Dynamical flexibility of torsionally vibrating mechatronic system

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Analysis and modelling

ABSTRACT

Purpose: of this paper is the application of the approximate method called Galerkin's method to solve the task of assigning the frequency-modal analysis and characteristics of a mechatronic system.

Design/methodology/approach: was the formulated and solved as a problem in the form of a set of differential equations of the considered mechatronic model of an object. To obtain the solution, Galerkin's method was used. The discussed torsionally vibrating mechatronic system consists of mechanical system, which is a continuous bar of circular cross-section, clamped on its ends. The electrical subsystem of the considered mechatronic system is a ring transducer to be perfectly bonded to the bar surface.

Findings: this study is that the parameters of the transducer have an important influence on the values of natural frequencies and on the form of the characteristics of the said mechatronic system. The results of the calculations were not only presented in a mathematical form but also as transients of the examined dynamical characteristic which are a function of frequency of the assumed excitation.

Research limitations/implications: is that the linear mechatronic system was considered, for this type of systems, such approach is sufficient.

Practical implications: of this researches was that another approach is presented, that means in the domain of frequency spectrum analysis. The method used and the obtained results can be of some value for designers of mechatronic systems.

Originality/value: of this paper is that the mechatronic system, created from mechanical and electrical subsystems with electromechanical bondage was examined. This approach is other than those considered elsewhere.

Keywords: Applied mechanics; Torsionaly vibrating shaft; Approximate method; Flexibility

1. Introduction

Graphs and structural numbers methods, was presented in the Gliwice Research Centre in [1-4,7,9-11], to solving the problem[1] to determine the dynamical characteristic of a longitudinally and torsionally vibrating continuous bar system and various classes of discrete mechanical systems in view of the frequency spectrum.

Challenging problems for scientific research are the requirements concerning mechatronic systems, for example their

exact positioning, working velocity, control and dimensions. The problems, cannot always be approached from the point of view of traditional principles of mechanics. Calculation of characteristics of the mechatronic systems need to investigate new possible methods for examination and analysis these systems.

For finding projects involving new construction solutions, a lot of attention has been given especially as far as the technology of drives based on the phenomenon of piezoelectricity and electrostriction is concerned [8,12-15,17,19,20]. To eliminate oscillation the piezoelectric elements are also used [16]. The mechatronic system, which has been clamped at one of its end (Fig. 1), has been considered in the paper [5]. The system was

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[1] Other diverse problems have been modelled by different kind of methods. Next for the last several years the problems were examined and analysed in the centre (e.g. [18, 21-25].

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excited by the harmonic electrical voltage from the electric side which was applied to the converter clips.
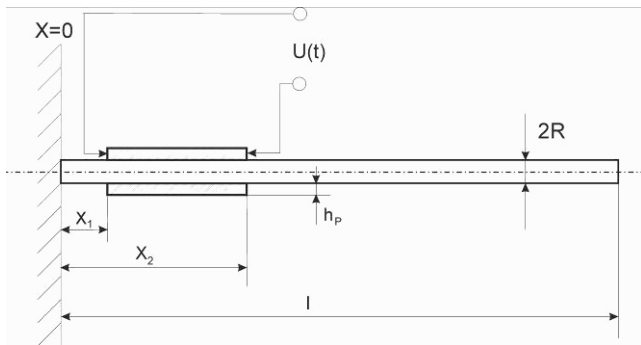

Fig. 1. The mechatronic system with electrical excitation

2. The torsionaly vibrating shaft with piezotransducer and shunting circuit

The torsionaly vibrating system clamped at of its ends is considered in this paper. An ideal piezotransducer ring is perfectly bonded at a certain position x_1, to the surface of the shaft (see Fig. 2).
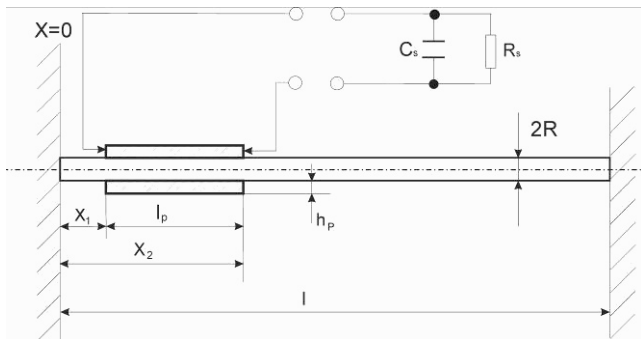

Fig. 2. Shaft with piezotransducer with mechanical exitatation and shunting circuit

The mechanical part of considered mechatronic system is the continuous elastic shaft with full section, constant along the whole length l. The shaft is made of a material with mass density ρ and Kirchoff's modulus G. The system was considered in [6], but in this paper the problem have only been signalized.

