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Design and examining sensitivity of machine driving systems with required frequency spectrum

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Analysis and modelling

ABSTRACT

Purpose: The paper describes the method of synthesis and sensitivity analysis of dynamics characteristics applicable for aiding the process of designing of machine driving systems.
Design/methodology/approach: In this work used method of polar graphs and their relationship with algebra of structural numbers, easy for programming.
Findings: Presented approach simplifies the process of selecting the dynamical parameters of machine drive systems in view of their dynamical characteristics.
Research limitations/implications: The scope of discussion is the synthesis and sensitivity of machine drive systems as discrete models of torsional vibrations.
Practical implications: High durability and reliability of drive systems is associated with proper setting of system parameters - inertial, elastical and damping. Proper setting of these parameters is made possible by applying synthesis and sensitivity techniques.
Originality/value: We should emphasize that the considered problem varies from other issues met in classic mechanics or control theory. The research has been undertaken on the basis of topological methods, developed in scholar environment of Gliwice.
Keywords: Constructional design; Dynamic flexibility; Polar graphs; Structural numbers

1. Introduction

The studies on vibrations constitute an essential branch of the dynamics of technical systems, including machines, mechanisms and equipment. The analysis of the vibrations of mechanical systems are an important issue at the design stage, as well as at the stage of adjusting already operating machines to the requirements of manufacturing processes.

Nowadays, machines are faced with increasing requirements as to their manufacturing properties, durability, energy-efficiency and safety, not to mention their hushed and steady operation mode. One of the most fundamental criteria in the design of modern mechanical structures are their dynamical properties, as they have a direct impact on the vibrations of the system, noise emission, fatigue resistance, controllability and stability.

Drive systems are basic elements of every machine. Technological advancement forced drive designers to provide a high level of durability and reliability under operating conditions. Therefore, potential issues that may cause disturbances in the operation of machines, leading to drastic deterioration of work conditions, need to be addressed as early as at the design stage. Solutions offered by classis static methods are insufficient. It is the development of algorithms of modeling, analysis and synthesis of vibrating systems that may guarantee the achievement of the design task. The issue of modeling dynamical systems has been a subject of interest in multiple research institutes. This interest was stimulated by the advancement of computer-aided simulation techniques. Simulation tests that are part of the concepts of theoretical research analyze the impact of selected parameters on the mechanical characteristics. They guarantee apparently scientific results, however, burdened by a certain insufficiency, which is the idealization of models in respect of real subjects of tests. Despite such deficiency, simulation tests have developed rapidly, partly due to lower costs of such research mode in comparison with experimental tests. At the same time, simulation tests enable the analyses of emergency failure states and shorten the design time. Accordingly, if an appropriate machine model is available, it is possible to consider many factors influencing the technical parameters of a machine at the design stage; whereas the quality of the model available to the designer, understood as the precision of representation of essential phenomena occurring in actually existing systems, exerts a major impact on the accuracy of decisions concerning the solution of optimal solutions, or suitable shape of construction, undertaken at the design stage.

The selection of the dynamical properties of machines is one of the methods enhancing their durability and reliability. Such task may be accomplished with the use of the analysis [17-20,24] and synthesis algorithm [3-16]. The scope of this paper is a method of selecting the dynamical parameters of machine drive systems on the grounds of the synthesis algorithm.

The determination of such structure of the system and its parameters that meets the requirements concerning the assumed dynamical phenomena is a task inverse to analysis, therefore, it is a synthesis. Such task may be regarded as a support of the stage of designing mechanical systems, where an essential element is the fulfillment of the required dynamical properties. These properties may be represented in a graphic or analytical form, or in a form of sequential zeros and poles, which shall be considered in the paper.

Modern information technologies enable the analysis and synthesis of mechanical systems on the grounds of algebraic methods, easy for programming. Such methods (apart from rigid final elements methods and boundary elements methods, on which the following packets are based: ADAMS, DADS, MESA, VERDE, SIMPAC, MEDYNA, COSMOS, BEASY, PRO-MES, ANSYS, ALGOR) include network methods of graph bonds, polar graphs, flow graphs, hybrid graphs and hyper-graphs and structural numbers [1,2]. The applications of graphs and structural numbers developed in Gliwice research institute [3-19, 21-24] have inspired the subject of this paper.

