Journal of Achievements in Materials and Manufacturing Engineering
VOLUME 31
ISSUE 2
December
2008

Analysis of mechanical systems with transversal vibrations in transportation

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Received 05.09.2008; published in revised form 01.12.2008

Analysis and modelling

ABSTRACT

Purpose: of this article are modelling and dynamic analysis of mechanical systems during the rotational movement. Nowadays technical problems are tied with high speeds of mechanisms, high precision of work, using lower density materials, and many other high demands for elements of work. Objective of this paper was the analysis with giving into consideration the interaction between working motion and local vibrations. The model is loaded by transverse forces and transformed to the global reference frame.

Design/methodology/approach: derived equations of motion were made by the Lagrange equations method with generalized coordinates and generalized velocities assumed as orthogonal projections of individual coordinates and velocities of each beam to axes of the global reference frame.

Findings: systems of equations of motion of transversally vibrating systems in two-dimensional motion will be put to use to derivation of the dynamical flexibility of these systems and complex systems. Those equations are the beginning of the analysis of complex systems. They can also be used to derivation of the substitute dynamical flexibility of n-linked systems.

Research limitations/implications: mechanical systems vibrating transversally in terms of two-dimensional motion were considered in the thesis. The consecutive problem of dynamical analysis is modelling of systems in spatial motion and also the analysis of systems loaded by longitudinal forces.

Practical implications: mathematical effects of this article can be put to use into many mechanisms and machines running in rotational transportation. For example applications are: high speed turbines, wind power plants, rotors, manipulators and in aerodynamics issues, etc. Of course results should be adopted and modified to appropriate system.

Originality/value: High demands for parameters of work of mechanisms and machines are the postulation for new research and new ways of modelling and analyzing those type systems. The example way of solution such systems is presented in this thesis. The transportation effect for models vibrating transversally was defined.

Keywords: Applied mechanics; Transversally vibrating systems; Manipulator; Transportation effect

1. Introduction

Mechanical systems are considered both in kinematical and dynamical sense with the criterion of full controlling. One of the most appropriate method for description of dynamic states of complex technical systems is the dynamic flexibility method. Local vibrations and working motion treated in this thesis as transportation in contemporary solutions have done separately. New models of analysis should take into consideration the flexibility of mechanism's links and should have ability to use in many applications where high precision of positioning are required [2-5]. In the literature there are many well-known positions where local vibrations superimpose on motion of a rigid system e.g. [1,14]. The superimpose produce the approximate solution in the global reference system. Practical using of those results can make mathematical simplification and general-purpose applications [10,11]. In this thesis there are considered both dynamical characteristics (such as dynamical flexibility) and equations of motion.

We can observe maintaining tendencies to optimization parameters of working machines and mechanisms in the technique. The cause of optimization is mainly lashed with permanently growing technical requirements. Main aspects of optimization concern positioning of manipulators and robots, maximal precision of working mechanical systems and also increase of reliability and quality. Except the method of superposition there is used more accurate ways of modelling mechanical systems. One of those ways there is presented in this article.

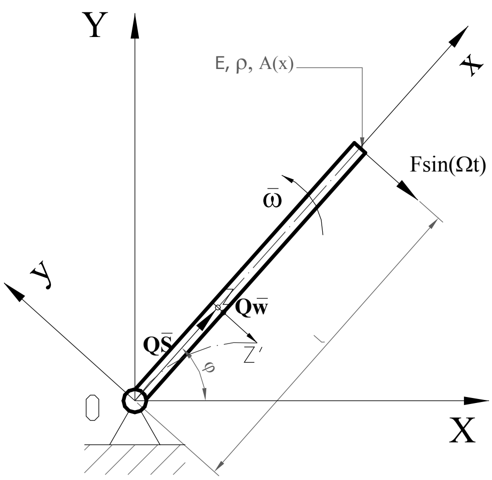

Fig. 1. The model of the rotating beam loaded by a transversal force

Main motion treated as transportation and working motion interacts with the amplitude of vibrations in our solutions. Systems were analyzed with taking into consideration in mathematical model the effect and interaction between main motion on local motion that is vibration. Work motion is considered as transportation whereas local vibrations is treated as relative motion. In this article there are concerned basic systems such as beams vibrating transversally in plane motion and in spatial motion. The thesis expresses mutual coupling between amplitude of vibrations and velocity of transportation. In literature e.g. [1,6-11,14-19] there are many articles and publications where the authors concern systems working with little velocities or analysis systems vibrating only in the local reference system. Those analyzed systems have well-known form of vibrations and are described by well-known equations of motion. Further analysis applies to derivation of dynamical flexibility of complex systems with an optional number of elements. In systems moving large velocities can put forward a phenomena of flatter or a phenomena of resonance. It applies to simple systems such as rods and beams but especially to complex systems because they are more susceptible. Phenomena of resonance manifest by growing amplitude of vibrations theoretically ad infinitum and practically to a moment of durable destroying mechanisms or growing amplitude to a level adequate to a velocity, so this is a reason of decreased durability of devices. Nowadays engineers search of new more detailed formulation of vibration's in the contemporary technique. All mentioned in this introduction things are the cause of searching new, more accurate ways of modeling. So the analysis of systems as rigid systems is too much simplification.

