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Dynamical flexibility of discrete-continuous vibrating mechatronic system

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Analysis and modelling

ABSTRACT

Purpose: The application of approximate method for solving the task of assignment the frequency-modal analysis and characteristics of flexibly vibrating mechatronic system, because for considered case of boundary conditions exact and approximate methods for the coordinates are equivalent.
Design/methodology/approach: Formulate and solve the problem in the form of a set of differential equations of motion and state equations of the considered mechatronic model of an object Galerkin's method was used. The considered flexibly vibrating mechanical system is a continuous beam, clamped at one of its ends. An integral part of the mechatronic system is a transducer perfectly bonded to the beam surface.
Findings: The parameters of the transducer exert an important influence on the values of natural frequencies and on the form of the characteristics of the discussed mechatronic system.
Research limitations/implications: The linear mechanical subsystem and linear electrical subsystem of the mechatronic system were analyzed and the theory Euler-Bernoulli is used for the beam; however, this approach is sufficient for such systems.
Practical implications: Global approach is presented in the domain of frequency spectrum analysis. The methods of analysis and the obtained results my give grounds for designing and investigating this type of mechatronic systems.
Originality/value: The mechatronic system created from mechanical and electric subsystems with electromechanical bondage has been considered. This approach is different from those considered so far.
Keywords: Applied mechanics; Beam; Piezotransducer; Galerkin's method; Dynamical characteristic

1. Introduction

The main interest of industry and scientists in the process of machine design is to consider energy conversion, efficiency and reliability. Many industrial branches focus on the problem of miniaturizing the existing systems and reducing their energy absorption. The crucial thing in this matter is to search for new solutions enabling the reduction of movable elements and complicated, long kinematical chains. Therefore, in the last few years specific development has occurred in the field of technologies based on piezoelectricity and electro and magnetostriction phenomena [5, 7, 8, 10-12, 14, 15].

The first attempt to solve this problem, involving the determination of the dynamical characteristics of a longitudinally and torsionally vibrating continuous bar system and various classes of discrete mechanical systems in view of the frequency spectrum, by means of graphs and structural numbers methods, and other diverse problems have been modelled, examined and analysed in Gliwice Research Centre in [1, 2, 13]. Torsionally vibrating mechatronic systems were discussed in [3, 4, 9]. The mechatronic system, clamped at one of its ends, was analyzed in [5]. The system was induced by harmonic electrical voltage from the electrical side attached to the converter clips.

This paper is focused on the dynamical characteristics of a flexibly vibrating continuous mechanical system combined with a piezoelectric converter into a mechatronic system, to examine the piezoelectric influence on the whole complex system. This kind of approach may provide grounds for examining flexibly vibrating mechatronic systems to generate vibrations with the assumed parameters.

2. Mechatronic system induced by harmonic force

The subject of deliberation is a homogeneous beam with a full section of area A and with area moment of inertia I, unchanged along its length l (Fig. 1). The beam was made of a material in consideration of Young Modulus E and mass density ρ. The beam was loaded with harmonic force. The piezoelectric converter was attached in an ideal way to the beam surface.

The mechatronic system analyzed in this paper was treated with bending stress. Therefore, the model of the piezoelectric converter was regarded as a bending actuator. The essential equation of the piezoelectric with changes in its stiffness under the influence of the electrical pole is given as [5, 7, 10-12, 14-15]:

$$\sigma_{11}=C_{11}\varepsilon_{11}-d_{31}\frac{U}{h_p}=C_{11}\frac{h_b}{2}\frac{\partial^2 y(x,t)}{\partial x^2}-U\frac{d_{31}}{h_p}, \tag{1}$$

where: C_{11} - elastic module of piezoelectric converter measured with certain value of excited voltage, d_{31} - piezoelectric converter constant.

The moment evoked by transverse electrical loading is described as follows [5, 15]:

$$M=\frac{\sigma_{11}I_a}{\frac{h_b}{2}+\frac{h_p}{2}} \tag{2}$$

The area moment of inertia of the piezoelectric converter after simplifications equals

$$I_a=\frac{b_p h_p}{2}\left(\frac{h_b}{2}+\frac{h_p}{2}\right)^2. \tag{3}$$

where: b_p - width of the beam section, h_b - depth of the beam section, h_b - depth of the piezoelectric converter section.