The scope of discussion is the synthesis of machine drive systems as models of torsional vibrations. Such vibrations are more difficult to detect than flexural ones, which are accompanied by noise and vibrations of the adjacent elements (for example, shaft frames). Due to the absence of symptoms, torsional vibrations are particularly dangerous, as they may be unnoticeable until the destruction of subsystems occurs. Therefore, the determination of basic frequencies of the drive system free vibration is of crucial importance, as it makes it possible to avoid the operation under resonance spheres which may hinder the durability and proper functioning of a machine.

The introduction to the synthesis of machine drive systems treated as torsionally vibrating systems was discussed in [16], where the presented method of the synthesis of the characteristics distribution into a continued fraction enables the derivation of the parameters and models of a uniaxial system. It should be emphasized, however, that the derived model may have a discrete, discrete-continuous or continuous form. In view of the form of the derive equations of the models and easiness of numerical solutions, the discrete distribution models have a wider practical application than the continuous and discrete-continuous distribution ones.

The derived models constitute the bases for further verification of complex models, and a starting point for the optimization of the dynamical properties of drive systems. Sensitivity analysis is a preliminary stage of optimization research. On the grounds of its results it is possible to assess the impact of the parameters on the change of the dynamical characteristics of the system.

The selection of the model from the synthesized group depends on the operating conditions of a machine. The operation of the machine in the vicinity of the resonance state is possible only if the value of the system internal damping is sufficiently high, due to the highest strain. In such case, damping plays a decisive role, as it essentially reduces the amplitude of vibrations. Another way of improving the operation of a machine is a proper selection of the natural frequency of the system, or its excitation frequency. To secure the exit from the resonance zone is a basic operational condition of a machine, although, it does not completely eliminate the problem of vibrations. However, if the machine operates outside the resonance zone, the dynamical calculations may be sufficient for the systems and the effect of damping may be disregarded.

Accordingly, the issue of the synthesis, enabling the determination of the parameters and structure of the systems in view of their dynamical characteristics, may be applied as a tool supporting the design process under any operating conditions. At the same time, a big quantity of systems and parameters obtained as a result of the synthesis, meeting the same dynamical properties in the form of vibrations, may have a big impact on a rational selection of the parameters of the discussed machine. Concurrently, it should be noticed that the synthesis methods enable the derivation of the parameters and models of multiaxial machine drives.

2. Immobility synthesis by means of the continued fraction distribution method

The continued fraction method of the synthesis of the dynamic characteristic distribution makes it possible to obtain cascade structures of the discrete system. The investigation into the structures of cascade systems was conducted by means of the software method for distributing the dynamic characteristics to the continued fraction based on graphs and structural numbers. Thanks to the computer program, the dependencies between the structure and the parameters of the synthesised discrete mechanical systems are designated.

The scope of the paper is to discuss the method of synthesising dynamic characteristics $U(s)$ to a continued fraction, assuming the odd number of the synthesised system elements; in such case dynamic characteristics $U(s)$ is as follows:

$$U(s)=\frac{d_l s^l+d_{l-2}s^{l-2}+\ldots+d_1 s}{c_{l-1}s^{l-1}+c_{l-3}s^{l-3}+\ldots+c_0}, \tag{1}$$

after distributing it to a continued fraction form:

$$U(s)=U_s^{(1)}(s)+\cfrac{1}{V_s^{(1)}(s)+\cfrac{1}{U_s^{(2)}(s)+\cfrac{1}{V_s^{(2)}(s)+\cfrac{\vdots}{+\cfrac{1}{V_s^{(k-1)}(s)+\cfrac{1}{U_s^{(k)}(s)}}}}}}, \tag{2}$$

where: $U^{(1)}(s)=J_U^{(1)}s,\; V^{(1)}(s)=\frac{s}{c_V^{(1)}},\ldots,\; V^{(k-1)}(s)=\frac{s}{c_V^{(k-1)}}$,

$U^{(k)}(s)=J_U{}^{(k)}s$.

The graphic representation of formula (2) is the graph presented in Fig.1, which has been defined by Berge [1], whereas the notations have been derived from Wojnarowski's formulation [21, 22]. The discrete mechanical system the graph of which is presented in Fig.1 has the form presented in Fig.2.