2. Modelling of the mechanical system vibrating transversally in transportation

This chapter reveals a new way of analyzing mechanical systems with giving into consideration the transportation. There were considered the models of the beam fixed in the origin of a global reference system and the free beam and also the two body manipulator in the thesis.

2.1. The model of transversally vibrating beam fixed in the origin in transportation

In Fig. 1 there is presented considered system that is the homogeneous flexible beam with a symmetrical section. The system is loaded by the harmonic changeable transversally force. The force is actuated into the tip of the beam. The constancy of sections was assumed in whole lengths of beams. Model is described in two-dimensional reference systems and it determines plane motion. Mechanism was described in the local reference frame concerns local vibrations and in the non-moveable global reference system. Local vibrations were transformed to the global reference frame. The beam is fixed both in the origin of the global reference frame and local reference frame.

2.2. The vibrating transversally free beam in transportation

The system vibrating in transportation is considered in this subchapter. The system consists of the homogenous elastic beam with a full cross-section A. Assumption was done that the cross-section into whole length of the beam l is constant (Fig. 2). The material of the beam has Young's modulus E and mass density ρ. The harmonic bending force is acting onto the beam. The mathematical model were derived in the global reference system The reference system is the planar system one.

2.3. The transversally vibrating two-linked system

Manipulator consisted of two vibrating links is considered. Cross sections of the beams are suitably A_1 as the section of first link and A_2 as the section of second link. The cross sections are constant on the whole length of beams appropriately for first link

l_{01} and in second link l_{12} (Fig. 3). Material of beams has Young's modulus E_1 for first beam and E_2 for second beam and suitably mass densities ρ_1 and ρ_2. Harmonic transverse forces are acted on the beam.

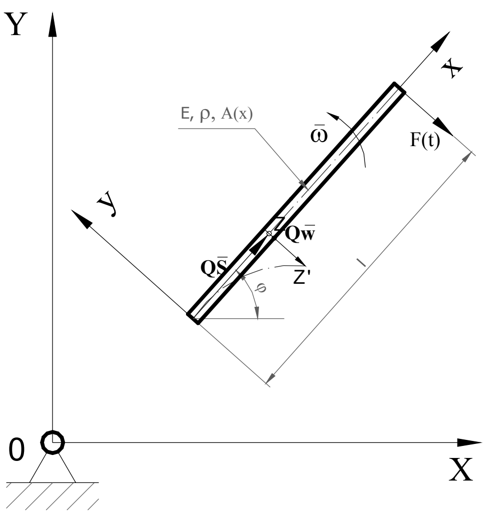

Fig. 2. The model of a vibrating beam in terms of plane motion and loaded by a harmonic bending force

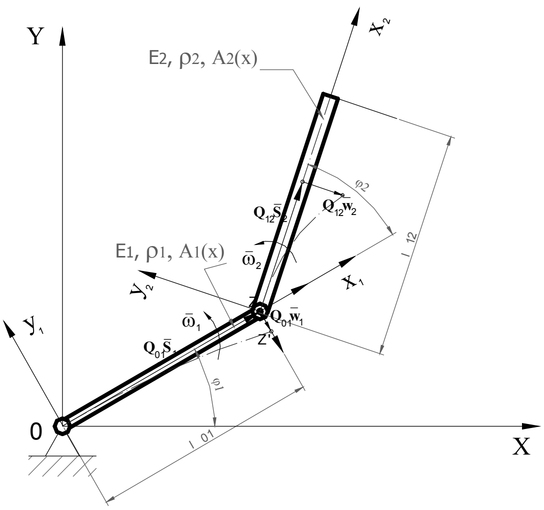

Fig. 3. The two-link model loaded by a transversal force in transportation

2.4. The transversally vibrating three-linked system

The tree-linked vibrating manipulator is considered. Beams from this system have cross sections constant on the whole length of beams (Fig. 4). Every beam rotate in the global reference system. The manipulator was loaded by a harmonic transversally acting force.

