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Investigation of piezoelectric influence on characteristics of mechatronic system

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Analysis and modelling

ABSTRACT

Purpose: The purpose of this paper is application of approximate method of solving the task of assignment the frequency-modal analysis and characteristics of mechatronic system.
Design/methodology/approach: The main approach of the subject was to formulate and solve the problem in the form of set of differential equation of motion and state equation of considered mechatronic model of object. Galerkin's method to solving has been used. The considered torsionally vibrating mechanical system is a continuous bar of circular cross-section, clamped at one of its end. Integral part of mechatronic system is a ring transducer, extorted by harmonic voltage excitation, to be perfectly bonded to the bar surface.
Findings: The parameters of the transducer have important influence of values of natural frequencies and on form of characteristics of the discused mechatronic system. The results of the calculations were not only presented in mathematical form but also as a transients of examined dynamical characteristic which were function of frequency of the excitation.
Research limitations/implications: In the paper the linear mechanical subsystem and linear electric subsystem of mechatronic system has been considered, however for this kind of systems the approach is sufficient.
Practical implications: The methods of analysis and obtained results can be base of design and investigation for this type of mechatronic systems.
Originality/value: The mechatronic system created from mechanical and electric subsystems with electromechanical bondage has been considered. This approach is different from those considered so far.
Keywords: Applied mechanics; Bar; Piezotransducer; Galerkin's method; Transmittance

1. Introduction

The object of special interest of researchers is lifting the machine's efficiency and reliability, during the process of designing the machines.

Many industry branches concentrate on minimizing energy-consumption of the existing systems and also on some problems of their miniaturizing.

A lot of attention during last years is paid to the researches connected with new construction solutions, especially as far as the technology of drives which lean on the phenomenon of piezoelectricity and electrostriction is concerned for example in [8,12-15,17,19-20]. The piezoelectrics are also used to eliminate the flexibly and torsionally vibration [16].

Using graphs models of continuous bar system and various class of discrete mechanical systems and structural numbers methods, one of the first attempt at the solution to this problem, that means to determining of dynamical characteristic, has been made in the Gliwice research centre in [1-5,10-11]. Other diverse problems have been modelled by different kind of methods, next the problems were examined and analysed in the centre for the last several years (e.g. [6,7,18,21-25]).

2. The mechatronic system with electric excitation

The torsionally vibration bar as a mechanical subsystem of mechatronic system has been considered The homogeneous elastical shaft is made by material with transverse - Kirchoff's modulus G and the mass density ρ (Fig. 1). In this mechatronic system, the mechanical subsystem i.e. the torsinally vibrating bar is clamped at one of its end, the second one is free. The system is not excited by any mechanic moments or forces. An ideal ring piezotransducer is attached to the surface of the bar. To the converter clips the harmonic electric voltage, which excites the system from electric side is applied. This voltage makes the deformation of the piezoelement, which interacts directly with the shaft.

The equation of motion of the mechanical part of mechatronic system (Fig. 1) takes the form [6,16]

$$\rho I_o \frac{\partial^2 \varphi}{\partial t^2} - G I_o \frac{\partial^2 \varphi}{\partial x^2} = \frac{-\lambda^*}{l} U\left[\delta(x-x_1) - \delta(x-x_2)\right] \tag{1}$$

or

$$\varphi_{,tt} - a^2 \varphi_{,xx} = bU\left[\delta(x-x_1) - \delta(x-x_2)\right], \tag{2}$$

where: $\lambda^* = \frac{2}{3}\pi G_p\left[\left(R+h_p\right)^3 - R^3\right]\frac{d_{15}}{l_p}$, $a = \sqrt{\frac{G}{\rho}}$, $b = \frac{-\lambda^*}{I_o l \rho}$.

The state equation of the transducer is given in the form:

$$\frac{\mathrm{d}U}{\mathrm{d}t} + \alpha_1 U(t) = -\frac{2\pi R^2 h_p d_{15} G_p}{l_p C_p}\frac{\partial \varphi}{\partial t}(l_p, t) \tag{3}$$

or, differently:

$$\dot{U} + \alpha_1 U(t) = -\alpha_2 \varphi_{,t}(l_p, t), \tag{4}$$

where: α_1 - constant measured in $\left[\frac{1}{s}\right]$, $\alpha_2 = \frac{2\pi R^2 h_p d_{15} G_p}{l_p C_p}$, $C_p = 2\pi R h_p \frac{e_1}{l_p}\left(1 - \frac{2 d_{15} G_p}{e_1}\right)$.