By replacing equations (1) and (3) with (2), and after some more transformations, the following form is derived

$$M=\left(C_{11}\frac{h_b}{2}\frac{\partial^2 y(x,t)}{\partial x^2}-U\frac{d_{31}}{h_p}\right)\cdot\frac{b_p h_p}{2}\left(\frac{h_b}{2}+\frac{h_p}{2}\right). \tag{4}$$

The inducement of the moment is combined with the piezoelectric converter and determined within the range of $x\in(x_1,x_2)$. By inserting Heaviside's function to the equation, the impact of the flexible moment is eliminated outside the range, leading to the following expression

$$M=\left(C_{11}\frac{h_b}{2}\frac{\partial^2 y(x,t)}{\partial x^2}-U\frac{d_{31}}{h_p}\right)\cdot\frac{b_p h_p}{2}\left(\frac{h_b}{2}+\frac{h_p}{2}\right)[H(x-x_2)-H(x-x_1)]. \tag{5}$$

In the case of the mechatronic system (Fig. 1), the equation of the flexibly vibrating beam is given as

$$\rho A\frac{\partial^2 y(x,t)}{\partial t^2}+EI\frac{\partial^4 y(x,t)}{\partial x^4}=\frac{\partial^2 M}{\partial x^2} \tag{6}$$

and, after inserting equation (6) assumes the following form

$$\frac{\partial^2 y}{\partial t^2}=-a^2\frac{\partial^4 y}{\partial x^4}+c\frac{\partial^4 y}{\partial x^4}H(\cdot)-dU\delta'(\cdot), \tag{7}$$

where: $a=\sqrt{\frac{EI}{\rho A}}$, $c=\frac{C_{11}h_p^2 b_p}{4\rho A}\left(\frac{h_b}{2}+\frac{h_p}{2}\right)$, $d=\frac{d_{31}}{\rho A h_p}\left(\frac{h_b}{2}+\frac{h_p}{2}\right)$, $H(\cdot)=[H(x-x_2)-H(x-x_1)]$ - Heaviside's function, $\delta(\cdot)$ - Dirac's function $\delta'(\cdot)=\frac{d\delta(\cdot)}{dx}=[\delta'(x-x_2)-\delta'(x-x_1)]$ - Dirac's function derivative.

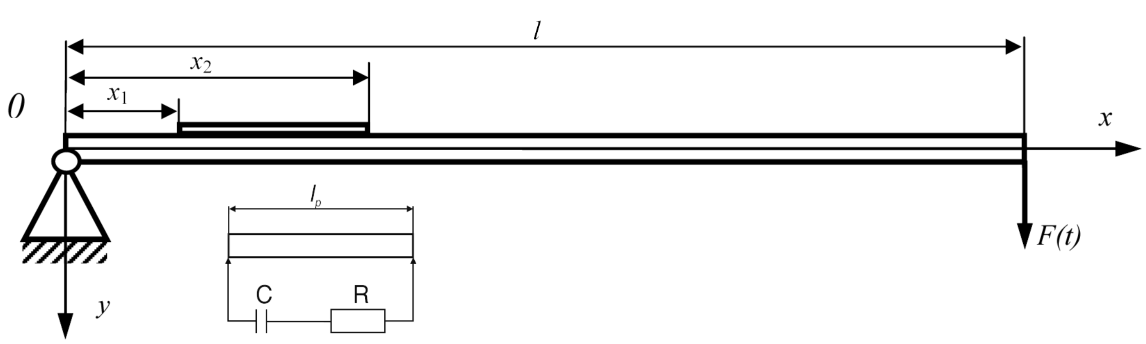

Fig. 1. Mechatronic system with mechanical excitation

The equation of the piezoelectric converter is given as

$$\frac{dU}{dt}+\frac{1}{RC}U=-\alpha\frac{\partial\dot{y}}{\partial x}(l_p,t)\,, \tag{8}$$

where: $\alpha=\frac{1}{2}d_{31}C_{11}b_ph_p$.

Thus, the considered mechatronic system (Fig. 1) is described by the following set of equations

$$\begin{cases}\dfrac{\partial^2 y}{\partial t^2}=-a^2\dfrac{\partial^4 y}{\partial x^4}+c\dfrac{\partial^4 y}{\partial x^4}H(\cdot)-dU\delta'(\cdot),\\[2mm] \dfrac{dU}{dt}+\dfrac{1}{RC}U=-\alpha\dfrac{\partial\dot{y}}{\partial x}(l_p,t).\end{cases} \tag{9}$$

Solution (9) must comply with the following boundary conditions

$$\begin{cases}y(0,t)=0,\;\; X(0)T(t)=0\rightarrow X(0)=0,\\[1mm] \dfrac{\partial y^2(0,t)}{\partial x^2}=0,\; X''(0)T(t)=0\rightarrow X''(0)=0,\\[1mm] \dfrac{\partial y^2(l,t)}{\partial x^2}=0,\; X''(l)T(t)=0\rightarrow X''(l)=0,\\[1mm] EI\dfrac{\partial y^3(l,t)}{\partial x^3}=F(t),\; EI\,X'''(l)T(t)=F(t),\end{cases} \tag{10}$$

where: $F(t)=F_0e^{i\omega t}$.