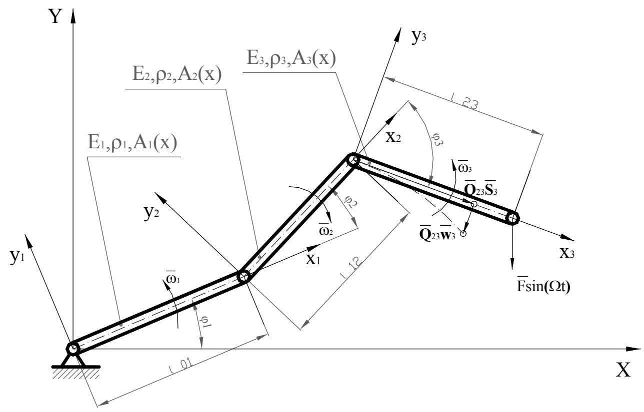

Fig. 4. The three-link model loaded by a transversal force in transportation

3. Mathematical model

The mathematical model of the analyzed systems were presented in this section. Both the free beam and the fixed beam are analyzed with different boundary conditions in the origin of the global reference system.

A vector of linear displacement of a cross-section in the beam is orthogonal to their center lines (w) in the local reference system (Fig. 1) and is as follow:

$$\overline{\mathbf{w}} = \begin{bmatrix} w & 0 & 0 \end{bmatrix}^T . \tag{1}$$

Body mass of the beam that is made from material with mass density ρ and a volume V and a cross-section A:

$$M = \int_V \rho dV = \int_0^s \rho \cdot A ds, \tag{2}$$

where:

$$dV = A \cdot dx. \tag{3}$$

Vibrations of the beam were analyzed in places along the axis x of the local reference system that is equal (s), so a position vector in that system is as follow:

$$\overline{\mathbf{S}} = \begin{bmatrix} s & 0 & 0 \end{bmatrix}^T . \tag{4}$$

Vibrations of the beam in planar transportation is analyzed. Generalized coordinates and generalized velocities were assumed as orthogonal projections of coordinates (r_X, r_Y) and velocities of the beam to axes of the global reference frame:

$$q_1 = r_X, \quad q_2 = r_Y, \tag{5}$$

$$\dot{q}_1 = \frac{dq_1}{dt} = \dot{r}_X = v_X, \quad \dot{q}_2 = \frac{dq_2}{dt} = \dot{r}_Y = v_Y. \tag{6}$$

The generalized forces acting in the system were defined as internal forces in the beam as follows:

$$\begin{aligned} P_X &= \frac{\partial P \cdot Q_{21} \cdot s_X}{\partial x}, \\ P_Y &= \frac{\partial P \cdot Q_{11} \cdot s_Y}{\partial x}. \end{aligned} \tag{7}$$

A vector of angular velocity of the first link and a vector of angular velocity of the second link are defined as:

$$\overline{\boldsymbol{\omega}} = \begin{bmatrix} 0 & 0 & \omega \end{bmatrix}^T. \tag{8}$$

A position vector of the vibrating points in the global reference system are presented as follow:

$$\overline{\mathbf{r}} = \overline{\mathbf{r}}_{\mathbf{X}} + \overline{\mathbf{r}}_{\mathbf{Y}} = \overline{\mathbf{i}} \cdot r_X + \overline{\mathbf{j}} \cdot r_Y = \mathbf{Q} \cdot \left(\overline{\mathbf{S}} + \overline{\mathbf{w}} \right). \tag{9}$$

A linear velocity of the vibrating points in the global reference system:

$$\begin{aligned} \dot{\overline{\mathbf{r}}} &= \overline{\mathbf{v}} = \overline{\mathbf{v}}_{\mathbf{X}} + \overline{\mathbf{v}}_{\mathbf{Y}} = \overline{\mathbf{i}} \cdot v_X + \overline{\mathbf{j}} \cdot v_Y = \\ &= \mathbf{Q} \cdot \overline{\boldsymbol{\omega}} \times \left(\overline{\mathbf{S}} + \overline{\mathbf{w}} \right) + \mathbf{Q} \cdot \dot{\overline{\mathbf{w}}}. \end{aligned} \tag{10}$$