Taking into consideration the equations (1-4), the mechatronic system (Fig.1) is described by set of equations in form

$$\begin{cases} \varphi_{,tt} - a^2 \varphi_{,xx} = U\left[\delta(x-x_1) - \delta(x-x_2)\right], \\ \dot{U} + \alpha_1 U(t) = -\alpha_2 \varphi_{,t}(l_p, t). \end{cases} \tag{5}$$

The solution of set (5) will be searched in order to the own sum function, which are strictly established and which realize the boundary conditions [6,7,16]. This approach is according to the Galerkin's discretisation of searching the solutions of differential equation system with partial derivative.

The boundary conditions on the shaft ends are given in form

$$\begin{cases} \varphi(0,t) = 0 \Rightarrow X(0)T(t) = 0 \Rightarrow X(0) = 0, \\ \left.\frac{\partial \varphi}{\partial x}\right|_{x=1} = 0 \Rightarrow X'(l)T(t) = 0 \Rightarrow X'(l) = 0. \end{cases} \tag{6}$$

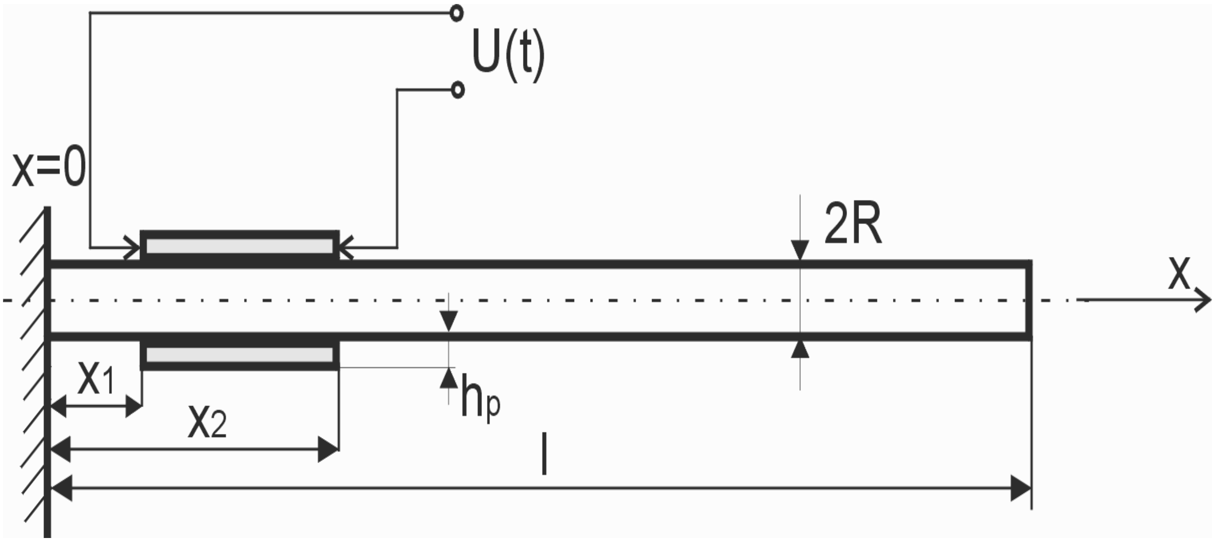

Fig. 1. The mechatronic system with electric excitation

It is accepted that the dislocation, that means angle of torsion of cross-section takes form:

$$\varphi(x,t)=\sum_{n=1}^{\infty}\varphi_n(x,t)=\sum_{n=1}^{\infty}A_n\sin\frac{(2n-1)\pi x}{2l}\cos\omega t\,. \tag{7}$$

