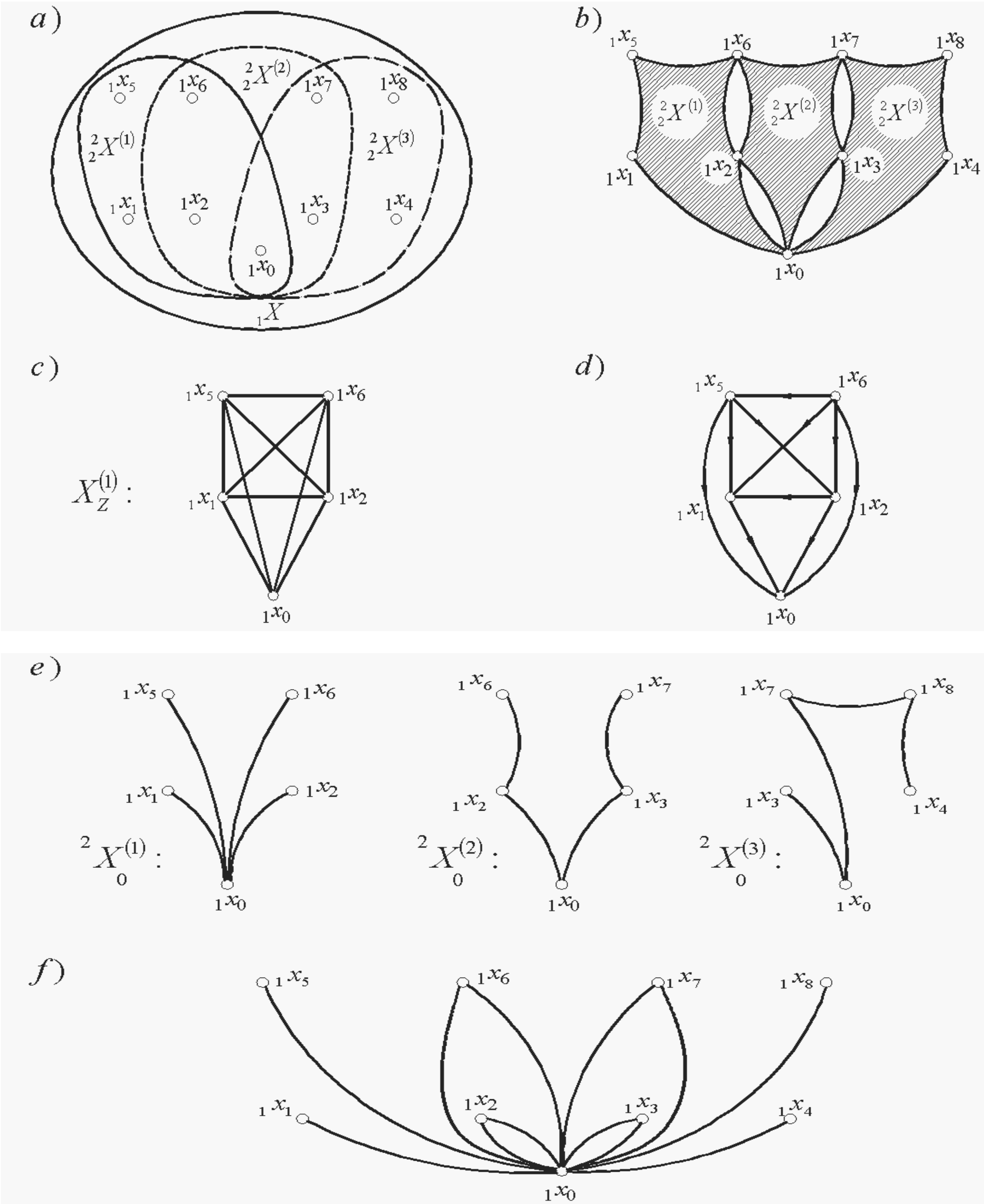

Fig. 1. Basic notions concerning the class of graphs, which are used throughout this paper: a) set of vertices of a hypergraph, b) graphical representation of a three-block graph, c) complete graphs of hypergraph blocks, d) complete oriented graphs of three-vertex blocks and of a two-vertex block, e) optionally selected tree-skeletons of hypergraph blocks, f) skeleton of hypergraph, g) Lagrange skeleton of hypergraph