When the excitation has a harmonic shape, the voltage generated on the piezoelectric clips has the same nature, leading to

$$U=Be^{i\left(\omega t-\frac{\pi}{2}\right)}\;. \tag{11}$$

3. Determination of dynamical flexibilities of a vibrating mechatronic system

The boundary problem of vibrating mechanical subsystem without any excitation, that means $F(t)=0$, is following

$$\begin{cases}X^{(IV)}(x)-k^4X(x)=0,\\ X(0)=0,\, X''(0)=0,\, X''(l)=0,\, X'''(l)=0.\end{cases} \tag{12}$$

As it is known the solution of (12) takes form

$\operatorname{tg} z=\operatorname{tgh} z\,,\;\; z=kl\,,\;\; k\cong(4n+1)\dfrac{\pi}{4l}\,.$

This considered beam has natural frequency equals zero, for k = 0, then own function takes form

$$X_0(x)=Cx\,, \tag{13}$$

and the the beam arounds the joint “0”.

The own functions for $k\neq 0$ following

$$X_n(x)=A_n\left(\sin\frac{z_n}{l}x+\frac{\sin z_n}{\sinh z_n}\sinh\frac{z_n}{l}x\right). \tag{14}$$

The functions (14) fulfill boundary conditions for the coordinates for $x=0$ and $x=l$ and then the solution of the set of equations (9) of considered mechatronic system (Fig. 1) can obtain using approximate method referred to as: Galerkin’s method. According the method, the solution is searched in the form of the function sums, which are the function of the time and the coordinates

$$y(x,t)=\sum_{n=1}^{\infty}y_n(x,t)=\sum_{n=1}^{\infty}A_n\sin kx\cdot e^{i\omega t}\,, \tag{15}$$

where: $\omega\cong\sqrt{\dfrac{EI}{\rho A}}\left((4n+1)\dfrac{\pi}{4l}\right)^2$.

3.1. The dynamical flexibility for the first, second and third vibration mode

When n=1 i.e. for the first vibration mode, deflection (15) takes the following form

$$y_1(x,t)=A_1\sin\frac{5\pi x}{4l}e^{i\omega t}\,. \tag{16}$$

The solution of the examined set of differential equations (9) is obtained by substituting the adequate derivatives, as follows

$$\begin{cases}\dfrac{dy_n}{dt}=A_ni\omega\sin\dfrac{5\pi x}{4l}e^{i\omega t},\;\dfrac{d^2y_n}{dt^2}=A_n(i\omega)^2\sin\dfrac{5\pi x}{4l}e^{i\omega t},\\[2mm] \dfrac{dy_n}{dx}=A_n\dfrac{5\pi}{4l}\cos\dfrac{5\pi x}{4l}e^{i\omega t},\;\dfrac{d^2y_n}{dx^2}=-A_n\left(\dfrac{5\pi}{4l}\right)^2\sin\dfrac{5\pi x}{4l}e^{i\omega t},\\[2mm] \dfrac{d^3y_n}{dx^3}=-A_n\left(\dfrac{5\pi}{4l}\right)^3\cos\dfrac{5\pi x}{4l}e^{i\omega t},\;\dfrac{d^4y_n}{dx^4}=A_n\left(\dfrac{5\pi}{4l}\right)^4\sin\dfrac{5\pi x}{4l}e^{i\omega t},\\[2mm] \dfrac{dU}{dt}=Bi\omega e^{i\left(\omega-\frac{\pi}{2}\right)}.\end{cases} \tag{17}$$

By substituting the derivatives (17) for n=1 to the set of equations (9) and by considering the boundary conditions, equation (10) is derived as

$$\begin{cases}A_1(i\omega)^2K_1e^{i\omega t}=-aA_1\left(\dfrac{5\pi}{4l}\right)^4K_1e^{i\omega t}-dBi\omega e^{i\left(\omega-\frac{\pi}{2}\right)}D+bF_0e^{i\omega t}{}_1,\\[2mm] Bi\omega e^{i\left(\omega-\frac{\pi}{2}\right)}+\beta Be^{i\left(\omega-\frac{\pi}{2}\right)}+\alpha A_1i\omega\left(\dfrac{5\pi}{4l}\right)C_1e^{i\omega t}=0\end{cases} \tag{18}$$