Moreover it is assumed that the system is excited with harmonic voltage as follow:

$$U(t)=U_0\sin\omega t\,. \tag{8}$$

When the excitation has the harmonic character (8) then the voltage, generated in transducer will have the same character

$$U=B\sin\omega t\,. \tag{9}$$

3. The dynamical characteristic for first second and third mode vibration

When $n=1$ i.e. for the first mode vibration, the angle of torsion (7) takes form

$$\varphi_1(x,t)=A_1\sin\frac{\pi x}{2l}\cos\omega t\,. \tag{10}$$

The solution of the examined set of differential equations (5), resolves to putting the adequate derivatives, as follow

$$\begin{cases}\varphi_{1,t}(x,t)=-A_1\,\omega\sin\frac{\pi x}{2l}\sin\omega t,\\ \varphi_{1,tt}(x,t)=-A_1\,\omega^2\cos\frac{\pi x}{2l}\cos\omega t,\\ \varphi_{1,xx}(x,t)=-A_1\left(\frac{\pi}{2l}\right)^2\cos\frac{\pi x}{l}\cos\omega t,\\ \dot{U}=-\omega B\sin\omega t.\end{cases} \tag{11}$$

Putting the derivatives (11) to the set of equation (5) is obtained

$$\begin{cases}A_1\sin\frac{\pi x}{2l}\cos\omega t\left[a^2\left(\frac{\pi}{2l}\right)^2-\omega^2\right]-Bb\left[\delta(x-x_1)-\delta(x-x_2)\right]\sin\omega t=0,\\ B\omega\cos\omega t-\alpha_2A_1\,\omega\sin\frac{\pi l_p}{2l}\sin\omega t=\alpha_1U_0\sin\omega t.\end{cases} \tag{12}$$

or

$$\begin{cases}A\sin\frac{\pi x}{2l}\cos\omega t\left[a^2\left(\frac{\pi}{2l}\right)^2-\omega^2\right]-Bb\left[\delta(x-x_1)-\delta(x-x_2)\right]\cos\left(\omega t-\frac{\pi}{2}\right)=0,\\ B\omega\cos\omega t-\alpha_2A\omega\sin\left(\frac{\pi}{2l}l_p\right)\cos\left(\omega t-\frac{\pi}{2}\right)=\alpha_1U_0\sin\omega t.\end{cases} \tag{13}$$

Using the Euler's theorem [7] the set of equations takes form

$$\begin{cases}A_1\sin\frac{\pi x}{2l}e^{i\omega t}\left[a^2\left(\frac{\pi}{2l}\right)^2-\omega^2\right]-Bb\left[\delta(x-x_1)-\delta(x-x_2)\right]e^{i\left(\omega t-\frac{\pi}{2}\right)}=0,\\ B\omega e^{i\omega t}-\alpha_2A_1\,\omega\sin\left(\frac{\pi}{2l}l_p\right)e^{i\left(\omega t-\frac{\pi}{2}\right)}=\alpha_1U_0e^{i\omega t}.\end{cases} \tag{14}$$

After transformations the set of equations (14) takes form

$$\begin{cases}A\sin\frac{\pi x}{2l}\left[a^2\left(\frac{\pi}{2l}\right)^2-\omega^2\right]-\frac{B}{e^{i\frac{\pi}{2}}}b\left[\delta(x-x_1)-\delta(x-x_2)\right]=0,\\ B\omega-\alpha_2\frac{A}{e^{i\frac{\pi}{2}}}\omega\sin\frac{\pi l_p}{2l}=\alpha_1U_0.\end{cases} \tag{15}$$

The equations (15) as far as the matrix shape is considered as

$$\mathbf{W}\,\mathbf{A}=\mathbf{F}\,, \tag{16}$$

where:

$$\mathbf{W}=\begin{bmatrix}\sin\frac{\pi x}{2l}\left[a^2\left(\frac{\pi}{2l}\right)^2-\omega^2\right] & -\frac{1}{e^{i\frac{\pi}{2}}}b\left[\delta(x-x_1)-\delta(x-x_2)\right]\\ -\alpha_2\frac{1}{e^{i\frac{\pi}{2}}}\omega\sin\frac{\pi l_p}{2l} & \omega\end{bmatrix},$$

$$\mathbf{A}=\begin{bmatrix}A_1\\ B\end{bmatrix},\quad \mathbf{F}=\begin{bmatrix}0\\ \alpha_1U_0\end{bmatrix}.$$

Main determinant of square matrix $\mathbf{W}$ is equal

$$\left|\mathbf{W}\right|=\sin\frac{\pi x}{2l}\left[a^2\left(\frac{\pi}{2l}\right)^2-\omega^2\right]\omega-\frac{b}{\left(e^{i\frac{\pi}{2}}\right)^2}\left[\delta(x-x_1)-\delta(x-x_2)\right]\alpha_2\omega\sin\frac{\pi l_p}{2l}\,. \tag{17}$$