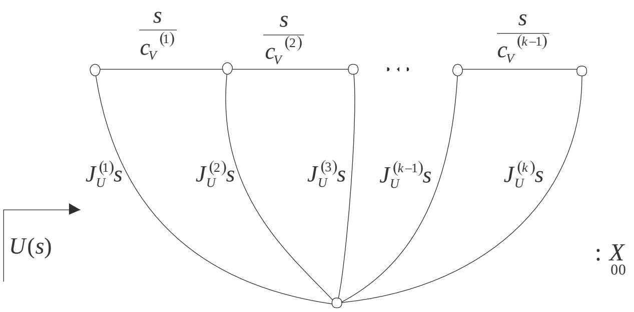

Fig. 1. Polar graph as an illustration of the implementation of equation (2)

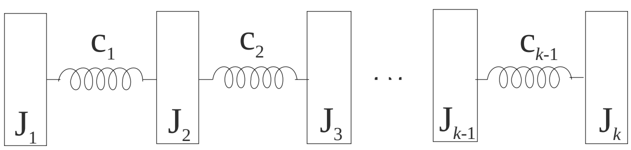

Fig. 2. Synthesised mechanical system

The synthesis of mechanical system with proportional damping to elastic parameters is

$$b_i=\lambda c_i, \tag{3}$$

where: b_i - damping parameter, c_i - elastic parameter, λ - proportionality factor. Value of parameter h has to be determination with:

$$\lambda=\frac{2h_n}{\omega_{bn}^2}\Rightarrow h_n=\frac{\omega_{bn}^2\cdot\lambda}{2}, \quad () \tag{4}$$

where: h_n - parameter answering for damping of system, having dimension of frequency , $\omega_{b1},\omega_{b2},\ldots,\omega_{bn}$ (2) resonance frequencies $(n=1,2,3,\ldots,k)$.

Having regard to the considered class of discrete systems with periodic movement (discrete vibrating systems with damping) it is necessary to determine the value of proportionality factor which should be selected from the interval:

$$0<\lambda<\frac{2}{\omega_n}, \tag{5}$$

where: $\omega_n\neq 0$ - max value of resonance frequencies or anti-resonance frequencies.

On the base of the relation (3÷5) it is possible to precise requested dynamic features which the searched system should meet.

1. To take values of resonant and anti-resonant frequencies in case of undamped vibration, i.e.:

$$\begin{cases}\omega_{b1},\omega_{b2},\ldots,\omega_{bn}-\text{resonance frequencies},\\ \omega_{z1},\omega_{z2},\ldots,\omega_{zn}-\text{anti-resonance frequencies}.\end{cases} \tag{6}$$

2. The value of proportionality factor λ has to be determination of Eq.5. The parameter h has to be determination of Eq.4:

$$h_n=\frac{\lambda\omega_{bn}^2}{2} \tag{7}$$

Graphical representation the synthesis in a form of a graph is shown in Fig. 3. The discrete mechanical system the graph of which is presented in Fig. 3 has the form presented in Fig. 4.

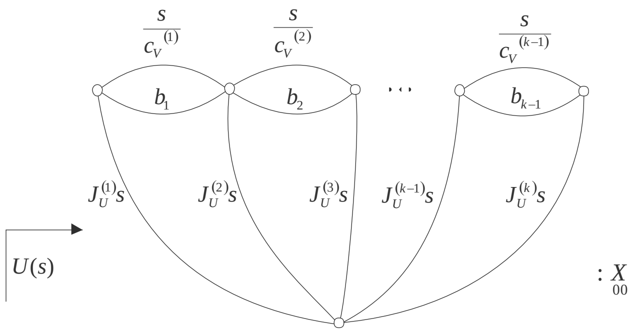

Fig. 3. Polar graph as an illustration of the synthesis of mechanical system with proportional damping to elastic parameters