The equation of the motion of the mechanical subsystem of the mechatronic system, in view of the given system, takes the following form [16].

$$\rho I_o \varphi_{,tt} - GI_o \varphi_{,xx} = \frac{-\lambda^*}{l} U\left[\delta(x-x_1) - \delta(x-x_2)\right] + \frac{M_o}{l} e^{i\omega t}\delta(x-l) \qquad (1)$$

or, differently:

$$\varphi_{,tt} - \frac{G}{\rho}\varphi_{,xx} = -\frac{\lambda^*}{I_o\rho l}U\left[\delta(x-x_1) - \delta(x-x_2)\right] + \frac{M}{I_0\rho l}\delta(x-l), (2)$$

where: $\lambda^* = \frac{2}{3}\pi G_p \left[(R+h_p)^3 - R^3\right]\frac{d_{15}}{l_p}$, G_p, l_p - transverse modulus and length of the piezoelement respectively, d_{15} the electromechanical coupling coefficient [15,16,19].

The equation of the of the electrical subsystem of mechatronic system, which is piezotransducer, is given in the form:

$$\dot{U} + \frac{1}{R_s C_p}U + \frac{2\pi R^2 h_p d_{15} G_p}{l_p C_p}\varphi_{,t}(l_p, t) = 0 \qquad (3)$$

or in a different way

$$\dot{U} + \alpha_1 U + \alpha_2 \varphi_{,t}(l_p, t) = 0, \qquad (4)$$

where: $C_p = 2\pi R h_p \frac{e_{15}}{l_p}\left(1 - \frac{2d_{15}G_p}{e_1}\right)$ $\alpha_1 = \frac{1}{R_s C_p}$, e_{15} - the dielectric constant [15, 16, 19].

Taking into consideration equations (1-4) the considered mechatronic system (Fig. 2) is described by the next set of equations in form

$$\begin{cases} \varphi_{,tt} - \frac{G}{\rho}\varphi_{,xx} + \frac{\lambda^*}{I_o\rho l}U\left[\delta(x-x_1) - \delta(x-x_2)\right] = \frac{1}{I_0\rho l}M\delta(x-l) \\ \dot{U} + \alpha_1 U + \alpha_2 \varphi_{,t}(l_p, t) = 0 \end{cases} \qquad (5)$$

The set of equations (5) will be a starting point of further considerations which can be derived.

The solution sought in this paper will involve the sum function, that means the function of the time and displacement variables, which are strictly determined and which fulfill the boundary conditions [6]. This approach is agreeable to Galerkin's discretisation of the solutions of the differential equation system with partial derivative.

The boundary conditions on the mechanical subsystem ends (Fig. 2) are given in the form of

$$\varphi(0,t) = 0; \quad \Phi(0)T(t) = 0 \rightarrow \Phi(0) = 0 \qquad (6)$$

and

$$\varphi(l,t) = 0; \quad \Phi(l)T(t) = 0 \rightarrow \Phi(l) = 0. \qquad (7)$$

The angle of the torsion of the cross-section takes the following form:

$$\varphi(x,t) = \sum_{j=1}^{\infty} \varphi_j(x,t) = \sum_{j=1}^{\infty} A_j \sin\frac{n\pi x}{l} e^{i\omega t}. \qquad (8)$$

Mechatronical system is additionally excited by moment as follows:

$$M = M_0 e^{i\omega t} \,. \tag{9}$$

The voltage generated in the transducer as a piezoelectric effect will have a harmonic character, because the mechanical excitation (9) has the same character, that means

$$U = Be^{i\left(\omega t - \frac{\pi}{2}\right)} \,. \tag{10}$$

3. Frequency-modal analysis of mechatronic system

3.1. The dynamical flexibility for the any vibration mode

For the any vibration mode, the angle of torsion (8) takes the form of

$$\varphi_j(x,t) = A_j \, \sin\frac{j\pi x}{l} e^{i\omega t} \,. \tag{11}$$

The solution of the examined set of differential equations (5), leads to appropriate derivative, as follow

$$\begin{cases} \varphi_{j,t}\,(x,t) = A_j\,\omega\sin\frac{\pi x}{l}e^{i\omega t}, \\[2mm] \varphi_{j,tt}\,(x,t) = -A_j\,\omega^2\sin\frac{\pi x}{l}e^{i\omega t}, \\[2mm] \varphi_{j,xx}\,(x,t) = -A_j\left(\frac{j\pi}{l}\right)^2\sin\frac{\pi x}{l}e^{i\omega t}, \\[2mm] \dot{U} = i\omega Be^{i\left(\omega t - \frac{\pi}{2}\right)}. \end{cases} \tag{12}$$