The couple

$$ {}^{2}_{1}X^{(i)} = \left[{}^{2}_{f}X^{(i)}, f_1 \right] \tag{6}$$

is called *weighted hypergraph,* where: f_1 is function which assigns to vertices ${}_1x_j^{(i)}$ of hypergraph ${}^{2}_{f}X^{(i)}$ the generalized displacements, that means deflections: ${}_1s_1^{(i)}$ and ${}_1s_2^{(i)}$ the slopes of the beam - ${}_1s_3^{(i)}$ and ${}_1s_4^{(i)}$ as:

$$ f_1\left({}_1x_j^{(i)}\right) = {}_1s_j^{(i)},\; j = 0,1,\ldots,4. \tag{7}$$

The graph ${}^{2}_{1}X^{(i)}$ as graphical representation of sentence (6) is shown in Fig. 3.

This attempt is an introduction to transformations of the five-vertex into three – vertex ones after analysis of the beams by means the approximate methods of analysis.

3. The exact and Galerkin's method of calculation of the dynamical flexibility of the clamped — free beam

In the case of vibration clamped - free beam as the subsystem of the beam-system its boundary conditions on the beam ends are following

$$ \begin{cases} y(0,t) = 0, \; y(0,t)_{,x} = 0, \\ y(l,t)_{,xx} = 0, \; EJ_Z\, y(l,t)_{,xxx} = -P(t), \end{cases} \tag{8}$$

where: l- length of the beam.

Solution $y(x,t)$ of equation of motion of transverse vibrating beam is the harmonic function in form

$$ y(x,t) = X(x)\sin\omega t \,. \tag{9}$$

Determining suitable derivatives of (9) and substituting them into the equation of motion of the beam, the set of equations, after transformations, was obtained in matrix form

$$ \mathbf{WA} = \mathbf{F}\,. \tag{10}$$

In (10) the matrixes are now following

$$ \begin{cases} \mathbf{W} = \begin{vmatrix} (\cosh kl - \cos kl), & (\sin kl + \sinh kl) \\ (\sinh kl - \sin kl), & (\cosh kl + \cos kl) \end{vmatrix}, \\ \mathbf{A} = \begin{vmatrix} A \\ B \end{vmatrix}, \quad \mathbf{F} = \begin{vmatrix} \dfrac{-P_0}{EJ_Z\, k^3} \\ 0 \end{vmatrix} \end{cases} \tag{11}$$

Main $|\mathbf{W}|$ determinant equals

$$ |\mathbf{W}| = 2(1 - \cos kl \cosh kl)\,. \tag{12}$$

To qualify constants *A, B,* should count following determinants

$$ |\mathbf{W}_A| = \begin{vmatrix} \dfrac{-P_0}{EJ_Z\, k^3} & (\sin kl + \sinh kl) \\ 0 & \cosh kl - \cos kl \end{vmatrix} = $$

$$ = -\frac{P_0}{EJ_Z\, k^3}(\cosh kl - \cos kl)\,. \tag{13}$$

$$ |\mathbf{W}_B| = \begin{vmatrix} (\cosh kl - \cos kl) & \dfrac{-P_0}{EJ_Z\, k^3} \\ (\sinh kl - \sin kl) & 0 \end{vmatrix} = $$

$$ = \frac{P_0}{EJ_Z\, k^3}(\sinh kl - \sin kl). \tag{14}$$

$$ A = -C = \frac{|\mathbf{W}_A|}{|\mathbf{W}|} = -\frac{P_0(\cosh kl - \cos kl)}{2EJ_Z\, k^3(1 + \cos kl \cosh kl)}\,. \tag{15}$$

$$ B = -D = \frac{|\mathbf{W}_B|}{|\mathbf{W}|} = \frac{P_0(\cos kl + \cosh kl)}{2EIk^3(1 + \cos kl \cosh kl)}\,. \tag{16}$$

