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Analysis of complex damped longitudinally vibrating systems in transportation

S. Żółkiewski *
Division of Mechatronics and Designing of Technical Systems, Institute of Engineering Processes Automation and Integrated Manufacturing Systems, Silesian University of Technology, ul. Konarskiego 18a, 44-100 Gliwice, Poland
* Corresponding author: E-mail address: slawomir.zolkiewski@polsl.pl

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Analysis and modelling

ABSTRACT

Purpose: of this thesis is dynamical analysis of complex systems in transportation. Analyzed systems are composed of rotatable rods. Transportation was defined as main motion of rods and the overall system.

Design/methodology/approach: The dynamical flexibility method is a leading methodology for dynamic analysis of considered systems. For solving equations of motion to dynamical flexibility the Galerkins method was used.

Findings: There were considered systems consisted of rods. Rods are rotated first round the origin of global reference frame simultaneously, the attached point and further ones round the end of the previous one. Charts of dynamic characteristics, in a form of dynamic flexibility as function of frequency and mathematical models were shown in this article.

Research limitations/implications: All multi-body systems components were simple linear homogeneous rods, the first one as the fixed rod and next ones treated as free-free rods. Transportation was limited to plane rotational motion round the Z axis of global reference frame. Future works would consider complex systems with geometrical and physical nonlinearity.

Practical implications: of presented analysis are derivation of multi-body rod systems of dynamic flexibility. Dynamic flexibility can be used in designing process. Presented mathematical models may be used for implementation in numerical applications and for automating some calculations in this type of systems.

Originality/value: In the mathematical model the damping forces were taken into consideration and the dynamic flexibility of complex systems was derived.

Keywords: Numerical techniques; Computational mechanics; Applied mechanics; Complex systems

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1. Introduction

Many technical problems are related to dynamic analysis of multi-body systems. This type of analysis considers a model of systems behaviour in changeable terms arising from mutual connection between individual subsystems. In this thesis this model was used to analyze interconnected rods in transportation. In the presented model, subsystems can move translational lengthwise and may undergo rotational displacements. Translational displacement is measured in the local reference frame and was assumed as longitudinal vibrations of analyzed system and rotational displacement was assumed as main and transportation motion. Many articles e.g. [1-4, 13-14, 20] concern the dynamic analysis of rod and beam systems in transportation and to differentiate it from articles concerning stationary rods and beams [5- 12, 15-19, 21] is an extension of the dynamic analysis. There are some works e.g. [1, 4] that draw attention to analysis of simple one element systems both fixed ones and free-free ones in transportation and this thesis is based on these works. The dynamic flexibility method used in this thesis can be based on matrix techniques and presented as a part of overall notation of equations of motion with mutual coupling.

The Galerkins method was chosen to solve equations of motion and search overall displacements, and after that dynamic flexibilities were derived by introducing to the dynamic flexibility definition. The dynamic behaviour of analyzed system was described by equations of motion and dynamic flexibilities. The meaning of dynamic analysis of multi-body is that each complex system formulation can be different and the superposition method is not valid, because it does not take mutual conjugations into consideration between subsystems and it does not take terms of continuity and inseparableness of displacements and forces into consideration.

2. Analyzed system's model

In this section the model of complex system in transportation was described (Fig. 1). The system is compounded from homogenises rods that are rotated round the origin of global reference frame and the end of each other. Rotation of subsystems and plane motion of the overall system is treated as transportation motion. Rods are loaded by axial harmonic forces providing longitudinal system vibrations.

In Figure 2 there is a presented model of the first subsystem of analyzed complex system. A system was described in global reference fixed frame where one is attached to the origin and local reference frame that is connected with rotation of the system.

In Figure 3, it is a model of consecutive subsystems shown from the second one to *n* one.

2.1. Description of the model

This subsection consists the way models are described. Model of the analyzed system was described by basic symbols:

$Y(\Omega)$ – the dynamic flexibility in function of frequency of extorted force,
η – mode of vibrations of a subsystem,
a – velocity of the wave propagation in the rod,

$$a_k = \sqrt{\frac{E_k}{\rho_k}}, \tag{1}$$

Ω – frequency of vibrations,
β_k – damping factor of k element,
E_k – the Young modulus of k element,
ρ_k – mass density of the k rod,
ω_k – angular velocity of the k rod,
φ_k – k element angle of rotation,
x_k – position of analyzed section of k element.
A_k – cross-section of the k rod,
l_k – length of the k rod,
t – time,

a vector of linear displacement of the k rod's section along centre line of the bar in the local reference system:

$$\bar{\mathbf{u}}_{\mathbf{k}} = \begin{bmatrix} u_k & 0 & 0 \end{bmatrix}^T, \tag{2}$$

a sample vector of linear displacement of the rod's in the global reference system:

$$\bar{\mathbf{u}} = \begin{bmatrix} u_X & u_Y & 0 \end{bmatrix}^T, \tag{3}$$

a position vector of the k element:

$$\bar{\mathbf{S}}_{\mathbf{k}} = \begin{bmatrix} s_k & 0 & 0 \end{bmatrix}^T. \tag{4}$$