By substituting the derivatives (12) in equation (5) the set algebraic equations is given in form

$$\begin{cases} -A_j\,\omega^2 K_1\,e^{i\omega t} + \dfrac{G}{\rho}A_j\left(\dfrac{j\pi}{l}\right)^2 Ke^{i\omega t} + Be^{i\left(\omega t-\frac{\pi}{2}\right)}\dfrac{\lambda^*}{I_o\rho l}D = \dfrac{1}{I_o\rho l}M_0 e^{i\omega t}E, \\[3mm] A_j\,i\alpha_2\omega Ce^{i\omega t} + Bi\omega e^{i\left(\omega t-\frac{\pi}{2}\right)} + B\alpha_1 e^{i\left(\omega t-\frac{\pi}{2}\right)} = 0 \end{cases} \tag{13}$$

or, in the form

$$\begin{cases} A_j Ke^{i\omega t}\left[\dfrac{G}{\rho}\left(\dfrac{\pi}{l}\right)^2 - \omega^2\right] + Be^{i\left(\omega t-\frac{\pi}{2}\right)}\dfrac{\lambda^*}{I_o\rho l}D = \dfrac{1}{I_o\rho l}M_0 e^{i\omega t}E, \\[3mm] A_j i\alpha_2\omega Ce^{i\omega t} + Bi\omega e^{i\left(\omega t-\frac{\pi}{2}\right)} + B\alpha_1 e^{i\left(\omega t-\frac{\pi}{2}\right)} = 0, \end{cases} \tag{14}$$

where: $K=\sin\dfrac{\pi x}{l}$, $C=\sin\dfrac{\pi}{l}l_p$, $D=\left[\delta(x-x_1)-\delta(x-x_2)\right]$, $E=\delta(x-l)$ for predetermined x, x_1 and x_2

After arrangement (13) takes character

$$\begin{cases} -A_j\omega^2 Ke^{i\omega t} + \dfrac{G}{\rho}A_1\left(\dfrac{\pi}{l}\right)^2 Ke^{i\omega t} + Be^{i\left(\omega t-\frac{\pi}{2}\right)}\dfrac{\lambda^*}{I_o\rho l}D = \dfrac{1}{I_0\rho l}M_0 e^{i\omega t}E, \\[3mm] A_j i\alpha\omega C_1 e^{i\omega t} + Bi\omega e^{i\left(\omega t-\frac{\pi}{2}\right)} = 0 \end{cases} \tag{15}$$

and after transformations the set of algebraic equations is given as follows

$$\begin{cases} A_j Ke^{i\omega t}\left[\dfrac{G}{\rho}\left(\dfrac{\pi}{l}\right)^2 - \omega^2\right]I_o\rho l + Be^{i\left(\omega t-\frac{\pi}{2}\right)}\lambda^* D = M_0 e^{i\omega t}E, \\[3mm] A_j i\alpha\omega Ce^{i\omega t} + Bi\omega e^{i\left(\omega t-\frac{\pi}{2}\right)} = 0. \end{cases} \tag{16}$$

To designate the dynamical characteristic, the time function must be eliminated from the set of equations (16), using Euler's theorem in form

$$e^{i\omega t} = \cos\omega t + i\sin\omega t \,. \tag{17}$$

Using (17) after transformation the set of equations (16) is obtained as follows

$$\begin{cases} A_j K\left[\dfrac{G}{\rho}\left(\dfrac{j\pi}{l}\right)^2 - \omega^2\right]I_o\rho l - Bi\lambda^* D = M_0 E, \\[3mm] A_j i\alpha\omega C_1 + B\omega = 0 \end{cases} \tag{18}$$