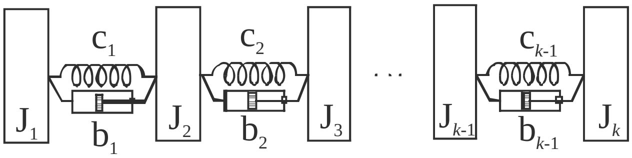

Fig. 4. Synthesised mechanical system

The method discussed in this chapter is used for synthesising both discrete and continuous systems [3 – 16]. Also, on the grounds of this method an attempt was made to synthesise of machine driving systems. In the course of the synthesis of the systems the values of the parameters are obtained. It can also be applied to the synthesis of machine driving systems if the results are transformed. Such transformation has a reciprocally unique character with regard to the structure, as well as to the values of the system elements. The synthesising of machine driving systems involves adopting the following transformation type:

$$\begin{cases} J_z = \sum_{i=1}^{r} J_i \left(\frac{\omega_i}{\omega_z} \right)^2, \\ c_z = c_r \left(\frac{\omega_r}{\omega_z} \right)^2, \\ b_z = b_r \left(\frac{\omega_r}{\omega_z} \right)^2, \end{cases} \tag{8}$$

where: J – polar inertial moment, c – elastic parameter, b – damping parameter, ω - velocity rotation, *z* – number of synthesis element , *r* – number of machine driving element.

The procedure for synthesising machine driving systems with torsionally structure is presented in Fig. 5.

In this paper, numerical examples of synthesized systems with constant segment intersection for *n* = 4 and a branched structure are provided, by the application of synthesizing program. The following requirements were subjected to synthesis:

- number of elements *n* = 5,
- furthermore, it was assumed that the resonance zones of the dynamical characteristic of the synthesized systems are in the neighborhood of poles, the values of which are: $\omega_0 = 0$ rad/s , $\omega_2 = 50$ rad/s , $\omega_4 = 100$ rad/s ;
- the anti-resonance zones of the dynamical characteristics of the synthesized systems are in the neighborhood of zeros, the values of the characteristic frequencies are:. $\omega_1 = 25$ rad/s , $\omega_3 = 75$ rad/s .

Such formulation of the conditions of resonance and anti-resonance zones imply the boundary conditions of the synthesised structures. It is inferred from the assumptions that the system in question is a free system vibrating torsionally. On the grounds of the input data, applying Synan's program [4, 8, 14] and Synteza's [8] program for designating inertial, elastic and damping elements of a discrete system, the following parameter values were derived, as well as the geometric representation of a discrete structure.

As the results of numerical calculations, the inertial, elastic and damping parameters are obtained (see tables 1).

Table 1.
Inertial, elastic and damping parameters of synthesized discrete system

i	$(c)_{(i)}$ [Nm/rac	$(b)_{(i)}$ [Nm/rad]	$(J)_{(i)}$ [kgm^2]
1			1.0E+00
1	6.31655E+03	3.05324E+01	
2z			2.22E+00
2z	4.91287E+03	1.93028E+01	
4z			3.89E+00

Applying the parameters of the discrete system (Table 1) it is possible to create two structures of a discrete system (see fig.5). The polar graphs of systems are presented in Fig.6, 7.

The obtained results should further be verified by both the theoretical and practical analysis. The values of parameters were verified by the same computer program. Numerical calculations of the theoretical analysis are illustrated in Fig.8.