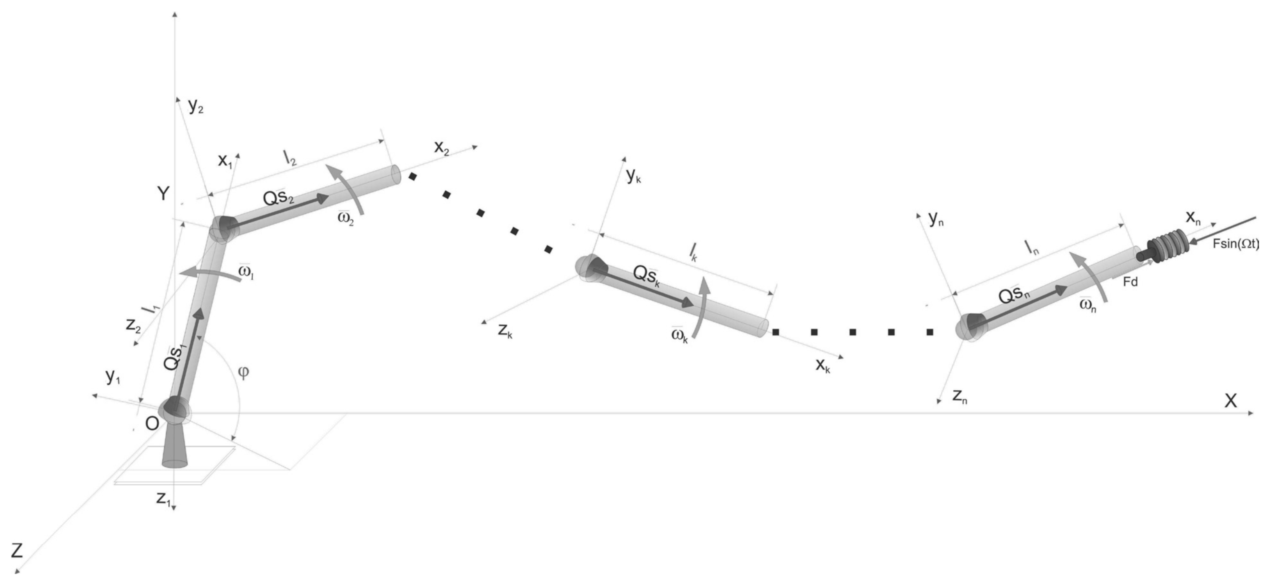

Fig. 1. The analyzed complex system

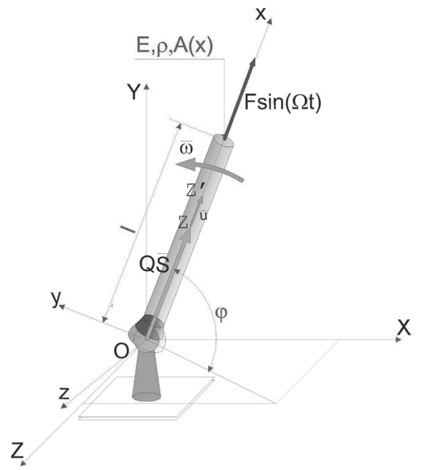

Fig. 2. The first subsystem attached to the origin of the global reference frame

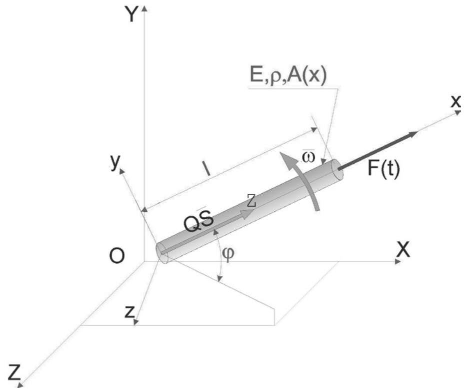

Fig. 3. Subsystem of the complex analyzed system

Generalized coordinates and generalized velocities were assumed as projections of coordinates and velocities of individual rods in the global reference frame and there are assumed the following notation:

- displacement for the X axis:

$$q_{2k-1} = r_{2k-1X}, \tag{5}$$

- displacement for the Y axis:

$$q_{2k} = r_{2kY}, \tag{6}$$

- velocity for the X axis:

$$\dot{q}_{2k-1} = \dot{r}_{2k-1X}, \tag{7}$$

- velocity for the Y axis:

$$\dot{q}_{2k} = \dot{r}_{2kY}. \tag{8}$$

where the mobility grade was expressed by a formula where the number of kinematic pairs (for example rotary ones, fifth class p_5 or fourth class p_4) and number of moveable elements signed by z is obtained:

$$n = 3 \cdot z - 2 \cdot p_5 - p_4. \tag{9}$$

Mass of the n element is equal:

$$M_n = \int_0^{l_{(k-1)k}} \rho_n \cdot A_n ds_n, \tag{10}$$

where: $k = 1,\ldots,n$

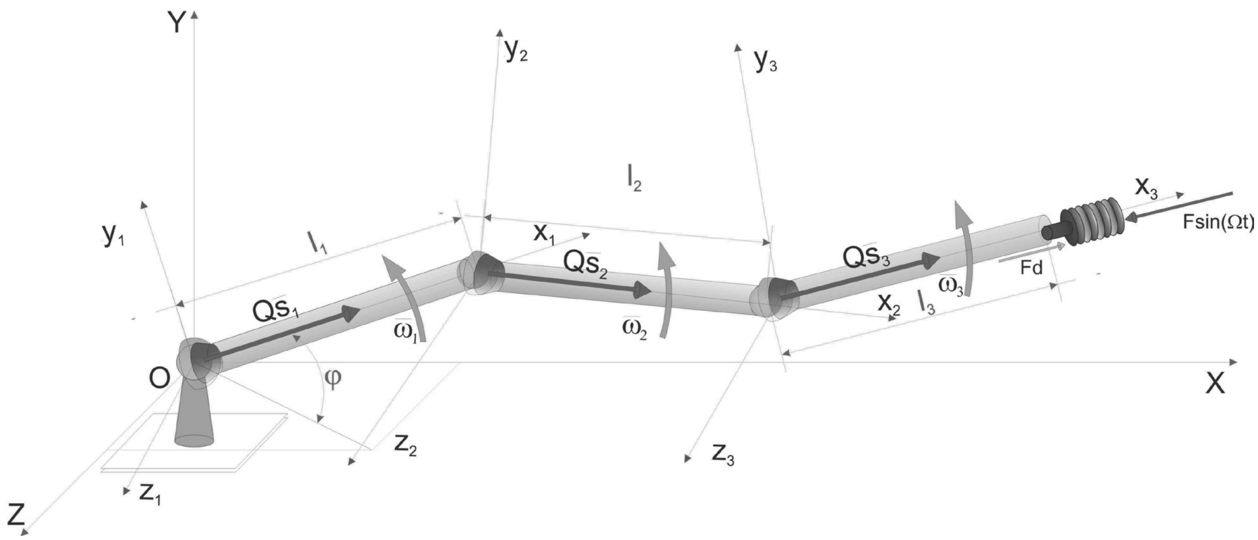

Fig. 4. The sample three-linked complex system (n=3)

Orientation of the elements was provided by the rotation in the form of:

$$\mathbf{Q}_{(k-1)k}=\begin{bmatrix}\cos\varphi_k & -\sin\varphi_k & 0\\ \sin\varphi_k & \cos\varphi_k & 0\\ 0 & 0 & 1\end{bmatrix} \tag{11}$$

and in the overall case:

$$\mathbf{Q}_{0n}=\prod_{k=0}^{n-1}\mathbf{Q}_{k(k+1)}. \tag{12}$$

In Figure 4 there is presented the sample three-linked complex system. This system is equipollent to complex system, where mobility grade is equal n=3.