or

$$\begin{cases}A_1\left[(i\omega)^2-a\left(\dfrac{5\pi}{4l}\right)^4\right]K_1e^{i\omega t}+Bde^{i\left(\omega-\frac{\pi}{2}\right)}D_1=bF_0e^{i\omega t},\\[2mm] A_1\alpha i\omega\left(\dfrac{5\pi}{4l}\right)C_1e^{i\omega t}+Bi\omega e^{i\left(\omega-\frac{\pi}{2}\right)}+B\beta e^{i\left(\omega-\frac{\pi}{2}\right)}=0,\end{cases} \tag{19}$$

where: $\sin\left(\frac{5\pi x}{4l}\right)=K_1$, $\cos\left(\frac{5\pi}{4l}l_p\right)=C_1$, $\delta'(x-x_2)-\delta'(x-x_1)=D$ (when $x=l, x_1=0{,}01l, x_2=0{,}03l$), $b=\frac{1}{\rho A}$, $\beta=\frac{1}{RC}$.

Putting in (19)

$$e^{i\omega t-i\frac{\pi}{2}}=\frac{e^{i\omega t}}{e^{i\frac{\pi}{2}}} \tag{20}$$

and dividing by $e^{i\omega t}$, the set of equations (19) after some transformations, takes the following form

$$\begin{cases} A_1\left[(i\omega)^2+a\left(\frac{5\pi}{4l}\right)\right]K_1+\frac{Bd}{e^{i\frac{\pi}{2}}}D=bF_0, \\ A_1\alpha i\omega\left(\frac{5\pi}{4l}\right)C_1+\frac{B}{e^{i\frac{\pi}{2}}}i\omega+\frac{B}{e^{i\frac{\pi}{2}}}\beta=0. \end{cases} \tag{21}$$

Using Euler's theorem in form of

$$e^{i\frac{\pi}{2}}=\cos\frac{\pi}{2}+i\sin\frac{\pi}{2}=0+\mathrm{i}=\mathrm{i}, \tag{22}$$

the set of equations (21) may by described as

$$\begin{cases} A_1\left[-\omega^2+a\cdot\left(\frac{5\pi}{4l}\right)^4\right]K_1+\frac{1}{i}BdD=bF_0, \\ A_1\alpha i\omega\left(\frac{5\pi}{4l}\right)C_1+\frac{1}{i}Bi\omega+\frac{1}{i}B\beta=0, \end{cases} \tag{23}$$

and, after transformations, as

$$\begin{cases} A_1\left[-\omega^2+a\left(\frac{5\pi}{4l}\right)\right]K_1-BdDi=bF_0, \\ A_1\alpha i\omega\left(\frac{5\pi}{4l}\right)C_1+B\omega-B\beta i=0. \end{cases} \tag{24}$$

Finally, the set of equations (24) in a matrix shape is as follows

$$\begin{bmatrix} \left\{-\omega^2+a\left(\frac{5\pi}{4l}\right)^4\right\}K_1 & -dDi \\ \alpha i\omega\left(\frac{5\pi}{4l}\right)C_1 & \omega-\beta i \end{bmatrix}\begin{bmatrix} A_1 \\ B \end{bmatrix}=\begin{bmatrix} bF_0 \\ 0 \end{bmatrix}, \tag{25}$$

leading to

$$\mathbf{WA}=\mathbf{F}. \tag{26}$$

The main determinant of square matrix **W** is equal to

$$|\mathbf{W}|=\begin{vmatrix} \left\{-\omega^2+a\left(\frac{5\pi}{4l}\right)^4\right\}K_1 & -dDi \\ \alpha i\omega\left(\frac{5\pi}{4l}\right)C_1 & \omega-\beta i \end{vmatrix}=$$

$$=[\omega-\beta i]\left\{-\omega^2+a\left(\frac{5\pi}{4l}\right)^4\right\}K_1-\left[\alpha\omega\left(\frac{5\pi}{4l}\right)C_1\right]dD. \tag{27}$$

Substituting the first column in square matrix **W** with matrix **F** the following form is obtained

$$|\mathbf{W}_A|=\begin{vmatrix} bF_0 & -dDi \\ 0 & \omega-\beta i \end{vmatrix}. \tag{28}$$

The determinant of matrix $\mathbf{W}_A$ equals

$$|\mathbf{W}_A|=bF_0[\omega-\beta i]. \tag{29}$$

The amplitude is obtained as

$$A_1=\frac{|\mathbf{W}_A|}{|\mathbf{W}|}=\frac{bF_0[\omega-\beta i]}{[\omega-\beta i]\left[-\omega^2+a\left(\frac{5\pi}{4l}\right)^4\right]K_1-\left[\alpha\omega\left(\frac{5\pi}{4l}\right)C_1\right]dD}. \tag{30}$$