An acceleration of the vibrating points in the global reference system is calculated from relationship (10) with taking into consideration the Coriolis acceleration and the normal acceleration and the tangential acceleration, so the following equation is obtained:

$$\begin{aligned} \ddot{\overline{\mathbf{r}}} &= \dot{\overline{\mathbf{v}}} = \dot{\overline{\mathbf{v}}}_{\mathbf{X}} + \dot{\overline{\mathbf{v}}}_{\mathbf{Y}} = \overline{\mathbf{i}} \cdot \dot{v}_X + \overline{\mathbf{j}} \cdot \dot{v}_Y = \\ &= \mathbf{Q} \cdot \dot{\overline{\boldsymbol{\omega}}} \times \left(\overline{\mathbf{S}} + \overline{\mathbf{w}} \right) + \mathbf{Q} \cdot \overline{\boldsymbol{\omega}} \times \overline{\boldsymbol{\omega}} \times \left(\overline{\mathbf{S}} + \overline{\mathbf{w}} \right) + \\ &+ \mathbf{Q} \cdot \overline{\boldsymbol{\omega}} \times \dot{\overline{\mathbf{w}}} + \mathbf{Q} \cdot \overline{\boldsymbol{\omega}} \times \dot{\overline{\mathbf{w}}} + \mathbf{Q} \cdot \ddot{\overline{\mathbf{w}}} = \\ &= \mathbf{Q} \cdot \dot{\overline{\boldsymbol{\omega}}} \times \left(\overline{\mathbf{S}} + \overline{\mathbf{w}} \right) + \mathbf{Q} \cdot \overline{\boldsymbol{\omega}} \times \overline{\boldsymbol{\omega}} \times \left(\overline{\mathbf{S}} + \overline{\mathbf{w}} \right) + \\ &+ 2 \cdot \mathbf{Q} \cdot \overline{\boldsymbol{\omega}} \times \dot{\overline{\mathbf{w}}} + \mathbf{Q} \cdot \ddot{\overline{\mathbf{w}}}. \end{aligned} \tag{11}$$

The rotation matrix is used to described the orientation of the beam with respect to the global reference frame and the matrix from the rotation round the Z axis of the global reference system is as follow:

$$\mathbf{Q} = \begin{bmatrix} \cos\varphi & -\sin\varphi & 0 \\ \sin\varphi & \cos\varphi & 0 \\ 0 & 0 & 1 \end{bmatrix}, \tag{12}$$

the rotation matrix of second link with respect to global reference system:

$$\begin{aligned} &\mathbf{Q}_{02} = \mathbf{Q}_{01} \cdot \mathbf{Q}_{12}, \\ &\mathbf{Q}_{02} = \begin{bmatrix} \cos\varphi_1 \cdot \cos\varphi_2 - \sin\varphi_1 \cdot \sin\varphi_2 & -\cos\varphi_1 \cdot \sin\varphi_2 - \sin\varphi_1 \cdot \cos\varphi_2 & 0 \\ \sin\varphi_1 \cdot \cos\varphi_2 + \cos\varphi_1 \cdot \sin\varphi_2 & \cos\varphi_1 \cdot \cos\varphi_2 - \sin\varphi_1 \cdot \sin\varphi_2 & 0 \\ 0 & 0 & 1 \end{bmatrix}. \end{aligned} \tag{13}$$

3.1. Kinetic energy of the beam

Kinetic energy based on the Koenig's law is defined as a function of generalized coordinates and generalized velocities and can be written in the form:

$$\begin{aligned} T &= \frac{1}{2} \cdot M \cdot \left[\overline{\boldsymbol{\omega}} \times \mathbf{Q} \cdot \left(\overline{\mathbf{S}} + \overline{\mathbf{w}} \right) \right]^T \cdot \left[\overline{\boldsymbol{\omega}} \times \mathbf{Q} \cdot \left(\overline{\mathbf{S}} + \overline{\mathbf{w}} \right) \right] \\ &+ \frac{1}{2} \cdot M \cdot \left(\mathbf{Q} \cdot \dot{\overline{\mathbf{w}}} \right)^2 = \frac{1}{2} \cdot M \cdot \left(\overline{\mathbf{i}} \cdot \dot{r}_X \right)^2 + \\ &+ \frac{1}{2} \cdot M \cdot \left(\overline{\mathbf{j}} \cdot \dot{r}_Y \right)^2 = \frac{1}{2} \cdot M \cdot \dot{r}_X^2 + \frac{1}{2} \cdot M \cdot \dot{r}_Y^2, \end{aligned} \tag{14}$$

where **i, j** are individual versors in the global reference system.

3.2. Equations of motion of the beam

In papers [2,3,5] there were derived the equations of motion of the beam by using the Lagrange's equations. Equations were presented as projections into axes of the global reference frame. The X axis projection of the global reference system is:

$$\frac{\partial^2 w_X}{\partial t^2} + \frac{E \cdot I_Z}{\rho \cdot A} \cdot \frac{\partial^4 w_X}{\partial x^4} = -\omega^2 \cdot \left(s_X - w_X \right) + 2 \cdot \omega \cdot \frac{\partial w_Y}{\partial t}. \tag{15}$$

and the Y axis of the global reference system projection is as follow:

$$\frac{\partial^2 w_Y}{\partial t^2} + \frac{E \cdot I_Z}{\rho \cdot A} \cdot \frac{\partial^4 w_Y}{\partial x^4} = -\omega^2 \cdot \left(s_Y - w_Y \right) - 2 \cdot \omega \cdot \frac{\partial w_X}{\partial t}. \tag{16}$$

3.3. Forms of vibrations of the fixed beam

There are presented first three forms of vibrations for the beam fixed in the origin of the global reference frame in a (Fig. 5). First form of vibrations was marked by a red line with only one node in place where the beam is fixed, the second one by a green line with two nodes and the third one by a blue line with three nodes.