2.2. Mathematical model

Individual vectors for individual elements without damping are as follow:

- for the first rod:

$$\overline{\mathbf{r}}_1=\mathbf{Q}_{01}\cdot\left(\overline{\mathbf{S}}_1+\overline{\mathbf{u}}_1\right), \tag{13}$$

- for k element:

$$\overline{\mathbf{r}}_k=\mathbf{Q}_{0k}\cdot\left(\overline{\mathbf{S}}_k+\overline{\mathbf{u}}_k\right), \tag{14}$$

- for n element:

$$\overline{\mathbf{r}}_n=\mathbf{Q}_{0n}\cdot\left(\overline{\mathbf{S}}_n+\overline{\mathbf{u}}_n\right). \tag{15}$$

Now after differentiation is done we, obtain acceleration of k element:

$$\begin{aligned}\ddot{\overline{\mathbf{r}}}&=\sum_{k=1}^{n}\ddot{\overline{\mathbf{r}}}_k=\sum_{k=1}^{n}\dot{\overline{\mathbf{v}}}_k=\sum_{k=1}^{n}\left(\dot{\overline{\mathbf{v}}}_{kX}+\dot{\overline{\mathbf{v}}}_{kY}\right)=\\&=\sum_{k=1}^{n}\left[\mathbf{Q}_{0k}\cdot\left(\sum_{m=1}^{k}\dot{\overline{\boldsymbol{\omega}}}_m\right)\times\left(\overline{\mathbf{S}}_k+\overline{\mathbf{u}}_k\right)\right]+\\&+\sum_{k=1}^{n}\left[\mathbf{Q}_{0k}\cdot\left(\sum_{m=1}^{k}\overline{\boldsymbol{\omega}}_m\right)\times\left(\sum_{m=1}^{k}\overline{\boldsymbol{\omega}}_m\right)\times\left(\overline{\mathbf{S}}_k+\overline{\mathbf{u}}_k\right)\right]+\\&+2\cdot\sum_{k=1}^{n}\left[\mathbf{Q}_{0k}\cdot\left(\sum_{m=1}^{k}\overline{\boldsymbol{\omega}}_m\right)\times\dot{\overline{\mathbf{u}}}_k\right]+\sum_{k=1}^{n}\left(\mathbf{Q}_{0k}\cdot\ddot{\overline{\mathbf{u}}}_k\right).\end{aligned} \tag{16}$$

The projection into the X axis of the global reference system:

$$\begin{aligned}&\frac{\partial^2 u_{1X}}{\partial t^2}-\frac{E_1}{\rho_1}\frac{\partial^2 u_{1X}}{\partial x^2}=\omega_1^2\cdot\left(l_1\cdot\cos\varphi_1+u_{1X}\right)+\\&+2\omega_1\frac{\partial u_{1Y}}{\partial t}-\beta_1\cdot\frac{\partial u_{1X}}{\partial t}+\beta_1\cdot\omega_1\cdot\left(l_1\cdot\sin\varphi_1+u_{1Y}\right)\end{aligned} \tag{17}$$

The projection into the Y axis of the global reference system:

$$\begin{aligned}&\frac{\partial^2 u_{1Y}}{\partial t^2}-\frac{E_1}{\rho_1}\frac{\partial^2 u_{1Y}}{\partial x^2}=\omega_1^2\cdot\left(l_1\cdot\sin\varphi_1+u_{1Y}\right)+\\&-2\omega_1\frac{\partial u_{1X}}{\partial t}-\beta_1\cdot\frac{\partial u_{1Y}}{\partial t}-\beta_1\cdot\omega_1\cdot\left(l_1\cdot\cos\varphi_1+u_{1X}\right)\end{aligned} \tag{18}$$

For k element the projection into the X axis of the global reference system:

$$\begin{aligned}&\frac{\partial^2 u_{kX}}{\partial t^2}-\frac{E_k}{\rho_k}\frac{\partial^2 u_{kX}}{\partial x^2}=\sum_{m=1}^{k}\omega_m^2\cdot\left(l_k\cdot\cos\varphi_k+u_{kX}\right)+\\&+2\cdot\sum_{m=1}^{k}\omega_m\cdot\frac{\partial u_{kY}}{\partial t}-\beta_k\cdot\frac{\partial u_{kX}}{\partial t}+\\&+\beta_k\cdot\sum_{m=1}^{k}\omega_m\cdot\left(l_k\cdot\sin\varphi_k+u_{kY}\right)\end{aligned} \tag{19}$$

The projection into the Y axis of the global reference system of the k element:

$$\begin{aligned}&\frac{\partial^2 u_{kY}}{\partial t^2}-\frac{E_k}{\rho_k}\frac{\partial^2 u_{kY}}{\partial x^2}=\sum_{m=1}^{k}\omega_m^2\cdot\left(l_k\cdot\sin\varphi_k+u_{kY}\right)+\\&-2\cdot\sum_{m=1}^{k}\omega_m\cdot\frac{\partial u_{1X}}{\partial t}-\beta_k\cdot\frac{\partial u_{kY}}{\partial t}+\\&-\beta_k\cdot\sum_{m=1}^{k}\omega_m\cdot\left(l_k\cdot\cos\varphi_k+u_{kX}\right)\end{aligned} \tag{20}$$