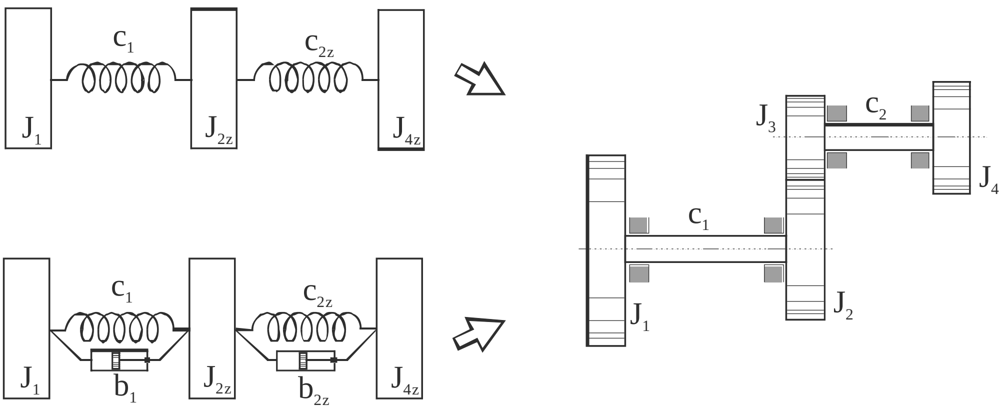

Fig. 5. Idea synthesis of machine driving systems

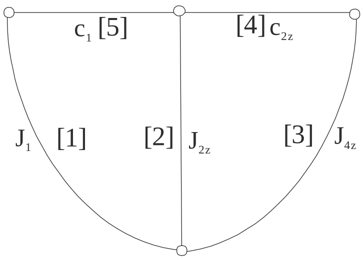

Fig. 6. Polar graph of vibration mechanical system without damping

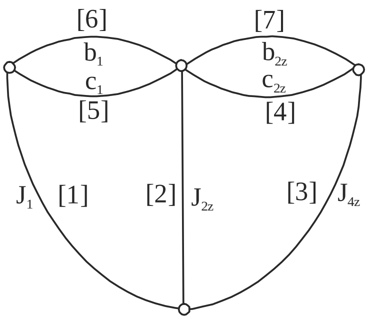

Fig. 7. Polar graph of vibration mechanical system with damping

3. The sensitivity of discrete systems represented by polar graph and structural numbers

The way of examining the sensitivity of obtained discrete system, with respect to values of received parameters, as results of synthesis-by means the graphs and structural numbers methods-have been presented. Problems of examining the sensitivity in polar graphs categories and the structural numbers in a regard to discrete structures has been introduced in [23].

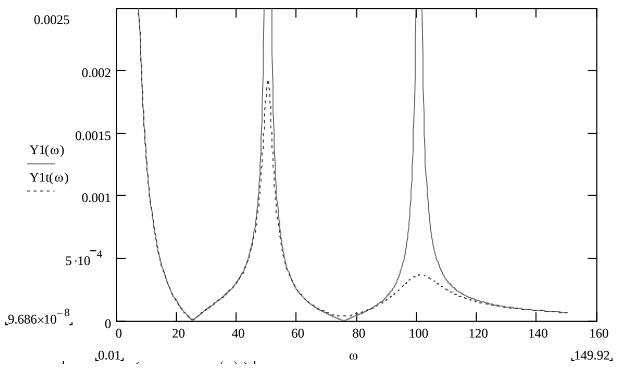

Fig. 8. FlexibilityY1 of system without damping and flexibility Y1t of system with damping

In this way the sensitive edge of graph as the model of mechanical system has been separated. The edge which includes variable parameter (inertial or elastic or damping) is called the sensitivity edge. Through the isolation from the graph of sensitivity edge the structural numbers of second category is:

$$ {}^2A = \begin{bmatrix} [a_r] & [\phi] \\ {}^2A_{a_r} & {}^2A^{a_r} \end{bmatrix} \tag{9} $$

${}^2A_{a_r} = \dfrac{\partial\, {}^2A}{\partial a_r}$ is the algebraic derivative of a structural number ${}^2A$ and when in the indication of the sensitive edge is concern, ${}^2A^{a_r} = \dfrac{\delta\, {}^2A}{\delta a_r}$ - is the anti derivative of an algebraic structural number ${}^2A$, when the sensitive edge considered.

The aim of isolation of the sensitive edge is to make a special transformation of a graph ${}^2X$. Such process is used to join the tops of an isolated, and to remove this edge from the graph. The mentioned structural number (9) stand for determinant function

$$ \underset{z}{\mathbf{det}}\, {}^2A = a_r\, \underset{z}{\mathbf{det}}\, {}^2A_{a_r} + \underset{z}{\mathbf{det}}\, {}^2A_{a_r} . \tag{10} $$

where: Z is a set of dynamical stiffness.

Taking into consideration the harmonic excitation, which affects on a k coordinate mass of the system, dynamical flexibility, of i mass can be obtained from the formula:

$$ Y_{ik}(p) = \frac{\underset{z}{Sim}\left({}^2A_{a_i}, {}^2A_{a_k}\right)}{\underset{z}{\mathbf{det}}\, {}^2A} . \tag{11} $$

where: $\underset{Z}{Sim}(\ldots)$ - is a simultaneous function of a derivative of a graph structural number:

$$ \begin{aligned} &\underset{Z}{Sim}\left({}^2A_{a_i}, {}^2A_{a_k}\right) = \\ &= a_r\, \underset{Z}{Sim}\left({}^2A_{a_r a_k}, {}^2A_{a_r a_i}\right) + \underset{Z}{Sim}\left({}^2A^{a_r}_{a_k}, {}^2A^{a_r}_{a_i}\right). \end{aligned} \tag{12} $$