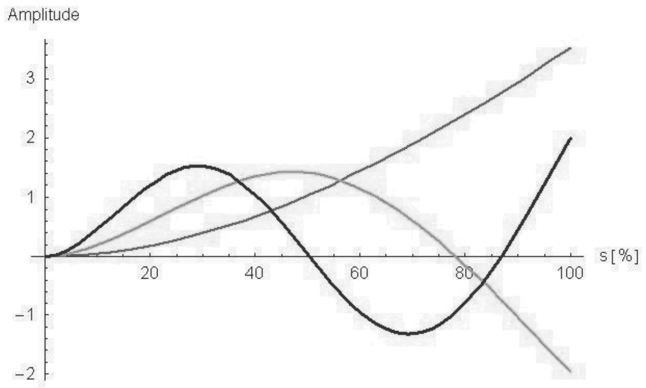

Fig. 5. The juxtaposition of first three forms of vibrations for the beam fixed in the origin of the global reference system

3.4. Forms of vibration of the free beam

In (Fig. 6) there are presented three forms of vibrations for the free beam loaded by harmonic transversal force. The Figure does not contain first form for zero natural vibration frequency that is a straight line.

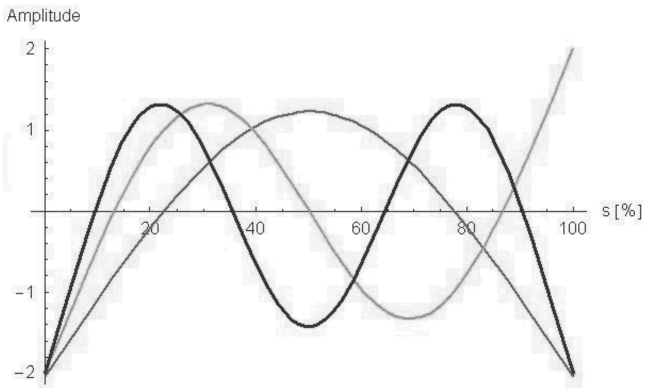

Fig. 6. The juxtaposition of three forms of vibrations for the free beam

First form of vibrations was marked by a red line with two nodes, the second form of vibrations by a green line with three nodes and the third form of vibrations by a blue line with four nodes.

3.5. Equations of motion of the two-linked manipulator vibrating transversally

Equations of motion of the two-linked manipulator are presented as the system of equations, there are equations tied with first beam and second beam, projected into axes of the global reference system. In the derived mathematical model equations are not coupled each other.

The projection into the X axis of the global reference system is as follow:

$$\begin{cases} \dfrac{\partial^2 w_{1X}}{\partial t^2} + \dfrac{E_1 \cdot I_{Z1}}{\rho_1 \cdot A_1} \cdot \dfrac{\partial^4 w_{1X}}{\partial x_1^4} = \\ = -\omega_1^2 \cdot \left(s_{1X} - w_{1X} \right) + 2 \cdot \omega_1 \cdot \dfrac{\partial w_{1Y}}{\partial t}, \\ \dfrac{\partial^2 w_{2X}}{\partial t^2} + \dfrac{E_2 \cdot I_{Z2}}{\rho_2 \cdot A_2} \cdot \dfrac{\partial^4 w_{2X}}{\partial x_2^4} = \\ = -\left(\omega_1 + \omega_2 \right)^2 \cdot \left(s_{2X} - w_{2X} \right) + 2 \cdot \left(\omega_1 + \omega_2 \right) \cdot \dfrac{\partial w_{2Y}}{\partial t}. \end{cases} \tag{17}$$