Substituting in square matrix $\mathbf{W}$ first column by the matrix is obtained

$$\mathbf{W}_A=\begin{bmatrix}0 & -\frac{1}{e^{i\frac{\pi}{2}}}b\left[\delta(x-x_1)-\delta(x-x_2)\right]\\ \alpha_1U_0 & \omega\end{bmatrix}. \tag{18}$$

The determinant of matrix $\mathbf{W}_A$ equals

$$\left|\mathbf{W}_A\right|=\alpha_1U_0\frac{b}{e^{i\frac{\pi}{2}}}\left[\delta(x-x_1)-\delta(x-x_2)\right]. \tag{19}$$

In the way the amplitude of dynamical characteristic is obtained as

$$A_1=\frac{\frac{1}{e^{i\frac{\pi}{2}}}b[\delta(x-x_1)-\delta(x-x_2)]\alpha_1}{\sin\frac{\pi x}{2l}\left[a^2\left(\frac{\pi}{2l}\right)^2-\omega^2\right]\omega-\frac{b}{\left(e^{i\frac{\pi}{2}}\right)^2}[\delta(x-x_1)-\delta(x-x_2)]\alpha_2\omega\sin\frac{\pi l_p}{2l}}U_0. \tag{20}$$

Putting the received amplitude A (20) to (10) the angle of torsion of cross-section for first mode vibration, that means n=1 and x=l, has been established.

$$\varphi_1(x,t)=\frac{\frac{1}{e^{i\frac{\pi}{2}}}b'\alpha_1\sin\left(\frac{\pi}{2l}x\right)}{\left[a^2\left(\frac{\pi}{2l}\right)^2-\omega^2\right]-\frac{b'}{\left(e^{i\frac{\pi}{2}}\right)^2}\alpha_2\omega\sin\left(\frac{\pi}{2l}l_p\right)}U_0\cos\omega t, \tag{21}$$

where: $b'=b\left[\delta(x-x_1)-\delta(x-x_2)\right]$.

Out of (21) the dynamical characteristic for the first mode vibration takes form

$$Y_{xl}=\frac{\frac{1}{e^{i\frac{\pi}{2}}}b'\alpha_1\sin\left(\frac{\pi}{2l}x\right)}{\left[a^2\left(\frac{\pi}{2l}\right)^2-\omega^2\right]-\frac{b'}{\left(e^{i\frac{\pi}{2}}\right)^2}\alpha_2\omega\sin\left(\frac{\pi}{2l}l_p\right)}, \tag{22}$$

Eliminating from (22) complex numbers the following transformations are used

$$e^{i\frac{\pi}{2}}=i,\ \frac{1}{e^{i\frac{\pi}{2}}}=-i,\ \left(e^{i\frac{\pi}{2}}\right)^2=-1 \tag{23}$$

and then (22) is given as

$$Y_{xi}=-i\frac{b'\alpha_1\sin\left(\frac{\pi}{2l}x\right)}{\left[a^2\left(\frac{\pi}{2l}\right)^2-\omega^2\right]\omega+b'\alpha_2\omega\sin\left(\frac{\pi}{2l}l_p\right)}. \tag{24}$$

Finally (23) the dynamical characteristic for the first mode vibration in the end of shaft, that means when x=l takes form

$$|Y_{l1}|=\left|-\frac{b'\alpha_1\sin\left(\frac{\pi}{2}\right)}{\left[a^2\left(\frac{\pi}{2l}\right)^2-\omega^2\right]\omega+b'\alpha_2\omega\sin\left(\frac{\pi}{2l}l_p\right)}\right|. \tag{25}$$

In Fig. 2 the transient of dynamical characteristic (25) has been shown. For the second mode vibration, that means when n=2 angle of torsion (7) takes form

$$\varphi_2(x,t)=A_2\sin\frac{3\pi x}{2l}\cos\omega t\ . \tag{26}$$

Putting the derivatives of sentences (26), similarly to (11), to the set of equation (5) the dynamical characteristic after steps (11-22) takes form

$$|Y_{l2}|=\left|-\frac{b'\alpha_1\sin\left(\frac{3\pi}{2}\right)}{\left[a^2\left(\frac{3\pi}{2l}\right)^2-\omega^2\right]\omega+b'\alpha_2\omega\sin\left(\frac{3\pi}{2l}l_p\right)}\right|. \tag{27}$$

The transient of expression (27) is shown. in Fig. 3.