Inserting the derived amplitude A_1 (30) to (136) the deflection of the beam cross-section for the first vibration mode is designated as

$$y_1(x,t)=\frac{b[\omega-\beta i]K_1}{[\omega-\beta i]\left[-\omega^2+a\left(\frac{5\pi}{4l}\right)^4\right]K_1-\left[\alpha\omega\left(\frac{5\pi}{4l}\right)C_1\right]dD}Fe^{i\omega t}. \tag{31}$$

On the grounds of equation (31), the dynamical characteristic for the first vibration mode in place $x = l$ takes the form of

$$Y_1=\frac{b[\omega-\beta i]K_1}{[\omega-\beta i]\left\{-\omega^2+a\left(\frac{5\pi}{4l}\right)^4\right\}K_1-\left[\alpha\omega\left(\frac{5\pi}{4l}\right)C_1\right]dD}, \tag{32}$$

and after transformation, the form of

$$Y_1=\frac{\omega bK_1-i\beta bK_1}{\omega\left[a\left(\frac{5\pi}{4l}\right)^4-\omega^2\right]K_1-\alpha d\omega\left(\frac{5\pi}{4l}\right)C_1D-i\beta\left[a\left(\frac{5\pi}{4l}\right)^4-\omega^2\right]K_1} \tag{33}$$

or

$$Y_1=\frac{\operatorname{Re}N+i\operatorname{Im}N}{\operatorname{Re}D+i\operatorname{Im}D}, \tag{34}$$

where: $\operatorname{Re} N = \omega b K_1$, $\operatorname{Re} D = \omega\left[a\left(\frac{5\pi}{4l}\right)^4 - \omega^2\right]K_1 - \alpha d\omega\left(\frac{5\pi}{4l}\right)C_1 D$,

$\operatorname{Im} N = -i\beta b K_1$, $\operatorname{Im} D = \beta\left[a\left(\frac{5\pi}{4l}\right)^4 - \omega^2\right]K_1$.

Further transformation of (31) leads to

$$Y_1 = \frac{\operatorname{Re} D \operatorname{Re} N + \operatorname{Im} D \operatorname{Im} N}{\operatorname{Re} D^2 + \operatorname{Im} D^2} + i\frac{\operatorname{Re} D \operatorname{Im} N - \operatorname{Im} D \operatorname{Re} N}{\operatorname{Re} D^2 + \operatorname{Im} D^2} = \tag{35}$$

$$= \operatorname{Re} Y_1 + i \operatorname{Im} Y_1.$$

The modulus of expression (35) is equal to

$$|Y_1| = \sqrt{(\operatorname{Re} Y_1)^2 + (\operatorname{Im} Y_1)^2}\,. \tag{36}$$

In Fig. 2 the increase of resonance zone at first natural frequency of the transients of dynamical characteristics (36) are shown.

For the second vibration mode, i.e. when n=2, the deflection (16) takes the following form

$$y_2(x,t) = A \sin\frac{9\pi x}{4l} e^{i\omega t}\,. \tag{37}$$

Likewise, by inserting the derivatives of expressions (37) to (17), the set of equations (9), after steps (18-31) takes the following form

$$Y_2 = \frac{\omega b K_1 - i\beta b K_1}{\omega\left[a\left(\frac{9\pi}{4l}\right)^4 - \omega^2\right]K_1 - \alpha d\omega\left(\frac{9\pi}{4l}\right)C_1 D - i\beta\left[a\left(\frac{9\pi}{4l}\right)^4 - \omega^2\right]K_1}\,. \tag{38}$$

After transforming (38) in accordance with (34-36), the increase of resonance zone at second natural frequency of the transient of expression (38), which is the absolute value (36) in the form of a complex number (35) is shown in Fig. 3.

For the third vibration mode, when n=3, the dynamical flexibility takes the following form

$$Y_3 = \frac{\omega b K_1 - i\beta b K_1}{\omega\left[a\left(\frac{9\pi}{4l}\right)^4 - \omega^2\right]K_1 - \alpha d\omega\left(\frac{9\pi}{4l}\right)C_1 D - i\beta\left[a\left(\frac{9\pi}{4l}\right)^4 - \omega^2\right]K_1}\,. \tag{39}$$

Using substitutions (34, 35), the dynamical flexibility is obtained as (36). The increase of resonance zone at third natural frequency of transients of the absolute value (36) of expression (39) are presented in Fig. 4.

A graphical representation of the dynamical flexibility determined for mechanical and mechatronic system, for the sum of i=1, 2, 3 vibration mode, is shown in Fig. 5.