The projection into the X axis of the global reference system:

$$\begin{cases} \dfrac{\partial^2 w_{1Y}}{\partial t^2} + \dfrac{E_1 \cdot I_{Z1}}{\rho_1 \cdot A_1} \cdot \dfrac{\partial^4 w_{1Y}}{\partial x_1^4} = \\ = -\omega_1^2 \cdot \left(s_{1Y} - w_{1Y} \right) - 2 \cdot \omega_1 \cdot \dfrac{\partial w_{1X}}{\partial t}, \\ \dfrac{\partial^2 w_{2Y}}{\partial t^2} + \dfrac{E_2 \cdot I_{Z2}}{\rho_2 \cdot A_2} \cdot \dfrac{\partial^4 w_{2Y}}{\partial x_2^4} = \\ = -\left(\omega_1 + \omega_2 \right)^2 \cdot \left(s_{2Y} - w_{2Y} \right) - 2 \cdot \left(\omega_1 + \omega_2 \right) \cdot \dfrac{\partial w_{2X}}{\partial t}. \end{cases} \tag{18}$$

where:

$$\begin{aligned} \overline{\mathbf{w}}_{\mathbf{1XY}} &= \overline{\mathbf{w}}_{\mathbf{1X}} + \overline{\mathbf{w}}_{\mathbf{1Y}} = \overline{\mathbf{i}} \cdot w_{1X} + \overline{\mathbf{j}} \cdot w_{1Y}, \\ \overline{\mathbf{w}}_{\mathbf{2XY}} &= \overline{\mathbf{w}}_{\mathbf{2X}} + \overline{\mathbf{w}}_{\mathbf{2Y}} = \overline{\mathbf{i}} \cdot w_{2X} + \overline{\mathbf{j}} \cdot w_{2Y}. \end{aligned} \tag{19}$$

3.6. Equations of motion of the three-linked manipulator

The mathematical model of the three-linked manipulator in form of the equations of motion is presented as system of equations. The equations are not coupled each other and first equation into X and Y projection is appropriate to first link and analogically the rest. The projection into the X axis of the global reference system (20) and the projection into the Y axis of the global reference system (21) are as follow:

$$\begin{cases} \dfrac{\partial^2 w_{1X}}{\partial t^2} + \dfrac{E_1 \cdot I_{Z1}}{\rho_1 \cdot A_1} \cdot \dfrac{\partial^4 w_{1X}}{\partial x_1^4} = \\ = -\omega_1^2 \cdot (s_{1X} - w_{1X}) + 2 \cdot \omega_1 \cdot \dfrac{\partial w_{1Y}}{\partial t}, \\ \dfrac{\partial^2 w_{2X}}{\partial t^2} + \dfrac{E_2 \cdot I_{Z2}}{\rho_2 \cdot A_2} \cdot \dfrac{\partial^4 w_{2X}}{\partial x_2^4} = \\ = -(\omega_1 + \omega_2)^2 \cdot (s_{2X} - w_{2X}) + 2 \cdot (\omega_1 + \omega_2) \cdot \dfrac{\partial w_{2Y}}{\partial t}, \\ \dfrac{\partial^2 w_{3X}}{\partial t^2} + \dfrac{E_3 \cdot I_{Z3}}{\rho_3 \cdot A_3} \cdot \dfrac{\partial^4 w_{3X}}{\partial x_3^4} = \\ = -(\omega_1 + \omega_2 + \omega_3)^2 \cdot (s_{3X} - w_{3X}) + \\ +2 \cdot (\omega_1 + \omega_2 + \omega_3) \cdot \dfrac{\partial w_{3Y}}{\partial t}. \end{cases} \quad (20)$$

$$\begin{cases} \dfrac{\partial^2 w_{1Y}}{\partial t^2} + \dfrac{E_1 \cdot I_{Z1}}{\rho_1 \cdot A_1} \cdot \dfrac{\partial^4 w_{1Y}}{\partial x_1^4} = \\ = -\omega_1^2 \cdot (s_{1Y} - w_{1Y}) - 2 \cdot \omega_1 \cdot \dfrac{\partial w_{1X}}{\partial t}, \\ \dfrac{\partial^2 w_{2Y}}{\partial t^2} + \dfrac{E_2 \cdot I_{Z2}}{\rho_2 \cdot A_2} \cdot \dfrac{\partial^4 w_{2Y}}{\partial x_2^4} = \\ = -(\omega_1 + \omega_2)^2 \cdot (s_{2Y} - w_{2Y}) - 2 \cdot (\omega_1 + \omega_2) \cdot \dfrac{\partial w_{2X}}{\partial t}, \\ \dfrac{\partial^2 w_{3Y}}{\partial t^2} + \dfrac{E_3 \cdot I_{Z3}}{\rho_3 \cdot A_3} \cdot \dfrac{\partial^4 w_{3Y}}{\partial x_3^4} = \\ = -(\omega_1 + \omega_2 + \omega_3)^2 \cdot (s_{3Y} - w_{3Y}) + \\ -2 \cdot (\omega_1 + \omega_2 + \omega_3) \cdot \dfrac{\partial w_{3X}}{\partial t}. \end{cases} \quad (21)$$