Equations (18), as far as the matrix shape is considered

$$\begin{bmatrix} K\left[\dfrac{G}{\rho}\left(\dfrac{j\pi}{l}\right)^2 - \omega^2\right]I_0\rho l & -i\lambda^* D \\[3mm] i\alpha\omega C_1 & \omega \end{bmatrix}\begin{bmatrix} A_j \\ B \end{bmatrix} = \begin{bmatrix} M_0 E \\ 0 \end{bmatrix} \tag{19}$$

or
$$\mathbf{W}\,\mathbf{A} = \mathbf{F} \,. \tag{20}$$

By substituting in square matrix $\mathbf{W}$, the first column by matrix $\mathbf{F}$, is obtained

$$\left|\mathbf{W}_{A_j}\right| = \begin{vmatrix} M_0 E & -i\lambda^* D \\ 0 & \omega \end{vmatrix} \,. \tag{21}$$

The determinant of matrix $\mathbf{W}_A$ equals to

$$\left|\mathbf{W}_{A_j}\right| = M_0 E\omega \,. \tag{22}$$

Thus, the amplitude of the dynamical characteristic is obtained as

$$A_j = \frac{\left|\mathbf{W}_{A_j}\right|}{|\mathbf{W}|} \,, \tag{23}$$

where: $|\mathbf{W}| = K\left[\dfrac{G}{\rho}\left(\dfrac{j\pi}{l}\right)^2 - \omega^2\right]I_0\rho l\omega - \alpha\omega C_1\lambda^* D$ - the main

determinant of square matrix $\mathbf{W}$

that means

$$A_j = \frac{M_0 E}{K\left[\dfrac{G}{\rho}\left(\dfrac{j\pi}{l}\right)^2 - \omega^2\right]I_0\rho l - \alpha C_1\lambda^* D} \; . \tag{24}$$

The angle of the torsion of the cross-section for the first vibration mode, i.e. $j=1$, after substituting (24) to (11) is determined

$$\varphi_1(x,t) = \frac{E\sin\dfrac{\pi x}{l}}{K_1\left[\dfrac{G}{\rho}\left(\dfrac{\pi}{l}\right)^2 - \omega^2\right]I_0\rho l - \alpha C_1\lambda^* D} M_0 e^{i\omega t} \; . \tag{25}$$

The dynamical flexibility for the first vibration mode, on the base (25) takes the form of

$$Y_{x1} = \frac{E\sin\dfrac{\pi x}{l}}{\left[\dfrac{G}{\rho}\left(\dfrac{\pi}{l}\right)^2 - \omega^2\right]I_0\rho l - \alpha C_1\lambda^* D} \; . \tag{26}$$

Finally (26), the dynamical flexibility for the first vibration mode at the end of the shaft, i.e. when $x=l$ takes the following form

$$|Y_{l1}| = \left|\frac{E}{\left[\dfrac{G}{\rho}\left(\dfrac{\pi}{l}\right)^2 - \omega^2\right]I_0\rho l - \alpha C_1\lambda^* D}\right| \; . \tag{27}$$

In Fig. 3 and 4 the transients of characteristics-dynamical flexibility are shown for the following parameters of bar: $l = 1\text{m}$, $R = 0{,}05\text{m}$, $G = 8{,}3\cdot10^{10}\,\dfrac{\text{N}}{\text{m}^2}$, $\rho = 7{,}8\cdot10^3\,\dfrac{\text{kg}}{\text{m}^3}$ and for piezotransducer: $h_p = 0{,}005\text{m}$, $l_p = 0{,}3\text{m}$, $d_{15} = 440\cdot10^{-12}\,\dfrac{\text{C}}{\text{N}}$, $S_{11} = 11{,}6\cdot10^{-12}\,\dfrac{\text{m}^2}{\text{N}}$, $e_1 = 9{,}8\,\dfrac{\text{C}}{\text{m}^2}$.

3.2. The dynamical flexibility for the second vibration mode

For the second vibration mode, i.e. when $n=2$, the angle of torsion (8) takes the form of

$$\varphi_2(x,t) = A_2\sin\frac{2\pi x}{l}e^{i\omega t} \; . \tag{28}$$

By substituting the derivatives of expressions (8), (10) and (28) to the set of equation (5) the dynamical characteristic, after steps (11-24) is derived as

$$|Y_{l2}| = \left|\frac{E}{\left[\dfrac{G}{\rho}\left(\dfrac{2\pi}{l}\right)^2 - \omega^2\right]I_0\rho l - \alpha C_1\lambda^* D}\right| \; . \tag{29}$$

The transient of expression (29) are shown in Fig. 5 and 6.