For the third mode vibration, that means when n=3 angle of torsion (7) takes form

$$\varphi_3(x,t)=A_3\sin\frac{5\pi x}{2l}\cos\omega t\ . \tag{28}$$

As formerly putting the derivatives of sentence (28) to (5) the dynamical characteristic after steps (12-23) is given as follow

$$|Y_{l3}|=\left|-\frac{b'\alpha_1\sin\left(\frac{5\pi}{2}\right)}{\left[a^2\left(\frac{5\pi}{2l}\right)^2-\omega^2\right]\omega+b'\alpha_2\omega\sin\left(\frac{5\pi}{2l}l_p\right)}\right|. \tag{29}$$

Graphical representation of equation (28) and a transient of sum of n=1, 2, 3 mode vibration are shown adequately in Fig. 4 and 5. For example the influence of thickness and length of piezoelectric on characteristics of mechatronic system are shown in Fig. 6 and Fig. 7.

4. Conclusions

In article other approach is presented, that means in domain frequency spectrum the analysis has been considered. Presented approach allows to look in a global way on the behavior of the mechatronic system. On the base of transients (Fig. 2-5) the poles of dynamical characteristic calculated by mathematical exact method and the Galerkin's method have different values. In these figures and in Fig. 6 and 7 the influence change of the values b and α, which directly depend on piezoelement's sort and its geometrical sizes according to the characteristics have been shown. The sort of vibrations of the mechatronic system, mainly as far as the piezoelectric converter "activation" is concerned, but the problems shall be discussed in further research works.

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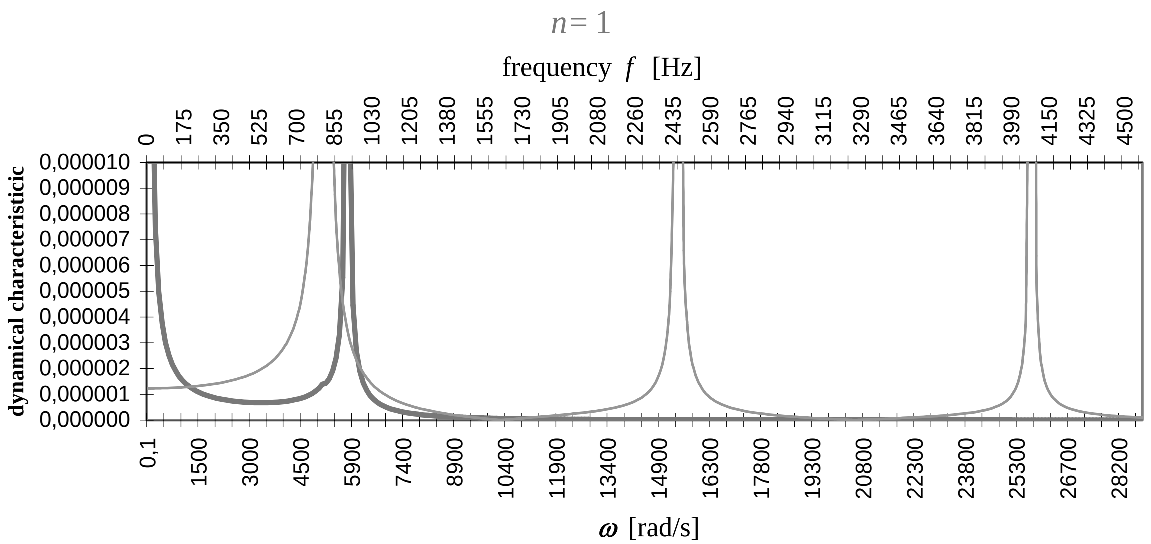