In Figs. 2-5 the transients of characteristics-dynamical flexibility are shown for the following parameters of mechatronic system: $b_b = 0{,}064\text{m}$, $h_b = 0{,}0016\text{m}$, $l = 0{,}2111m$, $b_p = 0{,}04\text{m}$, $h_p = 0{,}0004\text{m}$, $l_p = 0{,}02\text{ m}$, $E = 2{,}1\cdot 10^{11}\frac{\text{N}}{\text{m}^2}$, $\rho = 7{,}8\cdot 10^3\frac{\text{kg}}{\text{m}^3}$, $d_{31} = 3{,}0\cdot 10^{-12}\frac{\text{C}}{\text{N}}$, $C_{11} = 2{,}3\cdot 10^{11}\frac{\text{N}}{\text{m}^2}$.

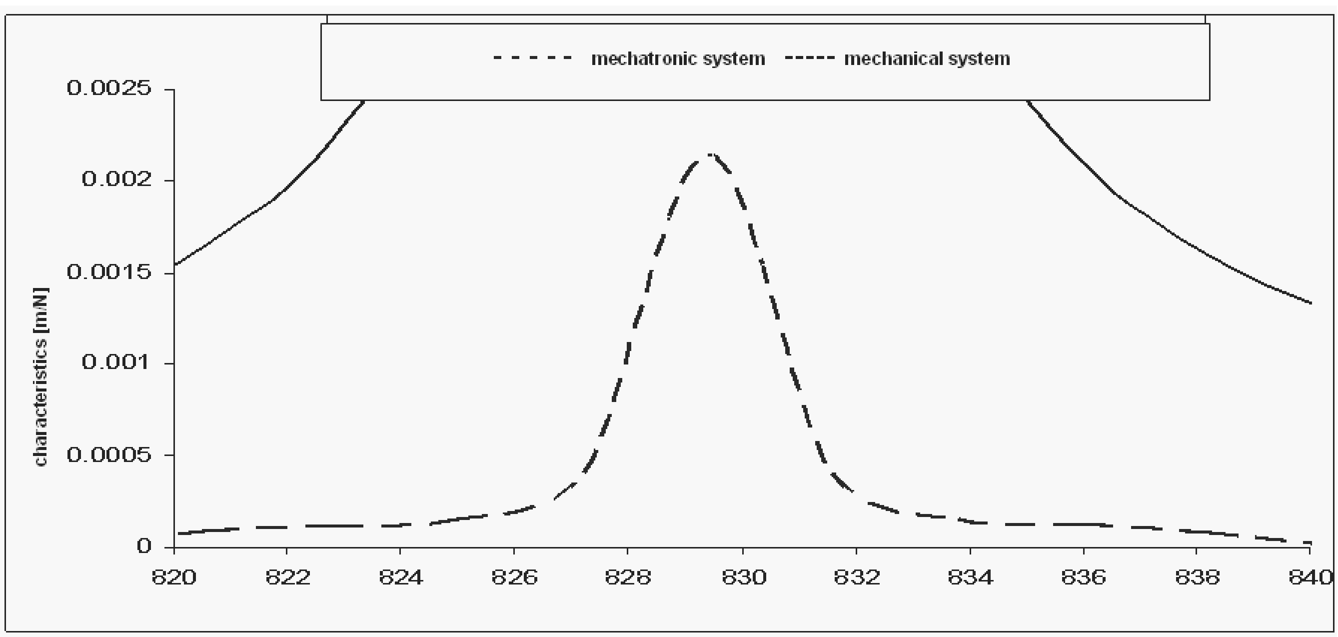

Fig. 2. Transient of characteristic - the increase of resonance zone at first natural frequency