Taking advantage of dependences (11) and (12) and definitions of sensitivities [23], sensitivity from dynamical flexibility Y_{ik}, according to the weight of sensitivity edge can be appointed in a following manner:

$$ \begin{aligned} S^{Y_{ik}(p)}_{a_r} &= a_r \left[\frac{\underset{Z}{Sim}\left({}^2A_{a_r a_k}, {}^2A_{a_r a_i}\right)}{\underset{Z}{Sim}\left({}^2A_{a_i}, {}^2A_{a_k}\right)} - \frac{\underset{Z}{\mathbf{det}}\, {}^2A_{a_r}}{\underset{Z}{\mathbf{det}}\, {}^2A} \right] = \\ &= \frac{\underset{Z}{\mathbf{det}}\, {}^2A^{a_r}}{\underset{Z}{\mathbf{det}}\, {}^2A} - \frac{\underset{Z}{Sim}\left({}^2A^{a_r}_{a_k}, {}^2A^{a_r}_{a_i}\right)}{\underset{Z}{Sim}\left({}^2A_{a_k}, {}^2A_{a_i}\right)} . \end{aligned} \tag{13} $$

In case of direct flexibility $Y_k(p)$ the formula number (13) can be written as:

$$S_{a_r}^{Y_k(p)} = a_r\left[\frac{\underset{Z}{\det}^2 A_{a_r a_k}}{\underset{Z}{\det}^2 A_{a_k}} - \frac{\underset{Z}{\det}^2 A_{a_r}}{\underset{Z}{\det}^2 A}\right] = \frac{\underset{Z}{\det}^2 A^{a_r}}{\underset{Z}{\det}^2 A} - \frac{\underset{Z}{\det}^2 A^{a_r}_{a_k}}{\underset{Z}{\det}^2 A_{a_k}}. \qquad (14)$$

The appointed functions of sensitivity (14) is equal to the parametrical function of sensitivity

$$S_{\alpha_r}^{Y_{ik}(p)} = S_{a_r}^{Y_{ik}(p)}, a_r = a_r(\alpha_r). \qquad (15)$$

where: α_r – is an inertial, elastic or damping parameter.

In this paper, it has been show how to nominate the sensitivity function in order to direct flexibility. The examination has been solved with a usage of graphs and their connections with structural numbers.

The sensitivity functions according to (14) for the edge of graphs, nominated as 1 it means the stiffness J_1 (see fig. 6 and 7) gets the shape of

$$S1 = S1t = [1]\left[\frac{\underset{z}{\det}\frac{\partial A}{\partial[1][1]}}{\underset{z}{\det}\frac{\partial A}{\partial[1]}} - \frac{\underset{z}{\det}\frac{\partial A}{\partial[1]}}{\underset{z}{\det} A}\right]. \qquad (16)$$

After following calculations, graphical representation of sensitivity function (16) is shown in fig. 9.

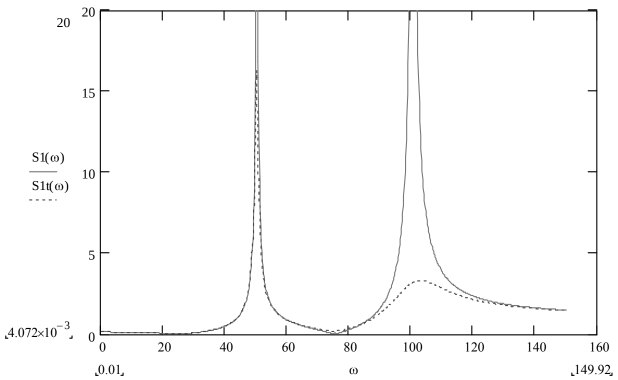

Fig. 9. Diagram of sensitivity function for J_1

The sensitivity functions according to (14) for the edge of graphs, nominated as 2 it means the stiffness J_{2z} gets the shape of

$$S2 = S2t = [2]\left[\frac{\underset{z}{\det}\frac{\partial A}{\partial[2][1]}}{\underset{z}{\det}\frac{\partial A}{\partial[1]}} - \frac{\underset{z}{\det}\frac{\partial A}{\partial[2]}}{\underset{z}{\det} A}\right]. \qquad (17)$$

After following calculations, graphical representation of sensitivity function (17) is shown in fig. 10.