Fig. 2. Transient of dynamical characteristic for the first mode vibration

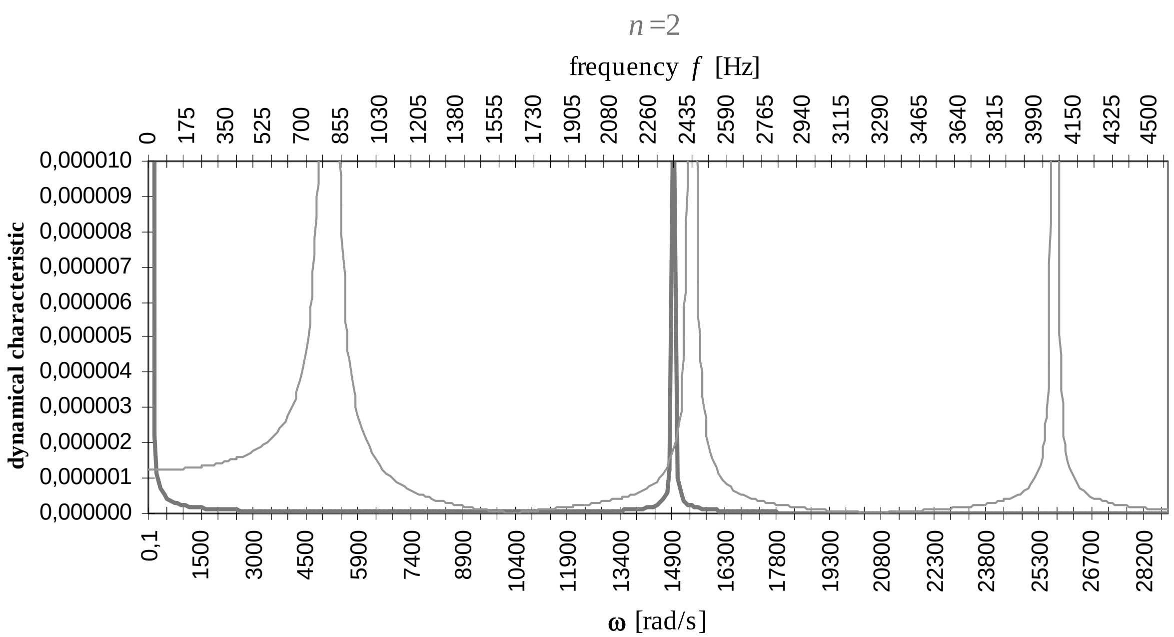

Fig. 3 Transient of dynamical characteristic for the first mode vibration

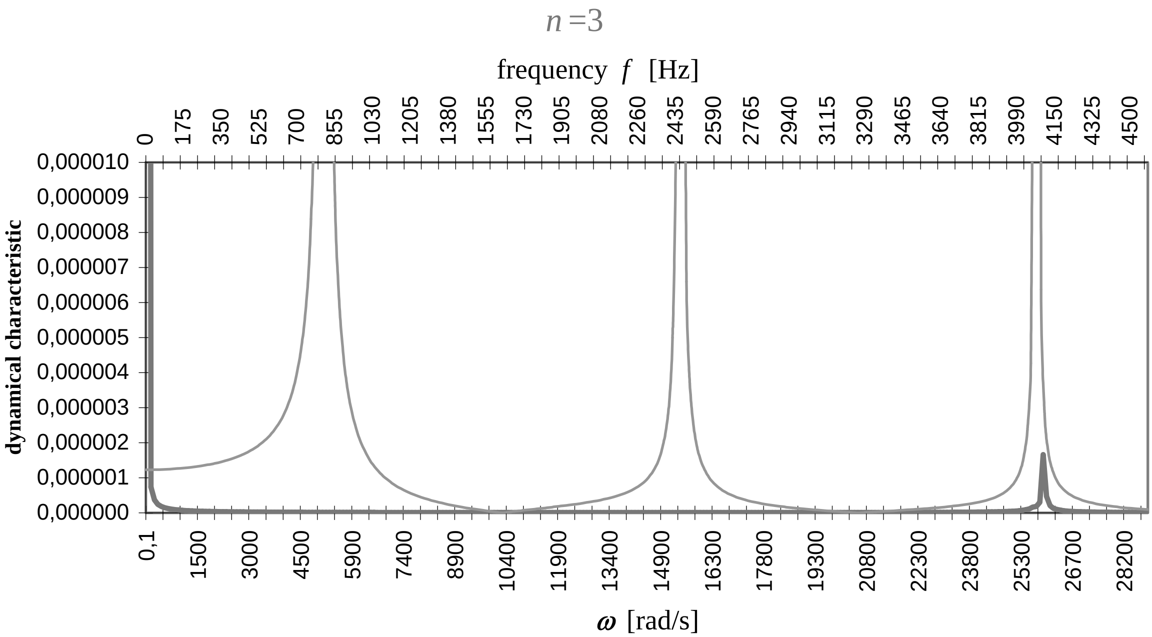