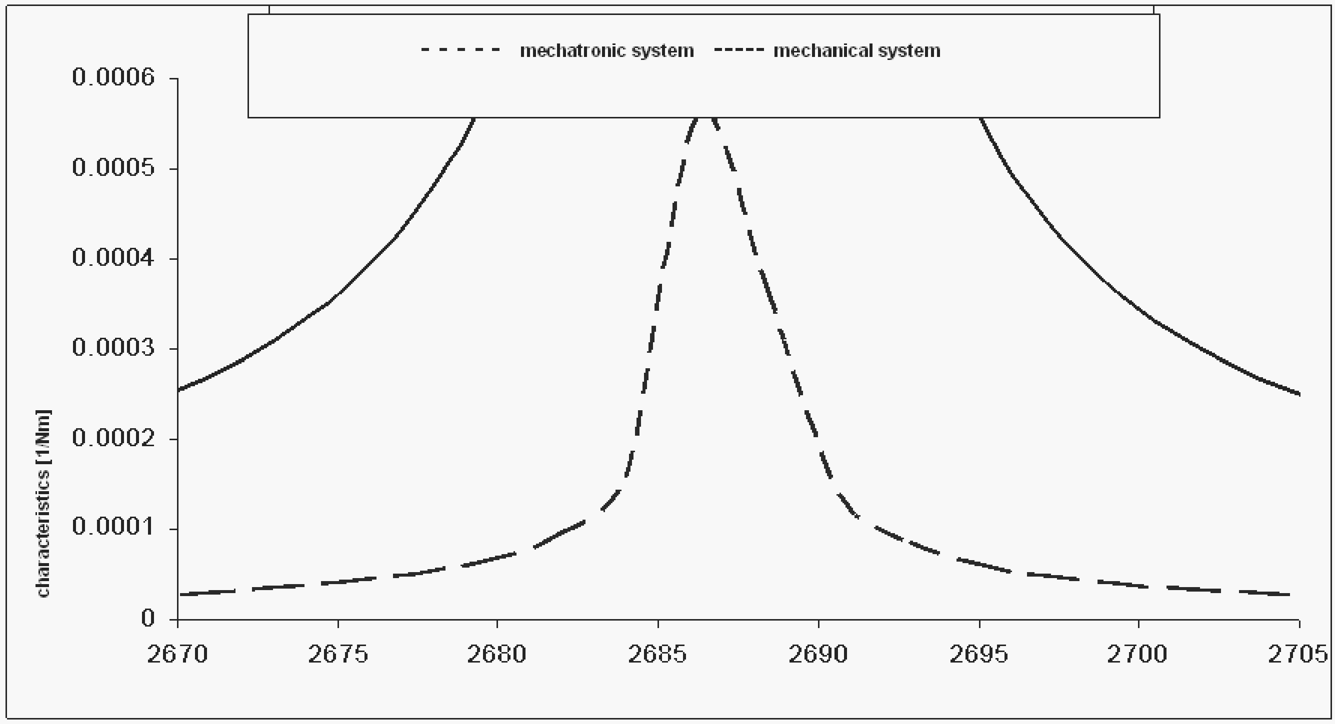

Fig. 3. Transient of characteristic - the increase of resonance zone at second natural frequency

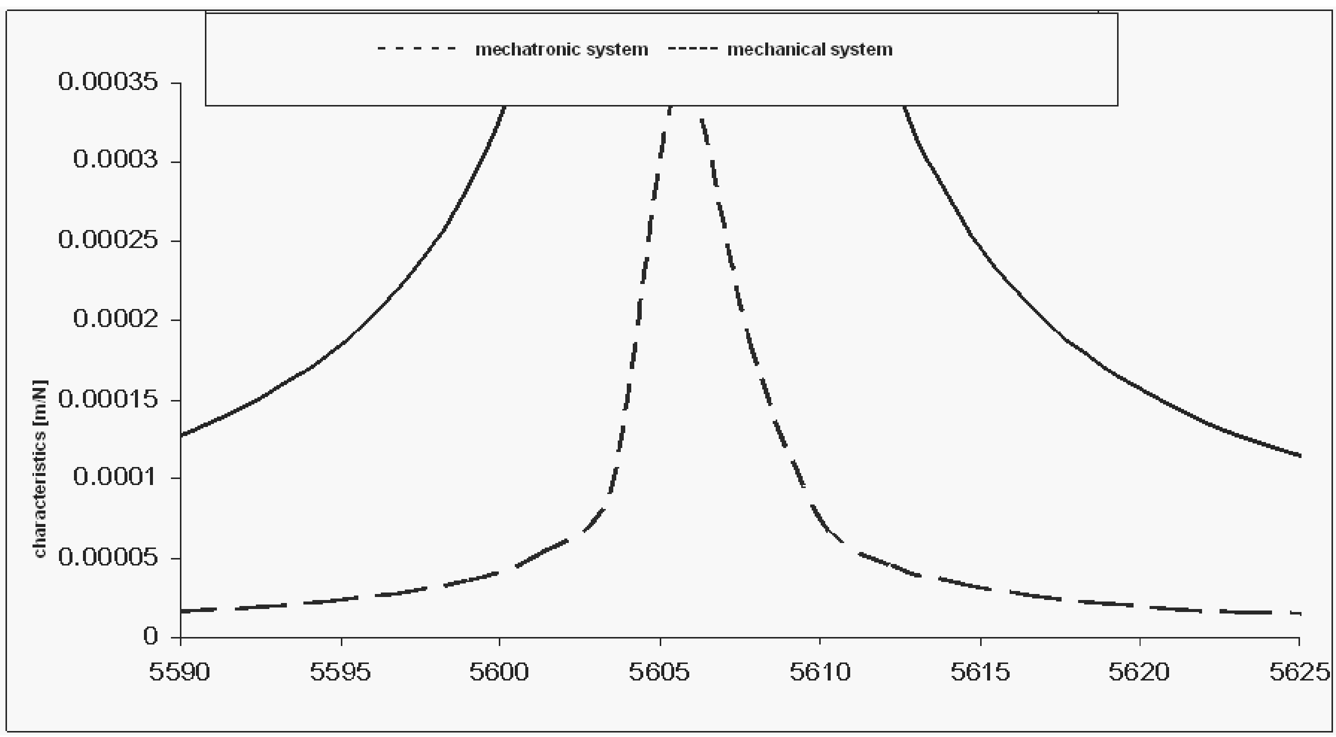

Fig. 4. Transient of characteristic - the increase of resonance zone at third natural frequency