For n element the projection into the X axis of the global reference system:

$$\begin{aligned}&\frac{\partial^2 u_{nX}}{\partial t^2}-\frac{E_n}{\rho_n}\frac{\partial^2 u_{nX}}{\partial x^2}=\sum_{m=1}^{n}\omega_m^2\cdot\left(s_n\cdot\cos\varphi_n+u_{nX}\right)+\\&+2\cdot\sum_{m=1}^{n}\omega_m\cdot\frac{\partial u_{nY}}{\partial t}-\beta_n\cdot\frac{\partial u_{nX}}{\partial t}+\\&+\beta_n\cdot\sum_{m=1}^{n}\omega_m\cdot\left(s_n\cdot\sin\varphi_n+u_{nY}\right)\end{aligned} \tag{21}$$

The projection into the Y axis of the global reference system of the n element:

$$\frac{\partial^2 u_{nY}}{\partial t^2}-\frac{E_n}{\rho_n}\frac{\partial^2 u_{nY}}{\partial x^2}=\sum_{m=1}^{n}\omega_m^2\cdot\left(s_n\cdot\sin\varphi_n+u_{nY}\right)+$$
$$-2\cdot\sum_{m=1}^{n}\omega_m\cdot\frac{\partial u_{nX}}{\partial t}-\beta_n\cdot\frac{\partial u_{nY}}{\partial t}+ \tag{22}$$
$$-\beta_n\cdot\sum_{m=1}^{n}\omega_m\cdot\left(s_n\cdot\cos\varphi_n+u_{nX}\right)$$

With equations (16-21) it is possible to derivate equations of motion for optional motility complex systems.

2.3. Boundary conditions

Boundary conditions of the first rod were assumed as follows:

$$\begin{cases} u_1(0,t)=0, \\ u_1(l_1,t)=u_2(0,t), \\ E_1A_1\dfrac{\partial u_1(l_1,t)}{\partial x}=E_2A_2\dfrac{\partial u_2(l_2,t)}{\partial x}, \end{cases} \tag{23}$$

the last rod has boundary condition as follow:

$$\begin{cases} E_nA_n\dfrac{\partial u_n(0,t)}{\partial x}=0, \\ E_nA_n\dfrac{\partial u_n(l_n,t)}{\partial x}=2\displaystyle\int_0^l F_0\delta(x_n-l_n)e^{j\Omega t}dx=1e^{j\Omega t}, \end{cases} \tag{24}$$

where the Dirac delta function was assumed as a distribution generalized function regarding a point of application of force. Force was assumed as harmonic one with unitary amplitude up to the dynamic flexibility definition.

For integrity of complex system assurance the continuity conditions must be performed:

$$\begin{cases} u_k(0,t)=u_{k-1}(l_{k-1},t), \\ u_k(l_k,t)=u_{k+1}(0,t), \\ E_kA_k\dfrac{\partial u_k(0,t)}{\partial x}=E_{k-1}A_{k-1}\dfrac{\partial u_{k-1}(l_{k-1},t)}{\partial x}, \\ E_kA_k\dfrac{\partial u_k(l_k,t)}{\partial x}=E_{k+1}A_{k+1}\dfrac{\partial u_{k+1}(0,t)}{\partial x}, \end{cases} \tag{25}$$

2.4. Eigenfunctions

The boundary problem was solved and the eigenfunction of displacement and eigenfunction of time variables were shown. The eigenfunction of displacement of each element without the first one can be written:

$$X_k(x_k)=\cos(\kappa x_k), \tag{26}$$

where:

$$\kappa=\frac{\eta\pi}{l},\quad \eta=0,\ 1,\ 2,\ 3,\ \ldots \tag{27}$$

and the first element of eigenfunction is as follows:

$$X_k(x_k)=\sin(\kappa x_k), \tag{28}$$

where:

$$\kappa=\frac{(2\eta+1)\pi}{2l},\quad \eta=0,\ 1,\ 2,\ 3,\ \ldots \tag{29}$$

3. Dynamic flexibility substitute

In this section the substitute dynamic flexibility of a complex system in transportation was presented.