3.3. The dynamical flexibility for the third vibration mode

For the third vibration mode, i.e. when $n=3$, the angle of torsion (8) takes the form of

$$\varphi_3(x,t) = A_3\sin\frac{3\pi x}{l}e^{i\omega t} \; . \tag{30}$$

As previously, by substituting the derivatives of expressions (8), (10) and (28) to (5), the dynamical characteristic after steps (11-24), has the following form

$$|Y_{l3}| = \left|\frac{E}{\left[\dfrac{G}{\rho}\left(\dfrac{3\pi}{l}\right)^2 - \omega^2\right]I_0\rho l - \alpha C_1\lambda^* D}\right| \; . \tag{31}$$

The graphical presentation of expression (31) are shown in Fig. 7 and 8.

4. Conclusions

On the base of transients of dynamical flexibilities the poles of the characteristic calculated with the use of mathematical exact method and Galerkin's method have approximately the same values. The presented frequency-modal approach makes it possible to consider the behavior of the mechatronic system in a global way.

Mathematical formulas, those which concern the dynamical characteristics-dynamical flexibilities make it possible to investigate the influence of the change in values parameters, which directly depend on the type of the piezoelement and on its geometrical size in view of the characteristics, the sort of vibrations of the mechatronic system, mainly as far as the piezoelectric converter "activation" is concerned, however the problems shall be discussed in further research works.
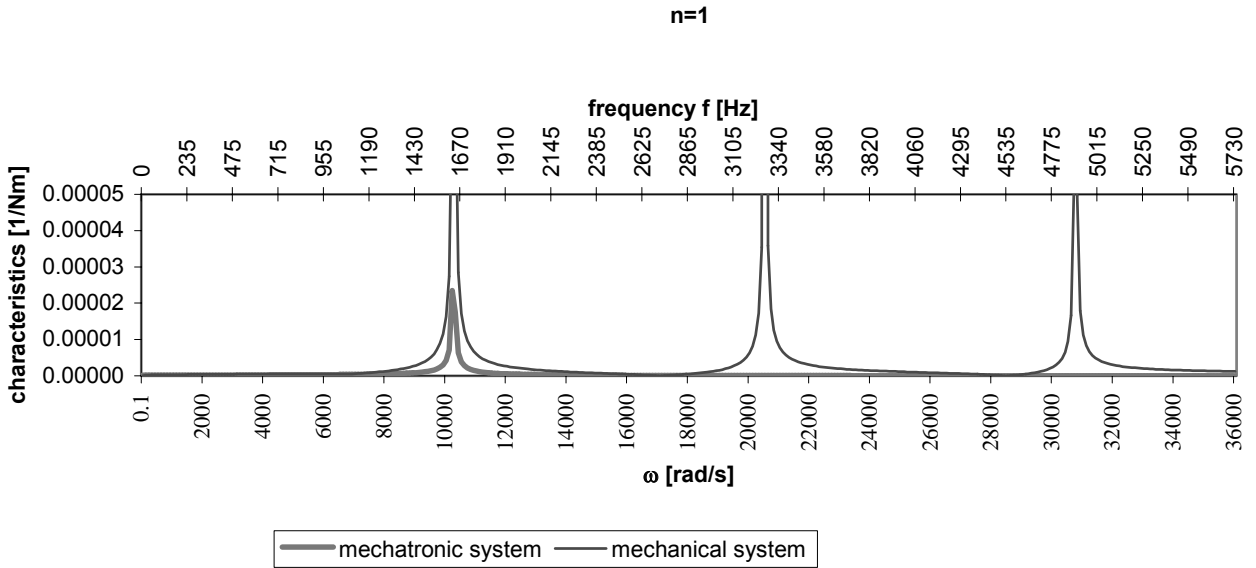
Acknowledgements