where:

$$\begin{aligned} \overline{\mathbf{w}}_{1XY} &= \overline{\mathbf{w}}_{1X} + \overline{\mathbf{w}}_{1Y} = \overline{\mathbf{i}} \cdot w_{1X} + \overline{\mathbf{j}} \cdot w_{1Y}, \\ \overline{\mathbf{w}}_{2XY} &= \overline{\mathbf{w}}_{2X} + \overline{\mathbf{w}}_{2Y} = \overline{\mathbf{i}} \cdot w_{2X} + \overline{\mathbf{j}} \cdot w_{2Y}, \\ \overline{\mathbf{w}}_{3XY} &= \overline{\mathbf{w}}_{3X} + \overline{\mathbf{w}}_{3Y} = \overline{\mathbf{i}} \cdot w_{3X} + \overline{\mathbf{j}} \cdot w_{3Y}. \end{aligned} \quad (22)$$

and

$$\begin{aligned} & w_{1X} = w_1 \cdot \cos\left(\varphi_1 - \frac{\pi}{2}\right) = w_1 \cdot \sin\varphi_1, \\ & w_{1Y} = w_1 \cdot \sin\left(\varphi_1 - \frac{\pi}{2}\right) = -w_1 \cdot \cos\varphi_1, \\ & w_{2X} = w_2 \cdot \cos\left(\varphi_1 + \varphi_2 - \frac{\pi}{2}\right) = w_2 \cdot \sin(\varphi_1 + \varphi_2), \\ & w_{2Y} = w_2 \cdot \sin\left(\varphi_1 + \varphi_2 - \frac{\pi}{2}\right) = -w_2 \cdot \cos(\varphi_1 + \varphi_2), \\ & w_{3X} = w_3 \cdot \cos\left(\varphi_1 + \varphi_2 + \varphi_3 - \frac{\pi}{2}\right) = \\ & = w_3 \cdot \sin(\varphi_1 + \varphi_2 + \varphi_3), \\ & w_{3Y} = w_3 \cdot \sin\left(\varphi_1 + \varphi_2 + \varphi_3 - \frac{\pi}{2}\right) = \\ & = -w_3 \cdot \cos(\varphi_1 + \varphi_2 + \varphi_3), \end{aligned} \quad (23)$$

3.7. Numerical examples of the dynamical flexibility of the free beam

There was assumed the description of the displacement function as the product of eigenfunctions series for the X axis *X(x)* of global reference frame with an amplitude A_X as follows:

$$w_X = \sum_{n=0}^{\infty} A_X \cdot X(x) \cdot \sin(\Omega t), \quad (24)$$

where Ω is a harmonic frequency and for the Y axis of global reference frame with an amplitude A_Y:

$$w_Y = \sum_{n=0}^{\infty} A_Y \cdot X(x) \cdot \cos(\Omega t), \quad (25)$$

where up to the boundary conditions n are forms of vibrations:

$$k = (2 \cdot n + 1) \cdot \frac{\pi}{2 \cdot l}, \quad n = 1, 2, 3, \ldots \quad (26)$$

Since the free beam has the additional natural frequency that equals zero we can assumed that:

$$k = 0, \quad n = 0. \quad (27)$$

Numerical examples are presented as a dynamical characteristic as dynamical flexibility. The dynamical flexibility of the free beam in transportation (Fig. 7) is as follow:

$$Y=\frac{-X(l)\cdot X\left(x_{k=0}\right)\cdot\left(\Omega^{2}+\omega^{2}\right)}{\rho\cdot A\cdot\gamma_{n}^{2}\cdot\left(\Omega^{2}-\omega^{2}\right)^{2}}+\sum_{n=1}^{\infty}\frac{-\left[c^{2}\cdot\left(\frac{2\cdot n+1}{2\cdot l}\cdot\pi\right)^{4}-\Omega^{2}-\omega^{2}\right]\cdot X(l)\cdot X(x)}{\rho\cdot A\cdot\gamma_{n}^{2}\cdot\left[\left(c^{2}\cdot\left(\frac{2\cdot n+1}{2\cdot l}\cdot\pi\right)^{4}-\Omega^{2}-\omega^{2}\right)^{2}-4\cdot\omega^{2}\cdot\Omega^{2}\right]}. \tag{28}$$