3.1. Searched displacement solution

Up to the Galerkin's method the solution for each element without the first one was assumed as:

For X axis:

$$u_{kX}=\sum_{\eta=0}^{\infty}A_{kX}\cdot\cos(\kappa x_k)\cdot e^{j\Omega t}, \tag{30}$$

and for Y axis of the global reference frame:

$$u_{kY}=\sum_{\eta=0}^{\infty}A_{kY}\cdot\cos(\kappa x_k)\cdot e^{j\Omega t}, \tag{31}$$

For the first element regard to X axis:

$$u_{1X}=\sum_{\eta=0}^{\infty}A_{1X}\cdot\sin(\kappa x_1)\cdot e^{j\Omega t}, \tag{32}$$

and for the first element Y axis of the global reference frame:

$$u_{1Y}=\sum_{\eta=0}^{\infty}A_{1Y}\cdot\sin(\kappa x_1)\cdot e^{j\Omega t}, \tag{33}$$

now displacements are searched as a sum of eigenfunction of displacement and eigenfunction of time.

3.2. Orthogonalisation

After orthogonalisation of equations of motion we can obtain equations for example for k element:

$$
\begin{aligned}
&\int_0^l \frac{\partial^2 u_{kX}}{\partial t^2}\cdot\cos(\kappa x_k)dx-\frac{E_k}{\rho_k}\cdot\int_0^l \frac{\partial^2 u_{kX}}{\partial x^2}\cdot\cos(\kappa x_k)dx=\\
&=\sum_{m=1}^{k}\omega_m^2\cdot\int_0^l\left(l_k\cdot\cos\varphi_k+u_{kX}\right)\cdot\cos(\kappa x_k)dx+\\
&+2\cdot\sum_{m=1}^{k}\omega_m\cdot\int_0^l\frac{\partial u_{kY}}{\partial t}\cdot\cos(\kappa x_k)dx+\\
&-\beta_k\cdot\int_0^l\frac{\partial u_{kX}}{\partial t}\cdot\cos(\kappa x_k)dx+\\
&+\beta_k\sum_{m=1}^{k}\omega_m\cdot\int_0^l\left(l_k\cdot\sin\varphi_k+u_{kY}\right)\cdot\cos(\kappa x_k)dx
\end{aligned}
\tag{34}
$$

and for second axis:

$$
\begin{aligned}
&\int_0^{l_k} \frac{\partial^2 u_{kY}}{\partial t^2}\cdot\cos(\kappa x_k)dx-\frac{E_k}{\rho_k}\cdot\int_0^{l_k} \frac{\partial^2 u_{kY}}{\partial x^2}\cdot\cos(\kappa x_k)dx=\\
&=\sum_{m=1}^{k}\omega_m^2\int_0^{l_k}\left(l_k\cdot\sin\varphi_k+u_{kY}\right)\cdot\cos(\kappa x_k)dx+\\
&-2\cdot\sum_{m=1}^{k}\omega_m\cdot\int_0^{l_k}\frac{\partial u_{kX}}{\partial t}\cdot\cos(\kappa x_k)dx+\\
&-\beta_k\cdot\int_0^{l_k}\frac{\partial u_{kY}}{\partial t}\cdot\cos(\kappa x_k)dx+\\
&-\beta_k\sum_{m=1}^{k}\omega_m\cdot\int_0^{l_k}\left(l_k\cdot\cos\varphi_k+u_{kX}\right)\cdot\cos(\kappa x_k)dx
\end{aligned}
\tag{35}
$$

3.3. Dynamic flexibility

There are few known methods of derivation of substitute dynamical flexibility of complex systems, for example making reference to Pasek [21] this dynamic flexibility can be written as:

$$
Y_z=\frac{Y_a^{(2)}\cdot Y_b^{(2)}+Y_b^{(1)}\cdot Y_b^{(2)}-Y_c^{(2)}\cdot Y_d^{(2)}}{Y_b^{(1)}+Y_a^{(2)}},
\tag{36}
$$

where:

Y_z - substituting dynamic flexibility ,

$Y_a^{(i)}$ - dynamic flexibility i rod loaded in section x=0, derived in section x=0,

$Y_b^{(i)}$ - dynamic flexibility i rod loaded in section x=l, derived in section x=l,

$Y_c^{(i)}$ - dynamic flexibility i rod loaded in section x=l, derived in section x=0,

$Y_d^{(i)}$ - dynamic flexibility i rod loaded in section x=0, derived in section x=l, i=1,2.

We can also use a matrix method. Equation (37) presents sample derived partial dynamical flexibility of k element that can be used to substitute dynamic flexibility formula.

4. Conclusions

Analysis of dynamic behaviour of the multi-body rod system was done in this thesis. Equations of motion of analyzed system were presented.

Dynamic flexibility substitute of the complex system compounded from rods in transportation was derived in this thesis. In the mathematical, model damping forces and Coriolis forces and centrifugal forces were taken into consideration. Rotational motion of the system and subsystems was treated in this thesis as transportation. Transportation was limited to plane motion.