n=1

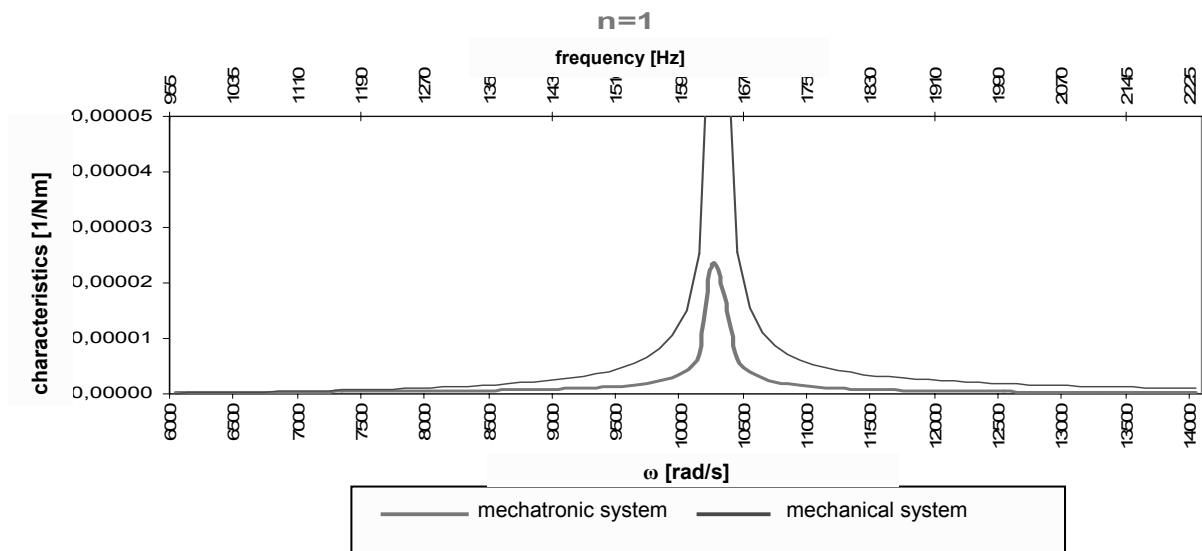Fig. 3. Transient of dynamical characteristic for the first mode vibration

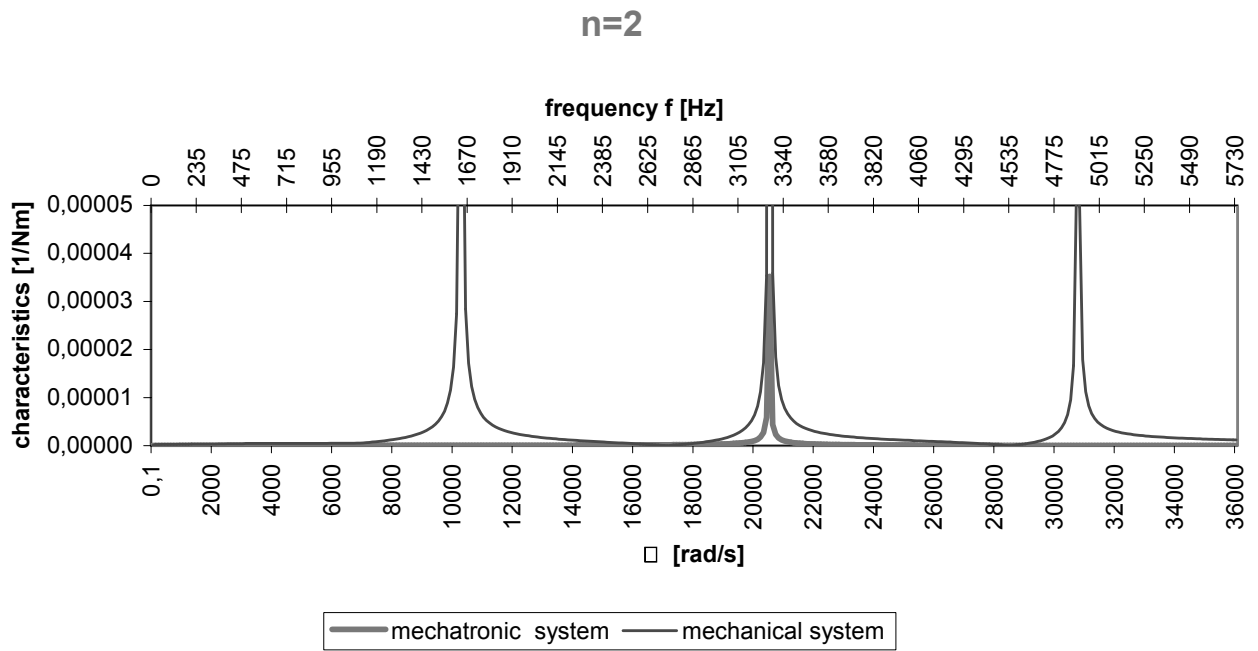Fig. 4. Transient of characteristic - the increase of resonance zone at first natural frequency