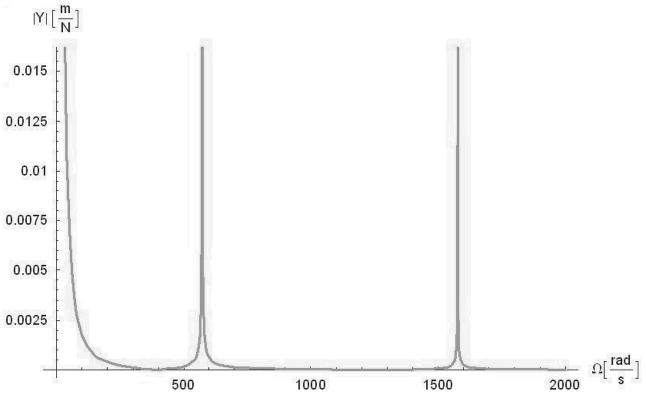

Fig. 7. Dynamical flexibility of the free beam vibrating longitudinally in transportation

3.8. Numerical examples of the dynamical flexibility of the fixed beam

There was assumed the description of the displacement function as the product of eigenfunctions series for the X axis of global reference frame as follows:

$$w_{X}=\sum_{n=1}^{\infty}A_{X}\cdot X(x)\cdot\sin(\Omega t), \tag{29}$$

and for the Y axis of global reference frame:

$$w_{Y}=\sum_{n=1}^{\infty}A_{Y}\cdot X(x)\cdot\cos(\Omega t), \tag{30}$$

where:

$$k=(2\cdot n-1)\cdot\frac{\pi}{2\cdot l},\quad n=1,2,3,\ldots \tag{31}$$

The dynamical flexibility of the beam fixed in the origin of the global reference system (Fig. 8) is as follow:

$$Y=\sum_{n=1}^{\infty}\frac{\left[c^{2}\cdot\left(\frac{2\cdot n-1}{2}\cdot\frac{\pi}{l}\right)^{4}-\Omega^{2}-\omega^{2}\right]\cdot X(l)\cdot X(x)}{\rho\cdot A\cdot\gamma_{n}^{2}\cdot\left\{\left[c^{2}\cdot\left(\frac{2\cdot n-1}{2}\cdot\frac{\pi}{l}\right)^{4}-\Omega^{2}-\omega^{2}\right]^{2}-4\cdot\omega^{2}\cdot\Omega^{2}\right\}}. \tag{32}$$

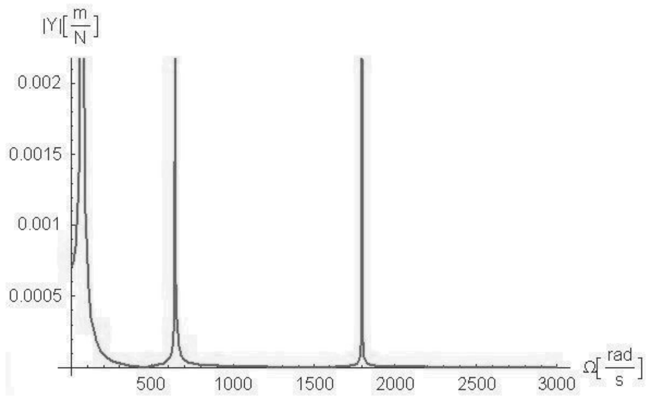

Fig. 8. Numerical calculation of the dynamical flexibility of the fixed beam vibrating longitudinally in transportation

4. Conclusions

The transversally vibrating systems were analyzed in this thesis. The new way of modelling those type systems was presented. Existing of unbalanced forces tied with transportation in the mathematical model was given into consideration. Elements express interactions between local displacements and main motion was emphasized. There were given into account the Coriolis' force and the centrifugal force in the mathematical model. Analysis was made by projection of the forces components into the individual axes of the global inertial system. The thesis can be considered as the introduction to the dynamic analysis of simple systems and complex systems in plane motion. This plane motion is treated as transportation. The material of beams in numerical examples was the aluminum alloy and the length of the beam was assumed as equal one meter. Equations of motion were derived by the Lagrange's equations methods. The transversally vibrating systems in planar transportation modeled in this article can be put to use to derivation of the dynamical flexibility of these systems, we can also use those equations to derivation of the substitute dynamical flexibility of n-linked systems.

Results of analysis can be put to use into mechanical devices in transportation such as wind power plants, high speed turbines, rotors, manipulators and in aerodynamics issues, etc. Model should be modified and adopted to appropriate applications. Mechanical systems vibrating transversally in terms of two-dimensional motion were considered. Successive problems of dynamical analysis are the analysis of systems in spatial motion. Simultaneously systems loaded by longitudinal forces were analyzed. Further plans of analysis concern the analysis of dynamical characteristics, mainly the dynamical flexibility.

Acknowledgements

This work was supported by Polish Ministry of Scientific Research and Information Technology (Polish State Committee for Scientific Research) Grant No. 4 T07A 009 28.

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