The sensitivity functions according to (14) for the edge of graphs, nominated as 3 it means the stiffness J_{4z} gets the shape of

$$S3 = S3t = [3]\left[\frac{\underset{z}{\det}\frac{\partial A}{\partial[3][1]}}{\underset{z}{\det}\frac{\partial A}{\partial[1]}} - \frac{\underset{z}{\det}\frac{\partial A}{\partial[3]}}{\underset{z}{\det} A}\right]. \qquad (18)$$

After following calculations, graphical representation of sensitivity function (18) is shown in fig. 11.

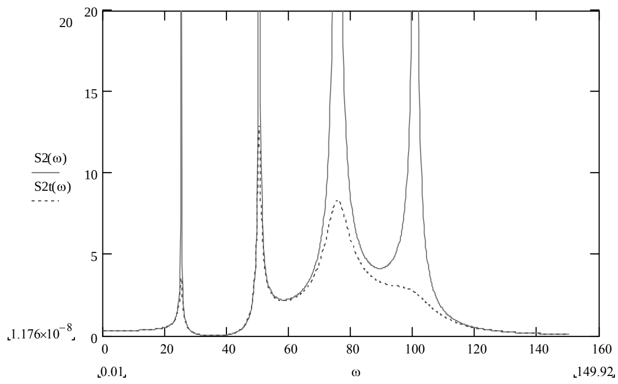

Fig. 10. Diagram of sensitivity function for J_{2z}

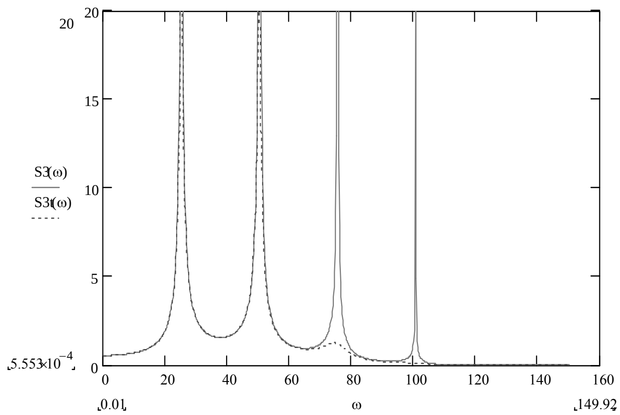

Fig. 11. Diagram of sensitivity function for J_{4z}

Analyzing the plot in Fig. 9-11 we can assume that the areas with the biggest values are covered with the resonance and anti-resonance sections of synthesized structure.

The sensitivity functions according to (14) for the edge of graphs, nominated as 4 it means the stiffness c_{2z} gets the shape of

$$S4 = S4t = [4]\left[\frac{\underset{z}{\det}\frac{\partial A}{\partial[4][1]}}{\underset{z}{\det}\frac{\partial A}{\partial[1]}} - \frac{\underset{z}{\det}\frac{\partial A}{\partial[4]}}{\underset{z}{\det} A}\right]. \qquad (19)$$

After following calculations, graphical representation of sensitivity function (19) is shown in fig. 12.

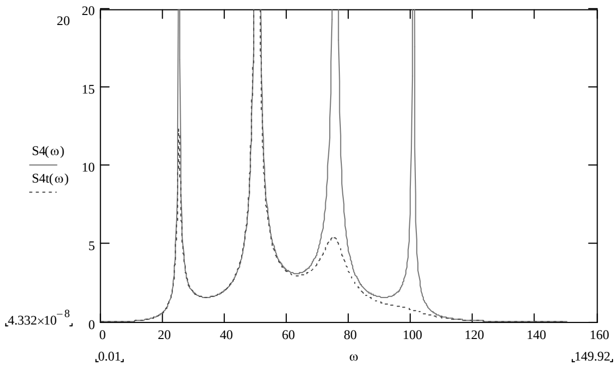

Fig. 12. Diagram of sensitivity function for c_{2z}

The sensitivity functions according to (14) for the edge of graphs, nominated as 5 it means the stiffness c_1 gets the shape of

$$S5 = S5t = [5]\left[\frac{\det\limits_{z} \frac{\partial A}{\partial [5][1]}}{\det\limits_{z} \frac{\partial A}{\partial [1]}} - \frac{\det\limits_{z} \frac{\partial A}{\partial [5]}}{\det\limits_{z} A}\right]. \quad (20)$$

After following calculations, graphical representation of sensitivity function (20) is shown in fig. 13.