There were presented sample dynamic characteristics (Fig. 5 and Fig. 6) in form of dynamic flexibility as function of frequency and mathematical models in this article. Derived mathematical model can be put to use to support designing process and to stabilize the analysis.

Characteristics were generated by Modyfit application (Modelling of dynamic flexible systems in transportation).

$$
\begin{aligned}
|Y_k| = \sum_{\eta=0}^{\infty}\Biggl\{\Biggl(&\frac{2\cos(\kappa l_k)\cos(\kappa x_k)}{\rho_k A_k l_k\left[\left(a_k^2\kappa^2-\left(\sum_{m=1}^{k}\omega_m\right)^2-\Omega^2\right)^2-(\beta_k\Omega)^2+\left(\beta_k\sum_{m=1}^{k}\omega_m\right)^2-4\left(\sum_{m=1}^{k}\omega_m\right)^2\Omega^2\right]^2+\left[2\beta_k\left(a_k^2\kappa^2-\left(\sum_{m=1}^{k}\omega_m\right)^2-\Omega^2\right)\Omega+4\beta_k\left(\sum_{m=1}^{k}\omega_m\right)^2\Omega\right]^2}\Biggr)^2\\
&\Biggl\langle\Biggl[\Biggl[\Biggl[a_k^2\kappa^2-\left(\sum_{m=1}^{k}\omega_m\right)^2-\Omega^2\Biggr]\Biggl\{1-\Biggl[a_k^2\kappa^2-\left(\sum_{m=1}^{k}\omega_m\right)^2-\Omega^2\Biggr]\Biggl[(\beta_k\Omega)^2+\left(\beta_k\sum_{m=1}^{k}\omega_m\right)^2-4\left(\sum_{m=1}^{k}\omega_m\right)^2\Omega^2\Biggr]\Biggr\}\Biggr]+\Biggl[2\beta_k\left(a_k^2\kappa^2-\left(\sum_{m=1}^{k}\omega_m\right)^2-\Omega^2\right)\Omega+4\beta_k\left(\sum_{m=1}^{k}\omega_m\right)^2\Omega\Biggr]\beta_k\Omega\Biggr]^2+\\
&+\Biggl[\Biggl[\left(a_k^2\kappa^2-\left(\sum_{m=1}^{k}\omega_m\right)^2-\Omega^2\right)^2-(\beta_k\Omega)^2+\left(\beta_k\sum_{m=1}^{k}\omega_m\right)^2-4\left(\sum_{m=1}^{k}\omega_m\right)^2\Omega^2\Biggr]\beta_k\Omega-\Biggl[2\beta_k\left(a_k^2\kappa^2-\left(\sum_{m=1}^{k}\omega_m\right)^2-\Omega^2\right)^2\Omega+4\left(a_k^2\kappa^2-\left(\sum_{m=1}^{k}\omega_m\right)^2-\Omega^2\right)\beta_k\left(\sum_{m=1}^{k}\omega_m\right)^2\Omega\Biggr]\Biggr]^2\Biggr\rangle\Biggr\}^{\frac{1}{2}}
\end{aligned}
\tag{37}
$$