Fig. 4. Transient of dynamical characteristic for the third mode vibration

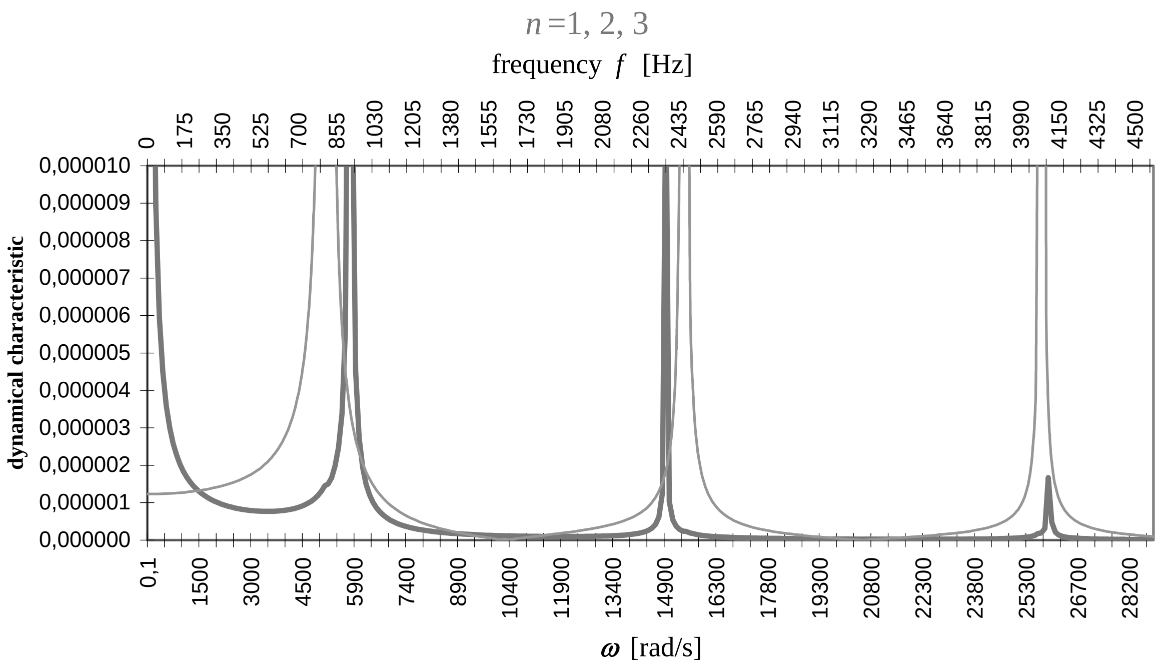

Fig. 5. Transient of the sum for n=1, 2, 3 mode vibration

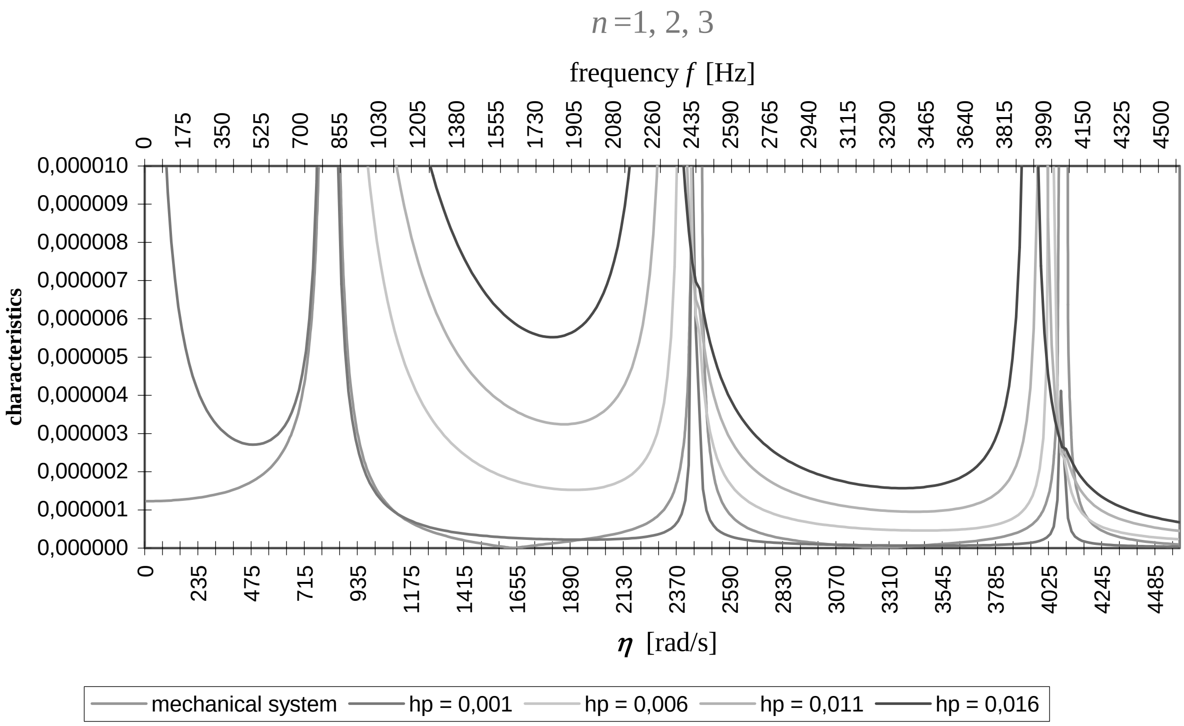

Fig. 6. Influence of thickness of piezoelectric h_p in [m] on characteristics of mechatronic system

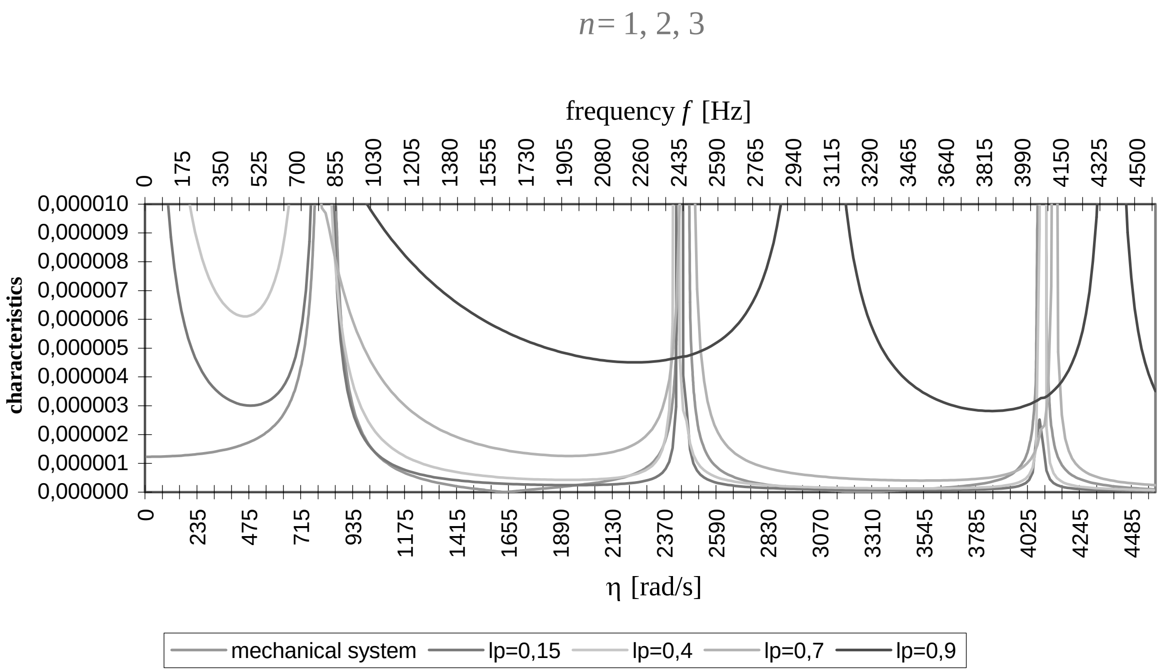

Fig. 7. Influence of length of piezoelectric $l_p = x_2 - x_1$ in [m] on characteristics of mechatronic system

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