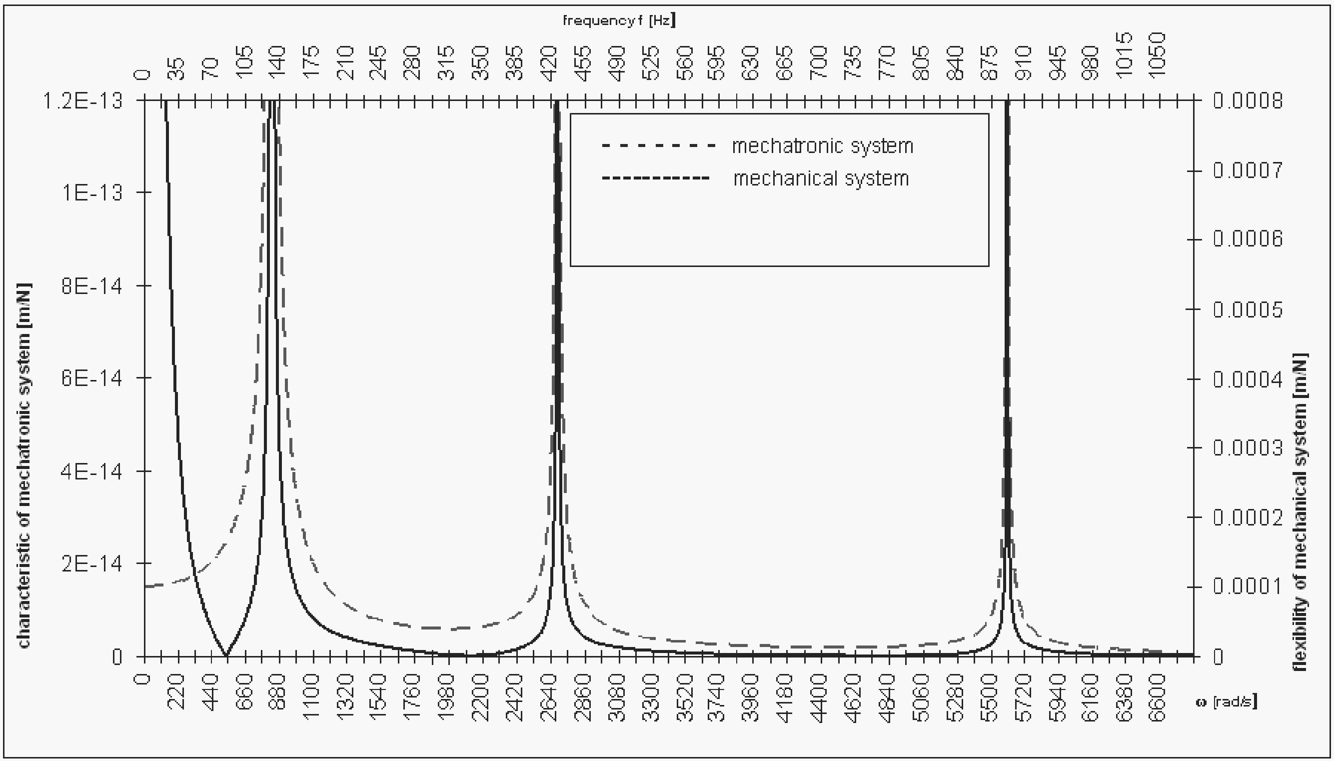

Fig. 5. Transients of the dynamical characteristics for the sum of n=1, 2, 3 vibration mode

4. Conclusions

An innovative approach is presented, involving the domain of the frequency spectrum analysis and enabling a global outlook on the behavior of a mechatronic system. On the grounds of the transients (Figs. 2-5), the poles of the dynamical characteristics calculated by the exact mathematical method and Galerkin's method have the same values. The derived mathematical formulas, concerning the dynamical characteristics make it possible to investigate the influence of changes in the values of the parameters that directly depend on the type of the piezoelement and on its geometrical size in view of the characteristics, mainly as far as the piezoelectric converter "activation" is concerned. The problems will be discussed in further research works.

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