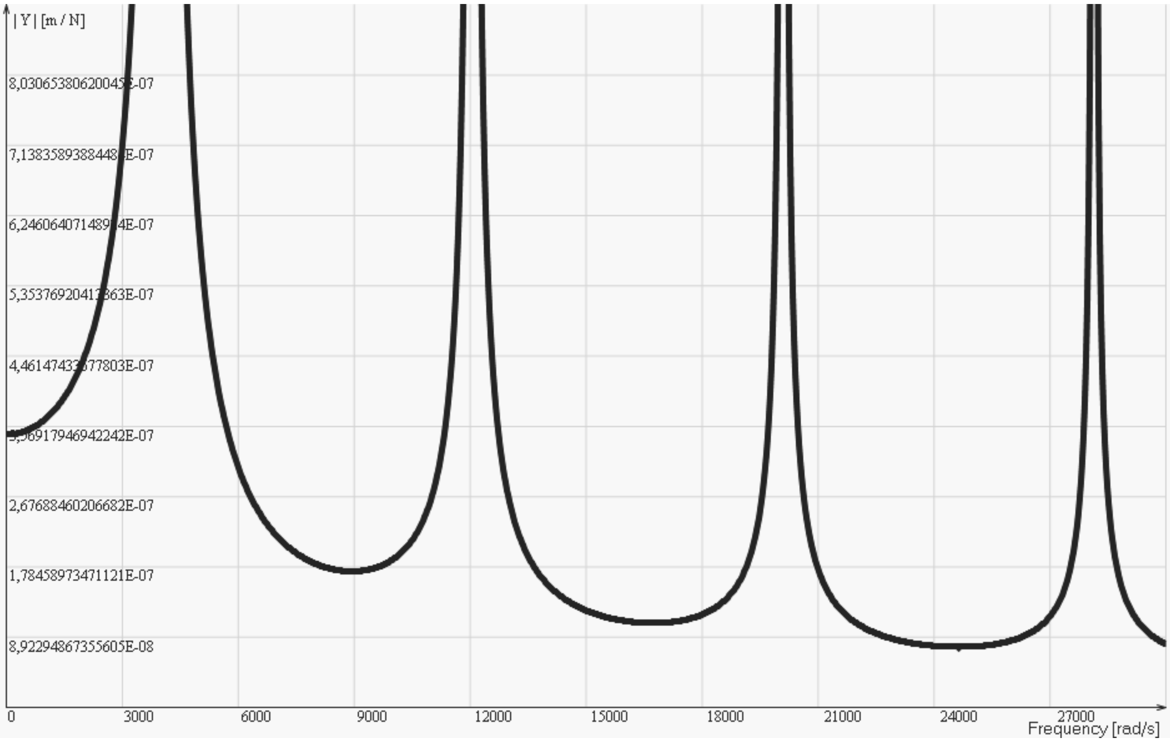

Fig. 5. Sample characteristic of absolute of substitute dynamic flexibility of the two-linked system

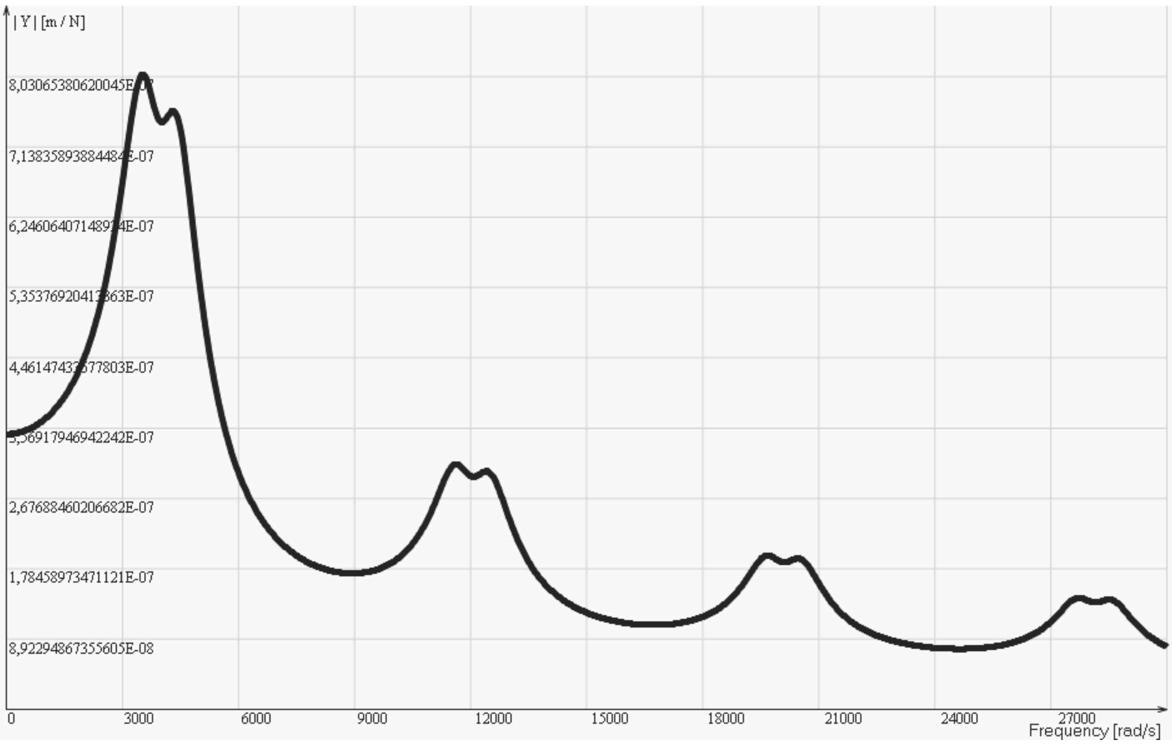

Fig. 6. Sample characteristic of absolute of substitute dynamic flexibility of the two-linked rotating system with damping

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