n=2

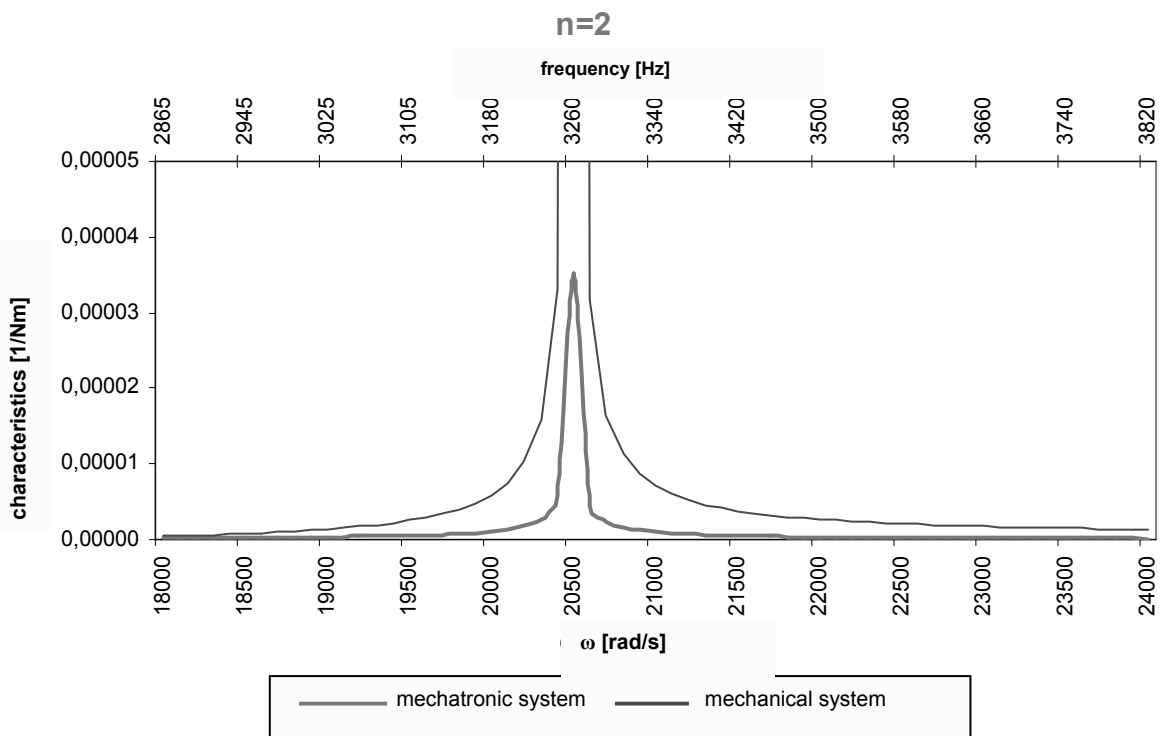Fig. 5. Transient of dynamical characteristic for the second mode vibration