Substituting expression (15) and (16) to (11) after transformations deflection beam is

$$ y(x,t) = -P_0 \sin\omega t \left[\frac{(\cosh kl - \cos kl)(\sin x + \sinh kx)}{2EJ_Z\, k^3(1 - \cos kl \cosh kl)} + \right. $$

$$ \left. + \frac{(\sinh kl - \sin kl)(\cos kx + \cosh kx)}{2EJ_Z\, k^3(1 - \cos kl \cosh kl)} \right]. \tag{17}$$

According to definition of dynamic flexibility, on the basis of (16), it takes form

$$ Y = -\frac{(\cosh kl - \cos kl)(\sin x + \sinh kx)}{2EJ_Z\, k^3(1 - \cos kl \cosh kl)} + $$

$$ + \frac{(\sinh kl - \sin kl)(\cos kx + \cosh kx)}{2EJ_Z\, k^3(1 - \cos kl \cosh kl)}\,. \tag{18}$$

The transient of absolute value of the dynamical flexibility (18) for x=l, that means $\alpha_Y = |Y|$ is drawn in Fig. 4 (red colour).

The defelection of clamped – free beam – using to the solution Galerkin's method - is given in shape of

$$ y(x,t) = \sum_{n=1}^{\infty} y_n(x,t) = \sum_{n=1}^{\infty} A_n \sin\left[(2n-1)\frac{\pi}{2l}x\right]\sin\omega t\,. \tag{19}$$

Substituting the following derivative of expression (19) to equation of motion of beam is obtained

$$EIA_n\left[(2n-1)\frac{\pi}{2l}\right]^4 \sin\left[(2n-1)\frac{\pi}{2l}x\right]\sin\omega t +$$

$$+\rho A A_n \omega^2 \sin\left[(2k-1)\frac{\pi}{21}x\right]\sin\omega t = P_0 \sin\omega t \tag{20}$$

The amplitude value A_n of the vibrations takes form of

$$A_n = \frac{P_0}{\rho A - EI\left[(2n-1)\frac{\pi}{21}\right]^4}. \tag{21}$$

Using the equation (21) and putting it to (19) the dynamical flexibility – after formal transformations - equals

$$Y_{xl}^{(n)} = \frac{\sin\left[(2n-1)\frac{\pi}{2l}x\right]}{\rho A\omega^2 - EI\left[(2n-1)\frac{\pi}{2l}\right]^4}. \tag{22}$$

In global case the dynamical flexibility at the end of the beam gets shape of

$$Y_{xl} = \sum_{n=1}^{\infty} Y_{xl}^{(n)} = \sum_{n=1}^{\infty} \frac{\sin\left[(2n-1)\frac{\pi}{2l}x\right]}{\rho A\omega^2 - EI\left[(2n-1)\frac{\pi}{2l}\right]^4}. \tag{23}$$

For sum k=1,2,3 the plot of value of dynamical flexibility defined by expression (23) is shown in Fig. 4 (blue colour).

.