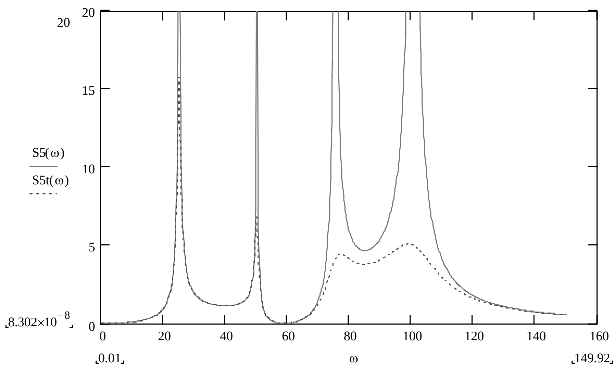

Fig. 13. Diagram of sensitivity function for c_1

Analyzing the plot in Fig. 12-13 we can assume that the sections of a highest value of a sensitivity function are covering with the strands of resonance and anti-resonance frequencies of a synthesis structure.

However, when we compare the plots, which were nominated in order to all elements (see fig.14 and 15) we can formulate following conclusions:

- the width of the resonance strand along the first frequency the biggest impact has the parameter J_{4z},
- comparing the width of an expanse of a sensitivity function along the second frequency it can be noticed that the parameter c_2 has a bigger influence on a width resonance section than the other parameters,
- the impact of all parameters on the width of the strand in order to the first and second frequency is comparable,
- observing the width of a section of a sensitivity function along the third frequency we can see that the more important parameter which has an influence on a width of a resonance strand along this frequency is stiffness J_2z,
- analyzing the width of the section of sensitivity function along which the fourth frequency we can notice that the most important parameter which has an impact on the width of this area has a parameter c_1, the other huge impact has the parameter J_1.

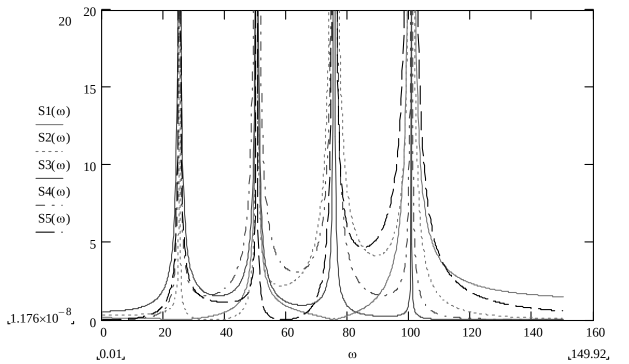

Fig. 14. The summary presentation of sensitivity function of examined mechanical system without damping

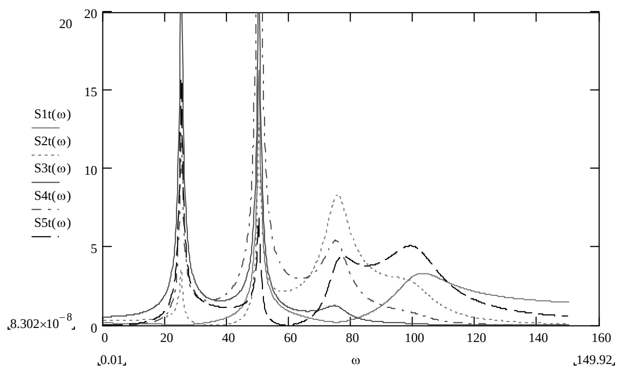

Fig. 15. The summary presentation of sensitivity function of examined mechanical system with damping

4. Conclusions

The greatest intensity of vibrations might be found in conditions of resonance. They can be avoided due to a proper selection of frequency free vibrations. Such task may be accomplished with the use of the synthesis algorithm.

Exiting the resonance zone is a crucial condition of the machine's work but it does not eliminate entirely the problem of vibrations. The free vibrations frequency appear in more than one machine. In these cases damping has a decisive role as it lowers the vibration amplitude in a serious way. That is why the paper concerns formulating and solving the problem of synthesis and sensitivity of machine driving systems with and without damping. The method of synthesising and examining sensitivity may influence considerably the research and selection of parameters of the analyzed machine. The derived parameters constitute the bases for further verification of complex models, and a starting point for the optimization.

Acknowledgements

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