n=2

Fig. 6. Transient of characteristic - the increase of resonance zone at second natural frequency
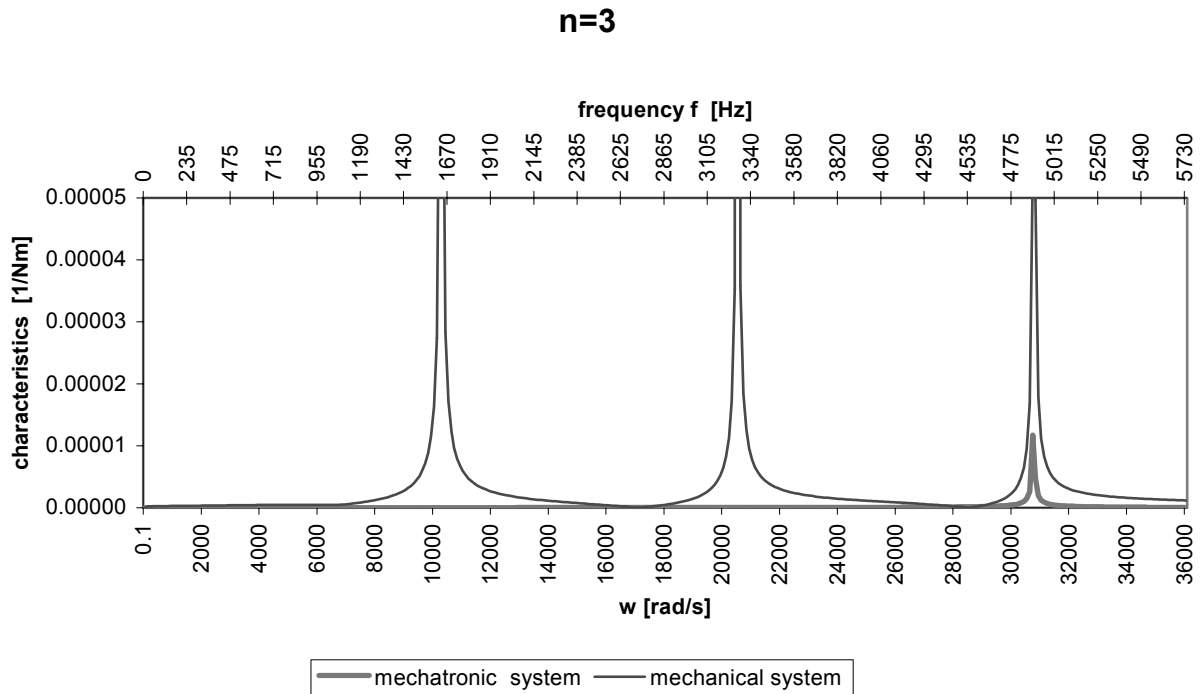
n=3

Fig. 7. Transient of dynamical characteristic for the third mode vibration
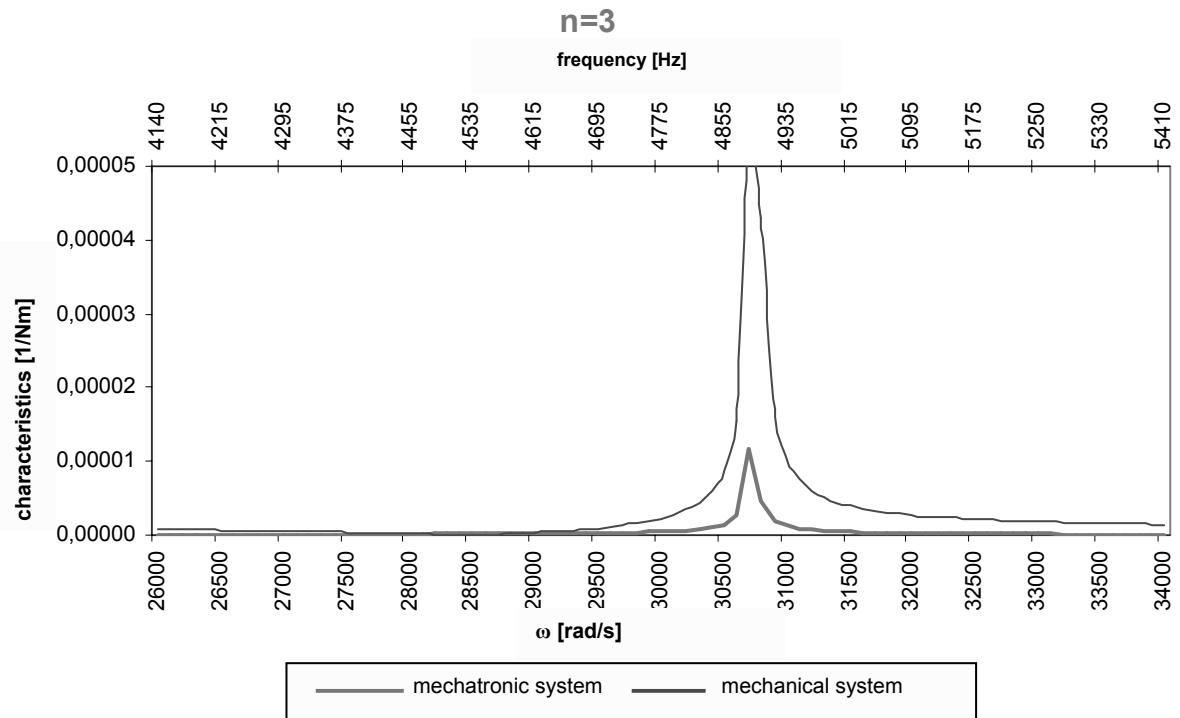

Fig. 8. Transient of characteristic - the increase of resonance zone at third natural frequency

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