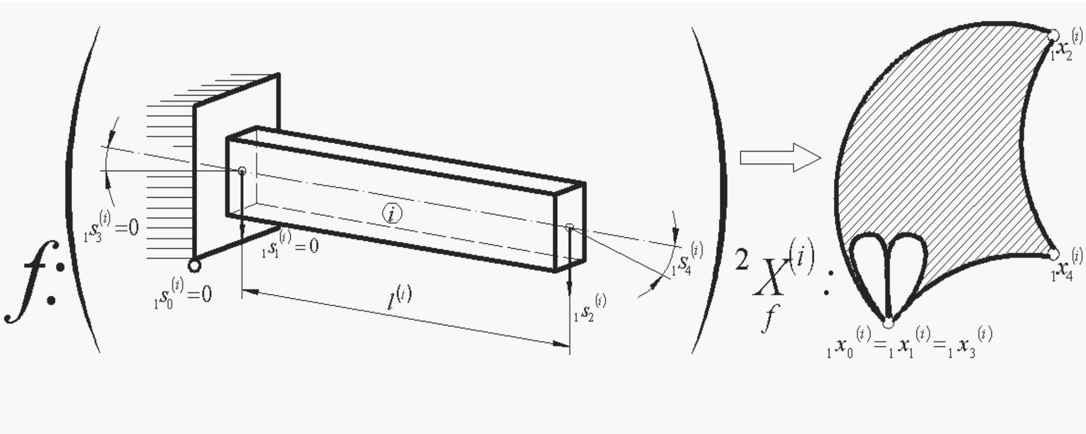

Fig. 2. Hypergraph of model of transverse vibrating clamped - free beam representation of tranformations (3) and (4)

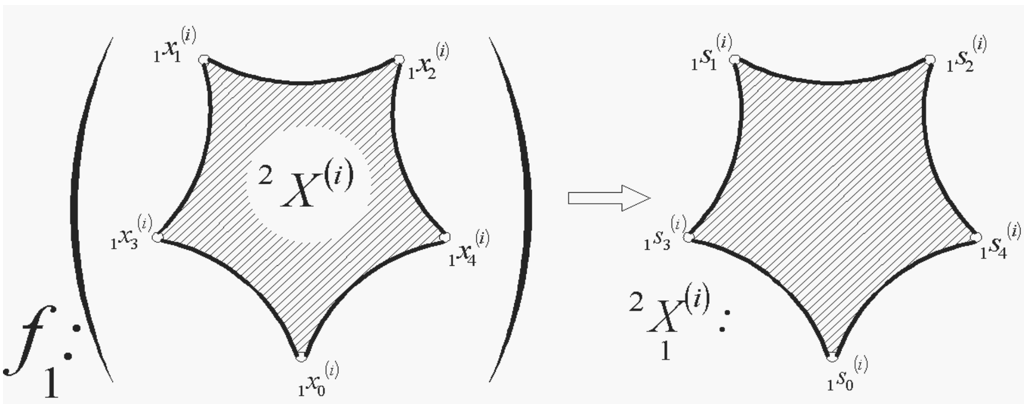

Fig. 3. Weighted hypergraph of transverse vibrating free –free beam as representation of transformation (6)

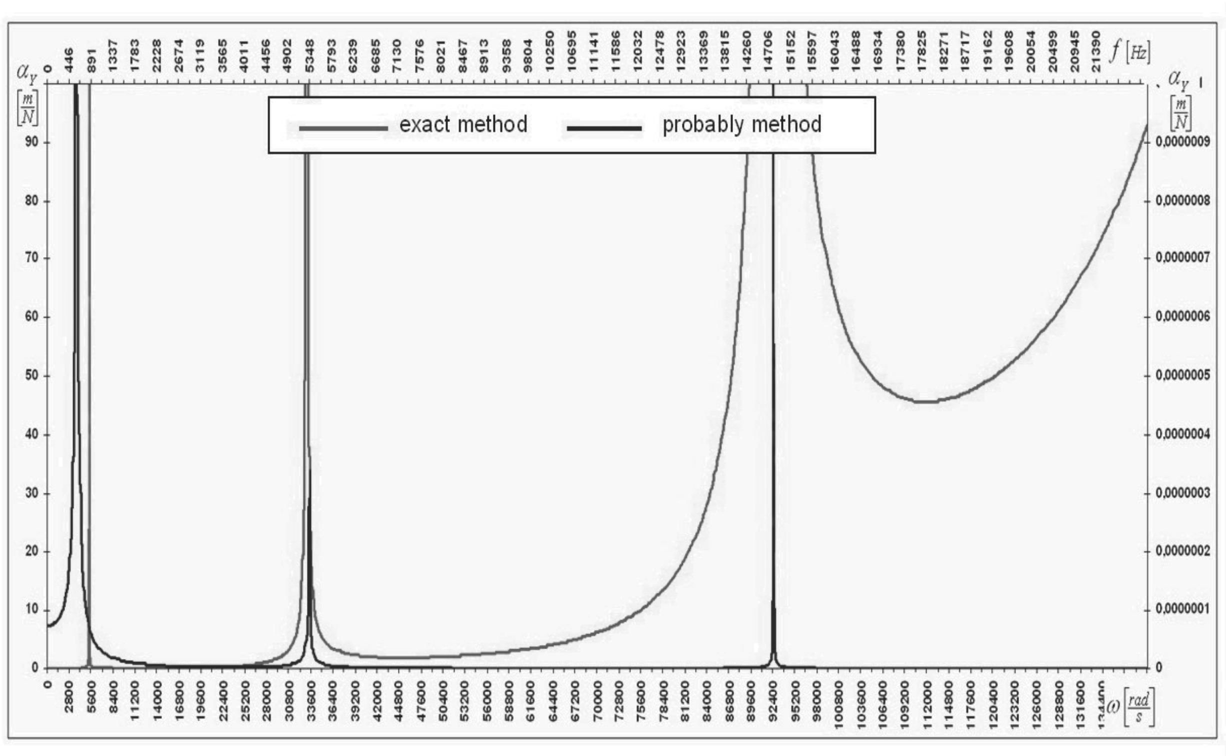

Fig. 4. The plot of absolute value of dynamical flexibility of the sum for *n*=1, 2, 3 mode vibration

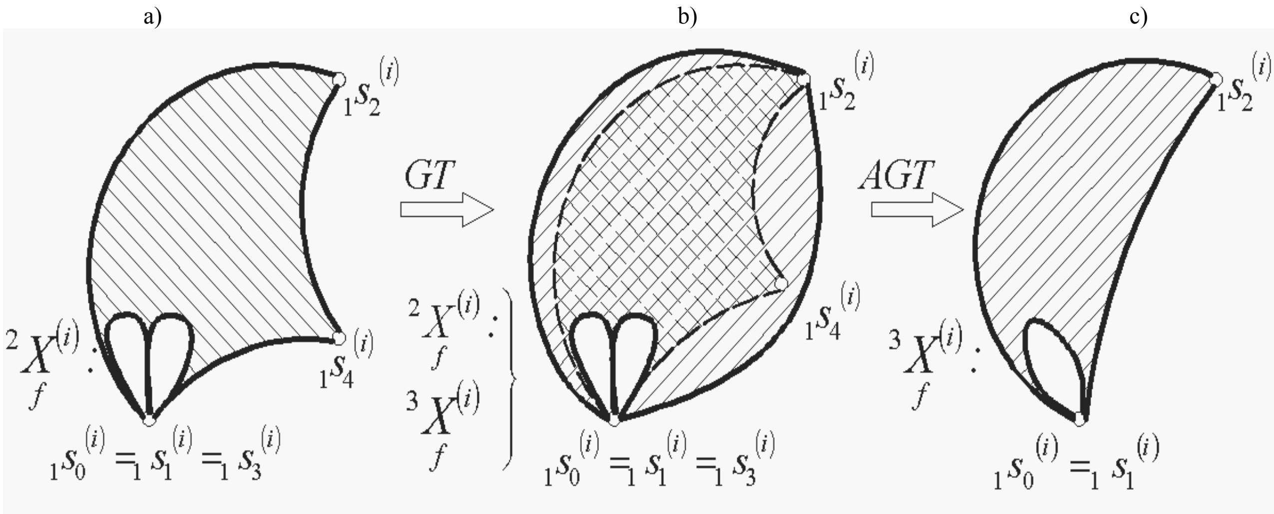

Fig. 5. The plot of absolute value of dynamical flexibility of flexibly vibrating free beam

4. Transformations of five-vertex into three-vertex hypergraphs as models of beams

On the base of Galerkin's transformation the five-vertex hypergraph (Fig. 5a) into three-vertex block graph (Fig. 5b,c) as model of free - free beam will be applied. This can be different approximate method of analysis for example the orthogonalization one (e.g. [14]).

In the case of synthesis of n-segment model of the system, composed of subsystems with constant section, vibrating flexibly, it is modelled by the loaded graph of the third category – after Galerkin's transformation - with n three-vertices-blocks, connected to those vertices to which the corresponding generalized coordinates are assigned (see i.e. [4,5]).

The use of a weighted hypergraph and its weighted subgraphs (as a model of flexibly vibrating system) in this way may provide the basis for the formalization which is the necessary condition of discretization of the considered class of continuous mechanical systems.

5. Last remarks

On the base of the obtained formulas, which were determined as the set of elements to modeling, synthesis and analysis of considered class of continuous mechanical systems. The problems will be presented in future works.

Acknowledgements

This work has been conducted as a part of research project N R03 0072 06//2009 supported by the Ministry of Science and Higher Education in 2009-2011.

References

[1] S. Bellert, H. Woźniacki, The analysis and synthesis of electrical systems by means of the method of structural numbers, WNT, Warsaw, 1968 (in Polish).

[2] C. Berge, Graphs and hypergraphs, American Elsevier Publishing Co., Inc., New York/ North Holland Publishing Co., Amsterdam-London, 1973.

[3] K. Białas, A. Buchacz, T. Dzitkowski, Synthesis of active mechanical systems with dumping inview of polar graphs and structural numbers, Monograph No. 230, Silesian University of Technology Press, Gliwice, 2009 (in Polish).

[4] A. Buchacz, Modelling, synthesis and analysis of bar systems characterized by a cascade structure represented by graphs, Mechanism and Machine Theory 30/7 (1995) 969-986.

[5] A. Buchacz, The Synthesis of Vibrating Bar-Systems Represented by Graph and Structural Numbers. Scientific Letters of Silesian University of Technology, Mechanics, vol. 104, Silesian University of Technology Press, 1991 (in Polish).

[6] A. Buchacz, A. Dymarek, T. Dzitkowski, Design and examining of sensitivity of continuous and discrete-continuous mechanical systems with required frequency spectrum represented by graphs and structural numbers, Monograph No. 88, Silesian University of Technology Press, Gliwice, 2005 (in Polish).

[7] A. Buchacz, The expansion of the synthesized structures of mechanical discrete systems represented by polar graphs, Journal of Materials Processing Technology 164-165 (2005) 1277-1280.

[8] A. Buchacz, K. Żurek, Reverse task of dynamice of active mechanical systems by means the graphs and structural numbers methods, Monograph No 81, Silesian University of Technology Press, Gliwice, 2005 (in Polish).

[9] A. Buchacz, Modelling, synthesis, modification, sensitivity and analysis of mechanic and mechatronic systems, Journal of Achievements in Materials and Manufacturing Engineering 24/1 (2007) 198-207.

[10] A. Buchacz, Dynamical flexibility of discrete-continuous vibrating mechatronic system, Journal of Achievements in Materials and Manufacturing Engineering 28/2 (2008) 159-166.

[11] A. Buchacz, Characteristics of discrete-continuous flexibly vibrating mechatronic system, Journal of Achievements in Materials and Manufacturing Engineering 28/1 (2008) 43-46.

[12] A. Buchacz, Calculation of Flexibility of Vibrating Beam as the Subsystem of Mechatronic System by Means the Exact and Approximate Methods, Proceedings in Applied Mathematics and Mechanics 9/1 (2009) 373-374.

[13] A. Buchacz, Transformations of Hypergraphs of Beams as Models in Synthesis of Flexibly Vibrating Continuous Mechanical Systems, Vestnik Krymskoy Akademii Nauk - Sevastopolskie otdelenie. Release 1. Sevastopol 2010, 35-38.

[14] A. Buchacz, Orthogonalization Method of Analysis of Mechanical And Mechatronic Systems. Les problèmes du technoshère et de la formation des cadres d'ingénieurs contemporains, Recueil des exposés des participants du III Séminaire international scientifique et méthodique, Sousse, Donetsk, 2009, 85-88.

[15] A. Buchacz, Comparision of Characteristics of Subsystems and Systems as Introduction to Synthesis of Torsionally Vibrating Mechatronic Systems. Les problèmes du technoshère et de la formation des cadres d'ingénieurs contemporain S, Recueil des exposés des participants du IV Séminaire international scientifique et méthodique, 2 à Hammamet, Donetsk, 2010, 85-88.